Rewriting Flash Memories and Dirty-Paper Coding

Brian M. Kurkoski

Japan Advanced Institute of Science and Technology

JAPAN ADVANCED INSTITUTE OF SCIENCE AND TECHNOLOGY 1990

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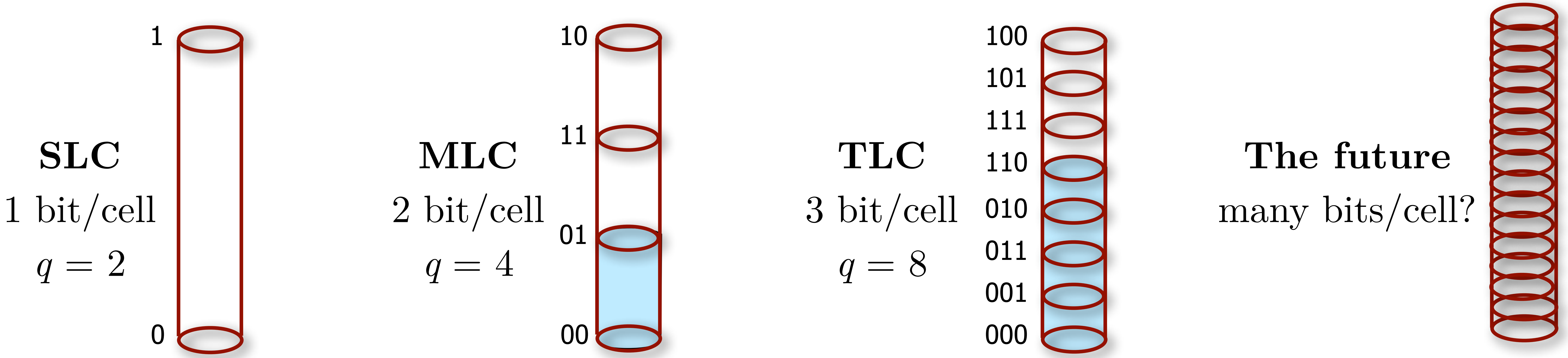
Flash Memories

Store charge on transistors called "cells." Increasing data storage density:

