Message-Passing Decoders and Their Application to Storage Systems

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Abstract — **Message-passing has been proposed for decoding parity check codes, especially Low Density Parity Check (LDPC) codes. Here we propose using message-passing detectors for partial response channels. Furthermore, we investigate how a single message-passing detector/decoder can be matched to a combination of a partial response channel and a LDPC code.**

I. Introduction

Modern digital storage systems, especially magnetic hard drives, utilize partial response equalization and a Viterbi detector, i.e., a maximum likelihood sequence detector (MLSD). The MLSD is a soft-input, hard-output device. The MLSD produces the most likely input sequence to this channel, given the observation of the noisy output of that channel.

There is a great deal of current interest in using the concatenation of a turbo code or a LDPC code with a partial response channel [1]. The detector/decoder for this combination usually is made up of two separate devices, a detector matched to the partial response channel and a decoder matched to the outer code. Current research has centered on improving performance by using these two devices in an iterative fashion. Efficient iterative detectors/decoders require soft-input, soft-output devices. This makes a MLSD unsuitable because of its hard output. Thus it has been proposed to use a soft-input, soft-output a posteriori probability (APP) detector in place of the MLSD. This APP detector is implemented using the Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm. A maximum a posteriori (MAP) algorithm consists of hard bit decisions made on the soft outputs of the APP detector.

A message-passing algorithm, such as the one proposed for decoding LDPC codes [2], is another type of soft-input, soft-output algorithm. It is known that this algorithm can applied to other types of systems [3] [4].

Here, we propose the use of a message-passing detector in place of the MLSD or APP detector for a partial response channel. We discuss its use for both a PR4 channel and an EPR4 channel. We then discuss the case where a single message-passing detector is matched to both the partial response channel and an LDPC outer code.

II. Message-Passing Detector for Partial Response Channels

We first discuss using a message-passing detector for the PR4 channel. The PR4 channel has binary inputs, $\{x_i\}$, and ternary outputs, $\{y_i\}$, where $y_i = x_i - x_{i-2}$. It is well known that the PR4 channel can be considered as two interleaved dicode channels with input-output equation $y_i = x_i - x_{i-1}$. Thus, in what follows, we will consider a dicode channel instead of a PR4 channel.

Message-passing decoders for LDPC codes are most often described by a Tanner graph with edges connecting two types of nodes, bit nodes and parity check nodes. The message-passing detector for the dicode channel also uses two types of nodes, nodes that represent input bits and nodes that represent output symbols. Edges connect the two specific input bits to a particular output symbol if the partial response polynomial indicates a direct input-output dependence. Messages of bit probability are passed along these edges between nodes. One message-passing iteration involves passing messages from the input nodes to the output nodes and then from the output nodes to the input nodes. Since all of the input nodes compute messages simultaneously, and likewise all the output nodes compute simultaneously, the message-passing schedule is parallel.

By computer simulation, it was found that for the dicode channel with additive white Gaussian noise (AWGN) at the output, the message-passing detector's performance approached that of a MAP decoder as the number of message-passing iterations became large.

We next discuss using message passing for the EPR4 channel. Again, the EPR4 channel has binary inputs, $\{x_i\}$, but has 5-level outputs, $\{y_i\}$, where $y_i = x_i + x_{i-1} - x_{i-2} - x_{i-3}$. Two types of message-passing decoders are discussed. The first is very similar to the detector previously discussed for dicode having two types of nodes, nodes that represent the input bits and nodes that represent the output symbols. Messages of bit probabilities are passed between the four specific input bits that directly depend upon a particular output symbol and vice versa. In computer simulations, the performance for this detector was found to be much poorer than the APP detector because of the many short cycles.

In a second type of message-passing detector for the EPR4 channel, state nodes are introduced that represent the state of the EPR4 channel at each time, i. This channel has 8 states, the state at time i being represented by the 3 previous channel inputs, $(x_{i-1}, x_{i-2}, x_{i-3})$. The states at time i and time $i+1$ are connected to the node representing the output at time i and messages of state probabilities are passed back and forth along these connections.

Simulations were run for the EPR4 channel with AWGN at the output. The performance of this system approaches that of an APP detector as the number of message-passing iterations gets large. For a small number of message-passing iterations, precoding improved the performance of the message-passing detector. The results of computer simulations showing this effect are shown in Figures 1 and 2. Here, the probability of bit error is plotted versus the signal-to-noise ratio (E_b/N_0) for various values of T, the number of message-passing iterations. The precoder used in Figure 2 was the inverse of the channel modulo 2. Also shown in Figure 1 are bounds that predict the performance in the non-precoded case for high signal-to-noise ratio and small T. Truncated union bounds are shown in the simulation results.