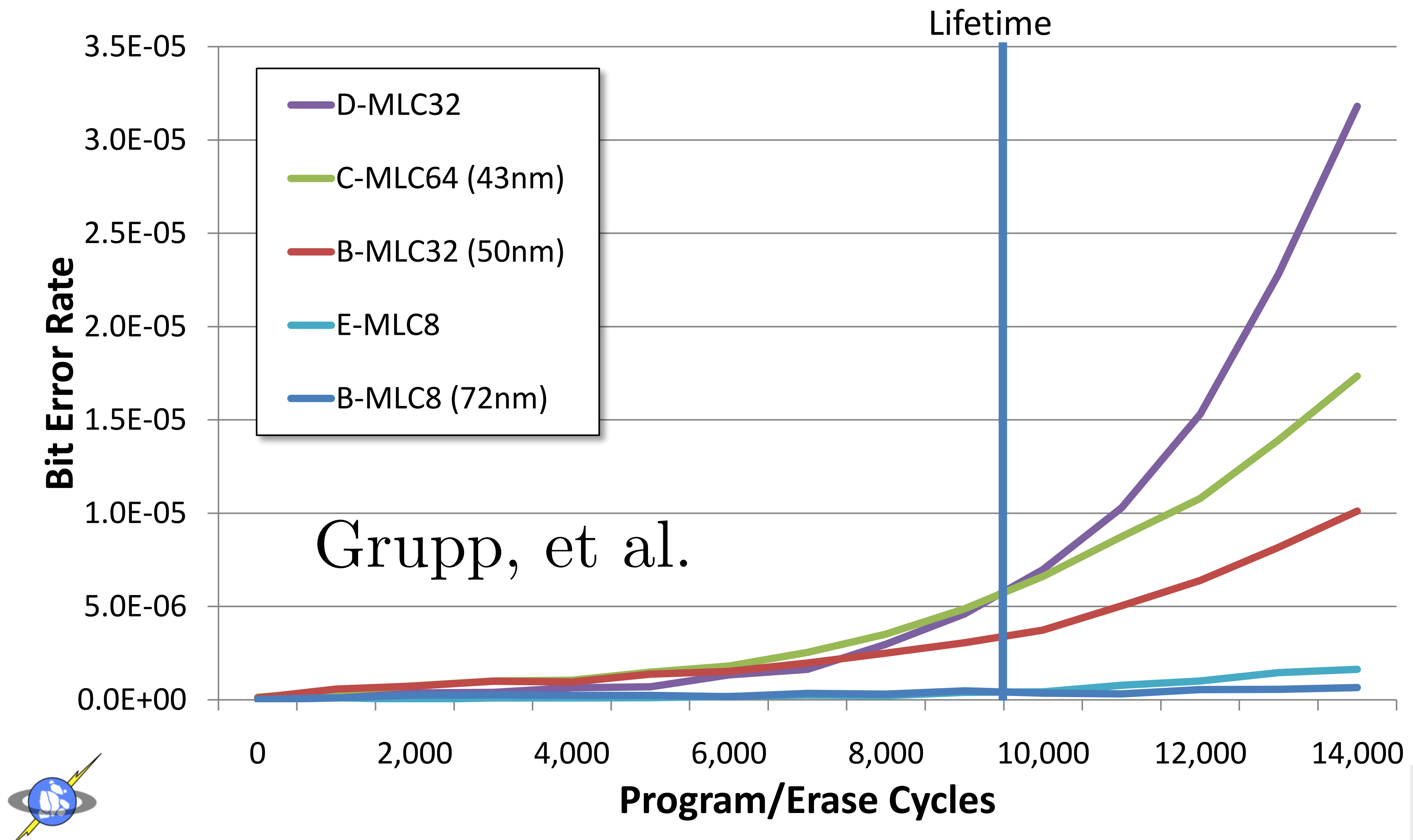


Flash Memories Wear Out

To re-write a memory, must first erase

With each cycle, the error rate increases

- Rewriting: Write-Once Memory (WOM) codes



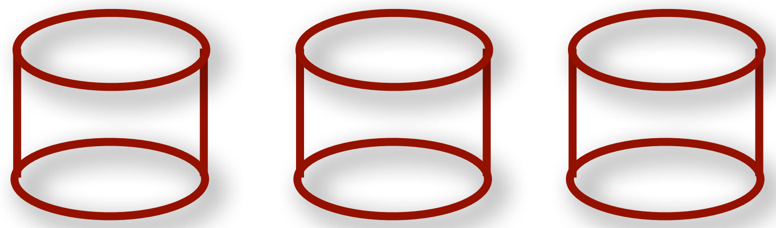
Codes for Write Once Memories

Binary, write once-memory: $0 \to 1$ allowed; $1 \to 0$ is illegal

How to write two times? Time sharing Rate $= 1/2$

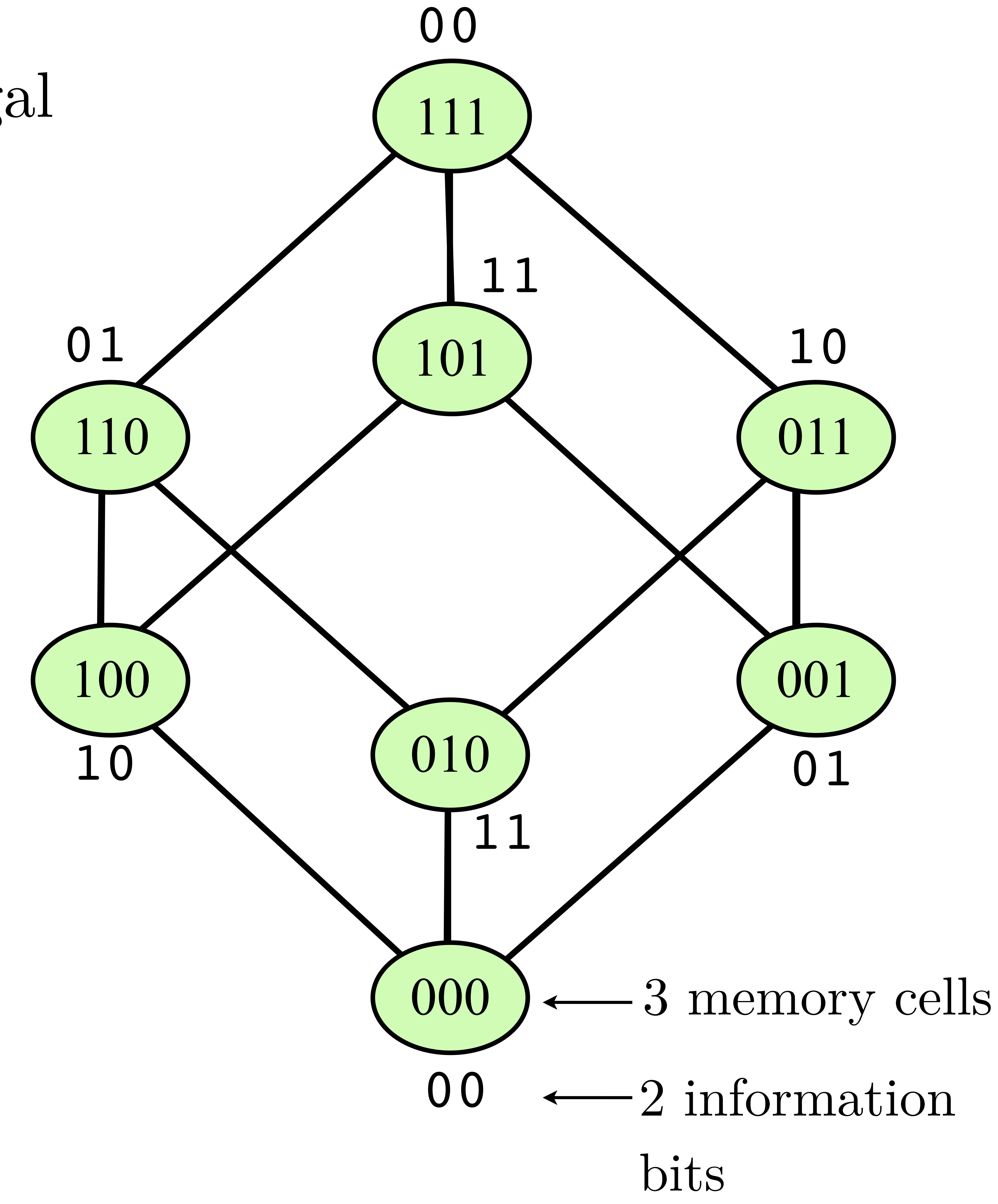
Toy example for 2 writes:

- 3 storage "cells", 2 bits of information
- Example: store 10, then store 00



Initial State:	**0**	**0**	**0**
Write 10:			
Write 00:			

Rate $2/3 > 1/2$



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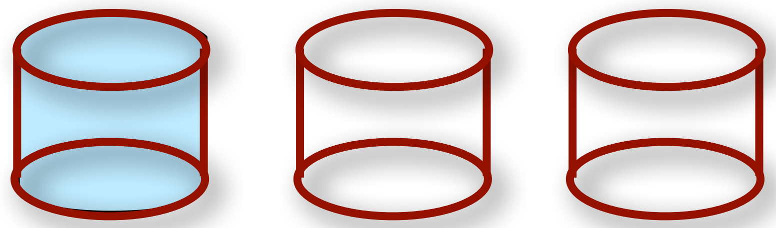

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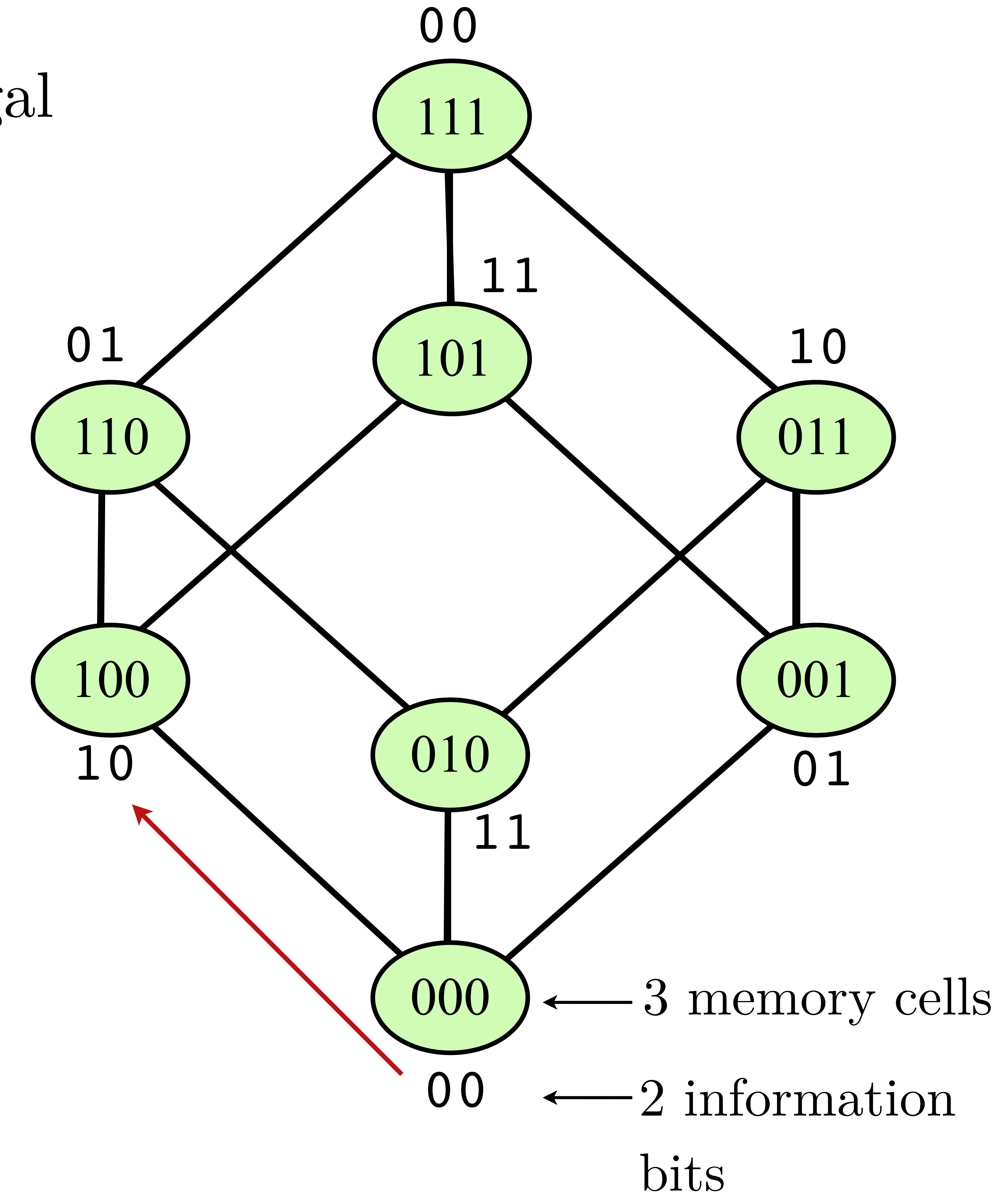

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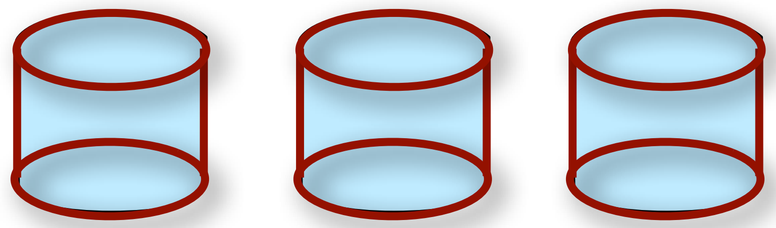

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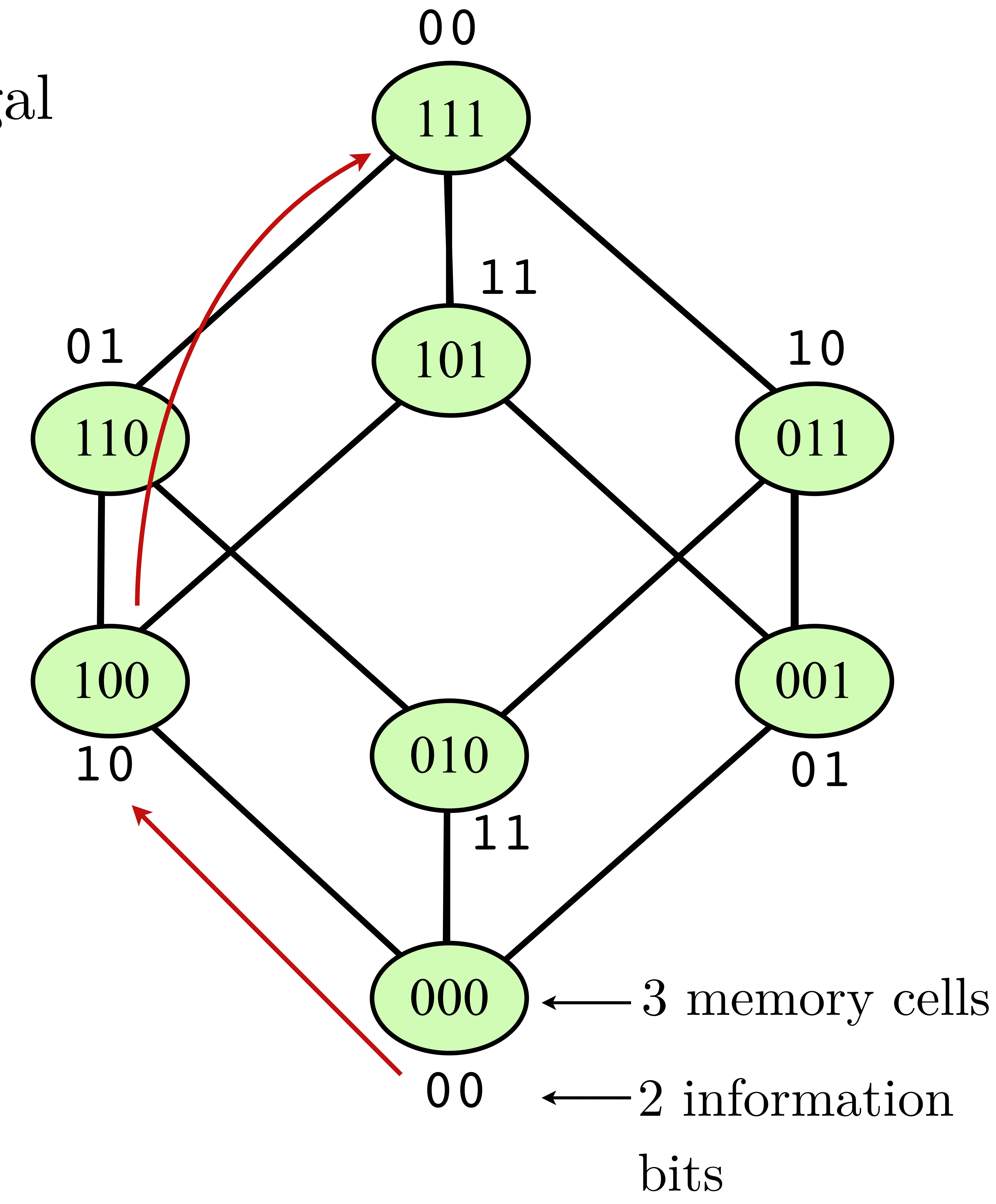