III. A Combined Message-Passing Detector/Decoder

We now consider the case where a dicode channel is used as an inner code and a rate 8/9, block length 495, LDPC code is used as the outer code. The output of the dicode channel is corrupted by AWGN.

A single message-passing detector/decoder was used, matched to both the partial response channel and the LDPC code. Now three types of nodes are used: nodes which represent the inputs to the dicode channel, nodes which represent the outputs of the dicode channel and nodes which represent the parity checks of the LDPC code that must be satisfied by the inputs to the dicode channel.

Although a parallel message-passing algorithm could be used that simultaneously detects and decodes, we chose instead to implement a serial algorithm. In this algorithm, T message-passing iterations first are used to detect the dicode channel, then S message-passing steps are used to decode the LDPC code and finally the process is repeated a total of U times.

Simulations were run to find the best values of the parameters T, S, and U when the total number of message-passing iterations, $(T+S)U$ was fixed at 24 or 25. The results are shown in Figure 3 where it is seen that $T=S=1$ and $U=12$ gave the lowest probability of bit error for the choices given. Similar results were found for a LDPC outer code used with an EPR4 inner code but now $T=2$, $S=1$ and $U=8$ was found to be best.

References

[1] J. L. Fan, A. Friedmann, E. Kurtas, and S. W. McLaughlin, "Low density parity check codes for magnetic recording," in *Proceedings 37th Annual Allerton Conference on Communication, Control and Computing*, pp.1314–1323, 1999.

[2] R. G. Gallager, "Low-density parity-check codes," Research Monograph 21, The M.I.T. Press, Cambridge, MA, USA, 1963.

[3] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Transactions on Information Theory*, vol. 47, pp. 498–519, February 2001.

[4] N. Wiberg, *Codes and Decoding on General Graphs.* PhD thesis Linköping University, S-581 83 Linköping, Sweden, 1996.

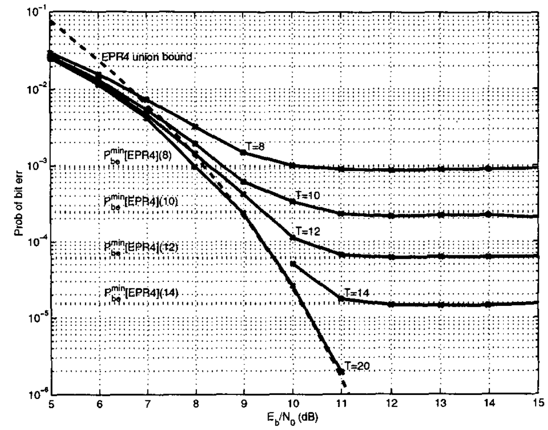

Figure 1: Bit error rate for state-based message passing on non-precoded EPR4, for various iterations.

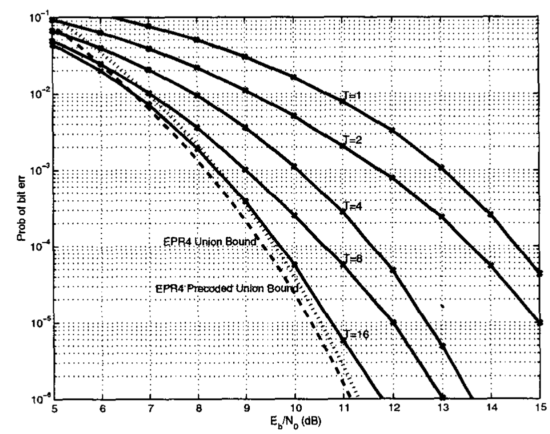

Figure 2: Bit error rate for state-based message passing on EPR4 precoded with $1 \oplus D \oplus D^2 \oplus D^3$, for various iterations.

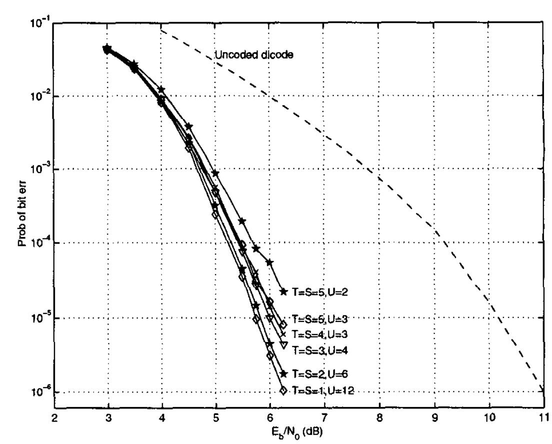

Figure 3: Simulation results for joint LDPC/dicode message-passing decoder (using bit-message passing for the dicode channel).