Lattice Strategies for WOM Codes

Lattices are codes over real numbers

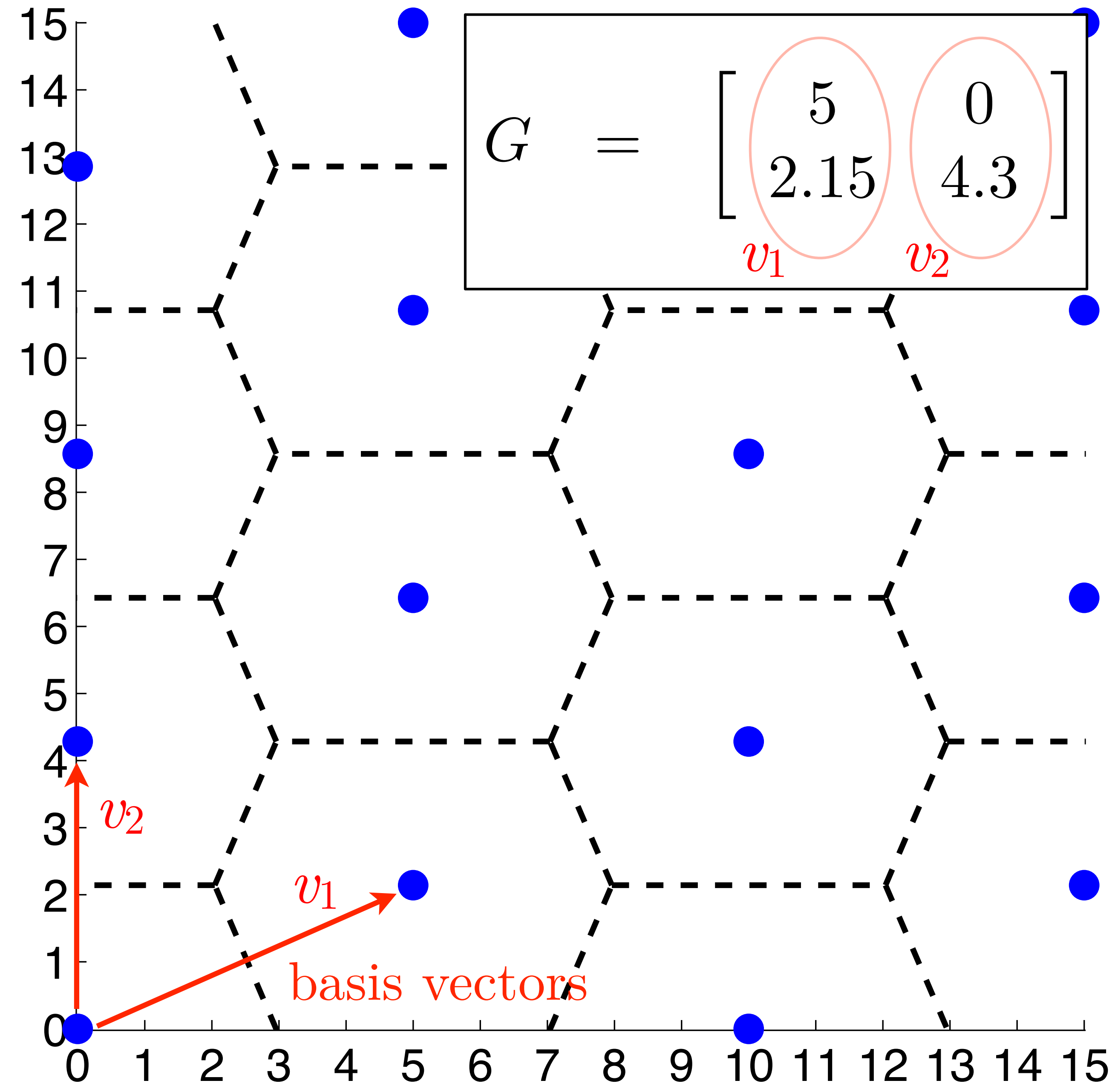
Lattices have inherent error-correcting capability

Numerous WOM code designs

- Many do not have error-correction capability

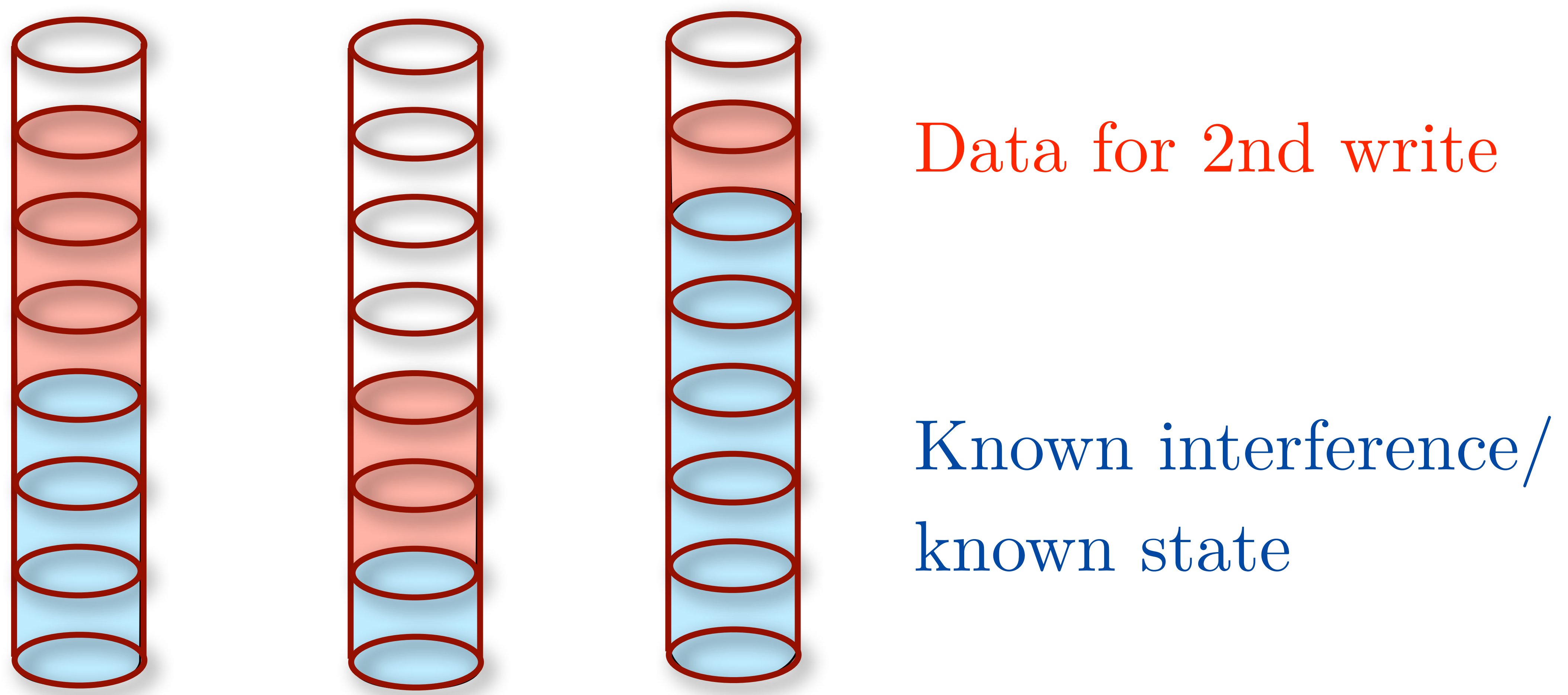
Lattice-based WOM code also corrects errors

- Hyperbolic lattice WOM Codes — Best rate, hard to encode
- **Cubic Lattice WOM — Easy to encode**



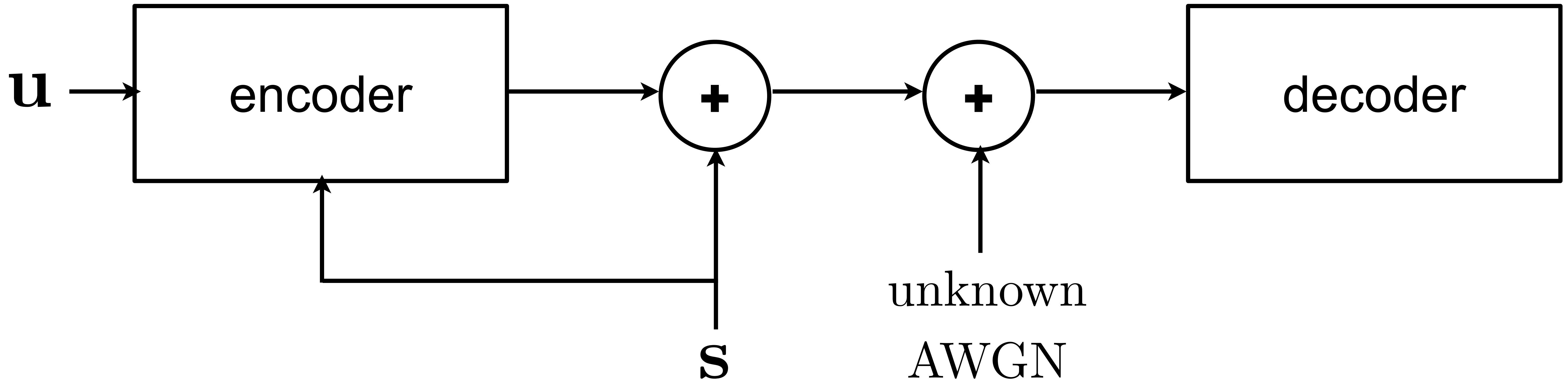
In This Talk...

1. Connection between dirty-paper coding and WOM codes
   - "known interference" = "current state of WOM"
   - connect with lattice strategies of Erez et al



2. Propose WOM code inspired by dirty paper coding
   - Specific code construction
   - using coset select bits to increase the average rate

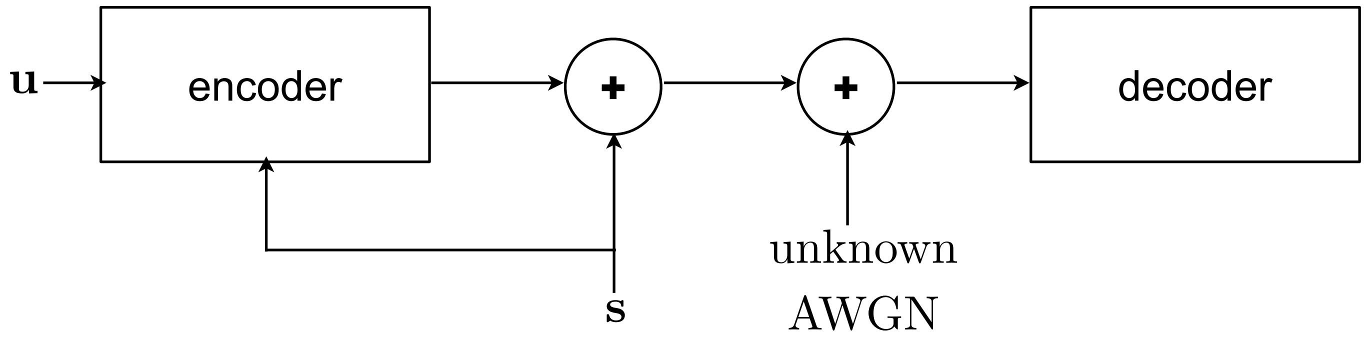
Dirty Paper Coding for AWGN Channel

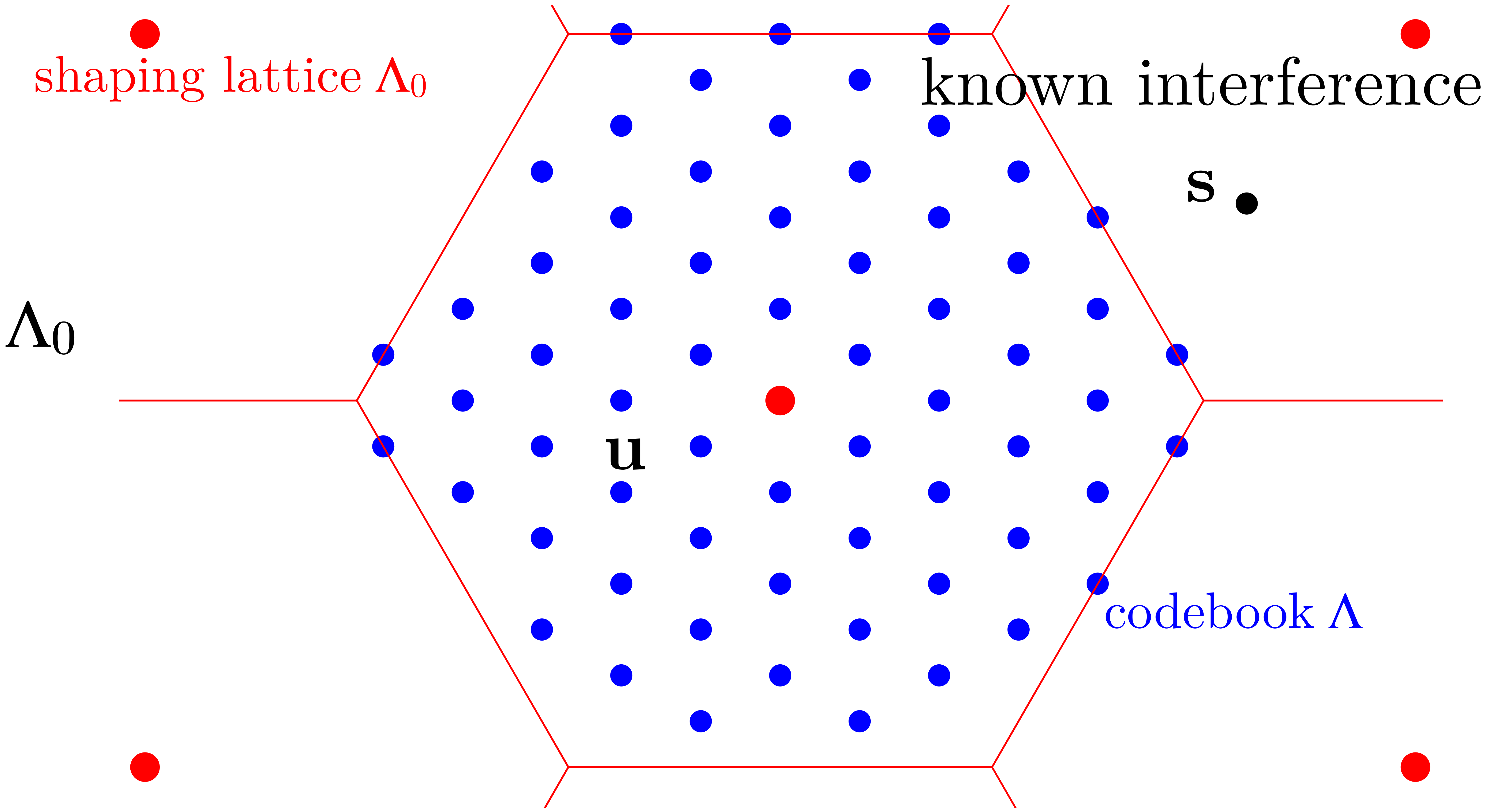


"Known interference **s** does not reduce capacity"

- Cannot pre-subtract **s**. Violates power constraint
- [Gelfand Pinsker, 1980]
- [Costa, 1983]

Dirty Paper Coding Using Lattices [Erez et al]





Lattice code Λ with power constraint Λ_0

Information $\mathbf{u} \in \Lambda$, interference $\mathbf{s}$

Transmit $\mathbf{u} - \mathbf{s} \bmod \Lambda_0$

$\mathbf{y} = \mathbf{u} - \mathbf{s} \bmod \Lambda_0 + \mathbf{s}$ is received,

Decoder computes $\mathbf{y} \bmod \Lambda_0$

Simple explanation captures main idea:

- Ignored lattice inflation and random dither.

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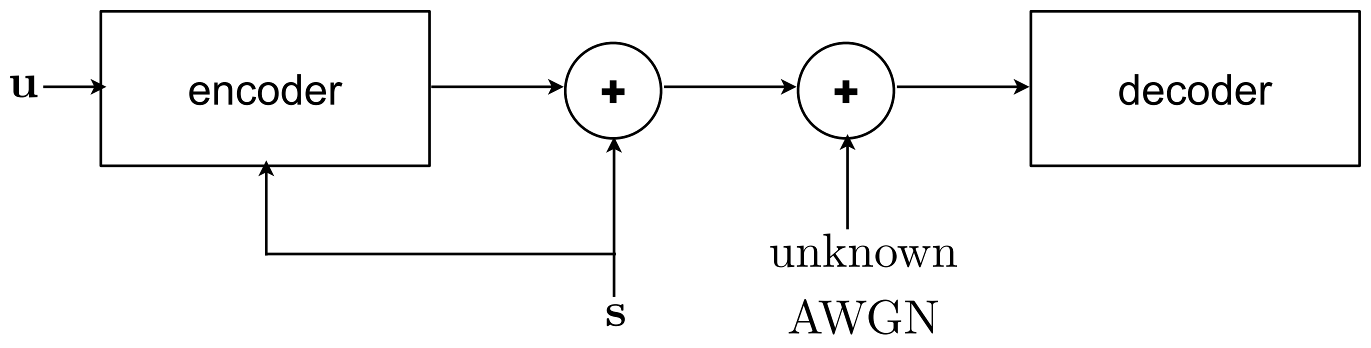

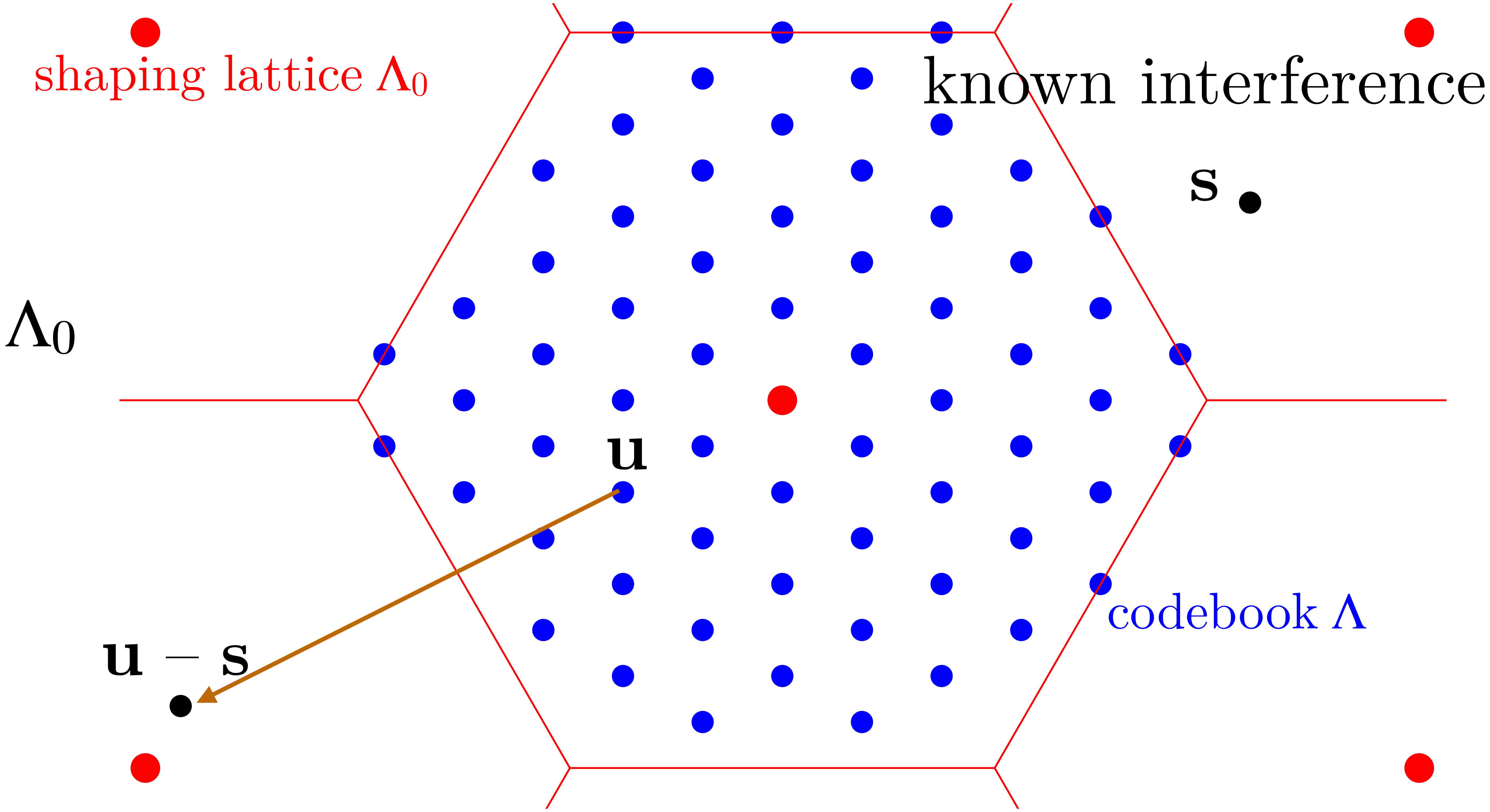

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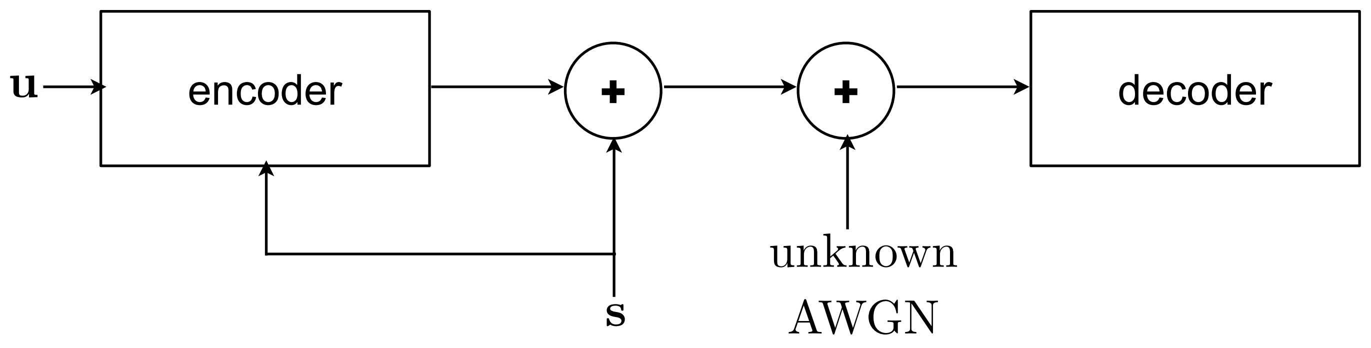

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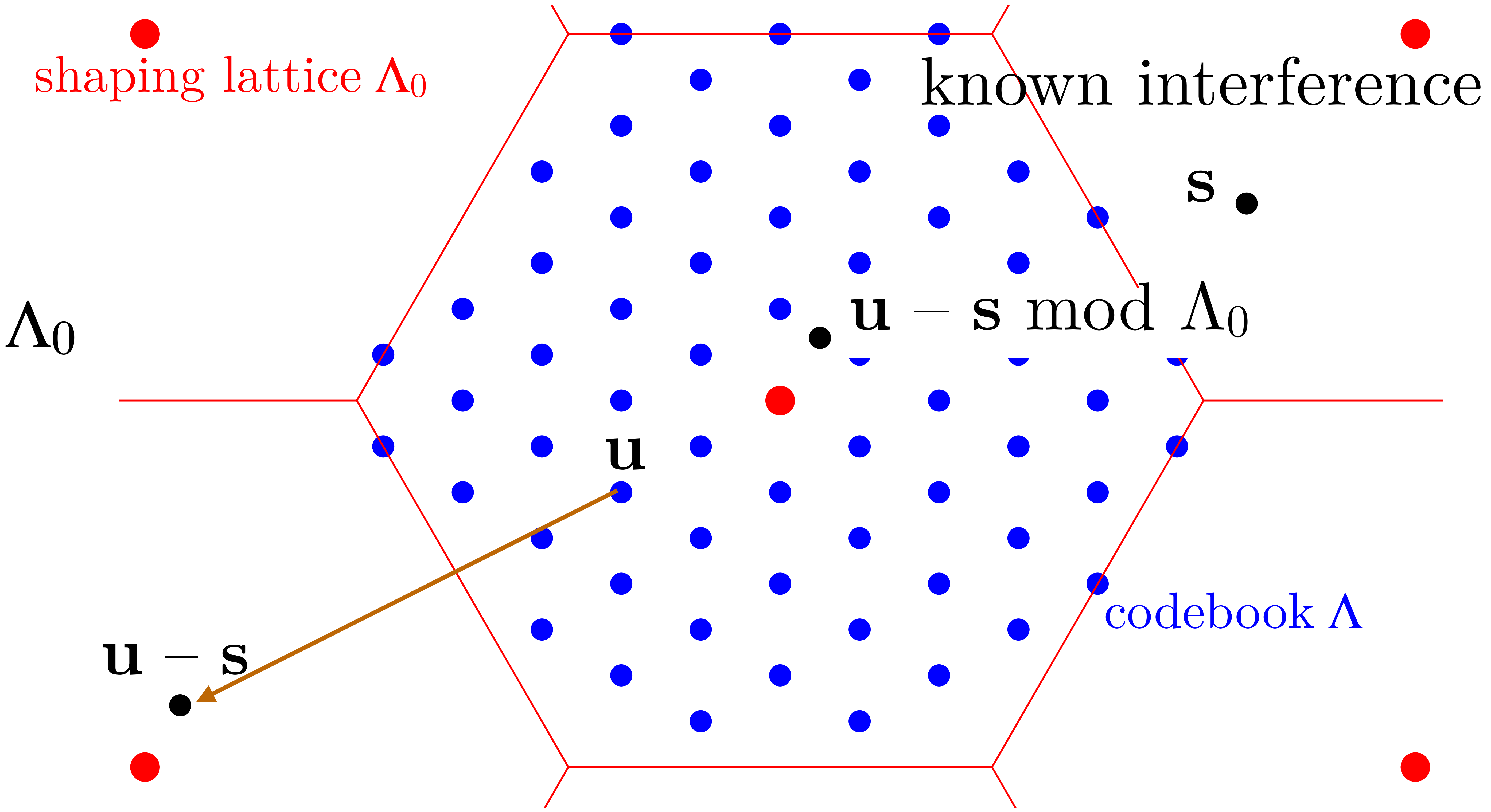


Dirty Paper Code for Rewriting Flash

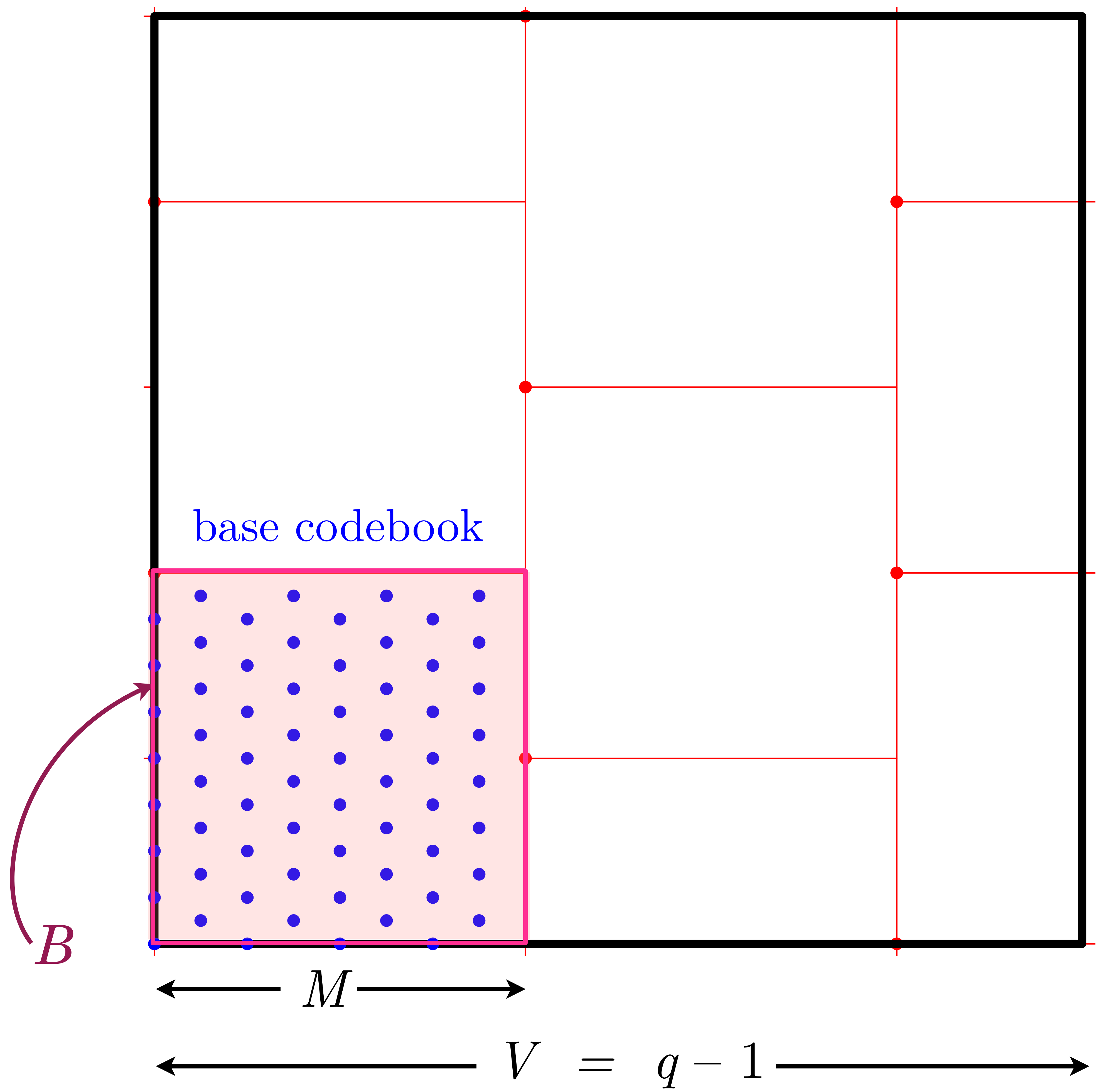
Flash memories:

- Power restriction is $[0, V]$
- Rewrite memory
  - Values can only increase

1st write: "base codebook"

- Shaping region B

2nd, 3rd,... : apply dirty-paper coding



Dirty Paper WOM

User data is $\mathbf{u}$

Current state of memory $\mathbf{s}$ "known interference"

Pre-subtract interference. Transmitted codeword:

$$\mathbf{u} - \mathbf{s} \bmod B$$

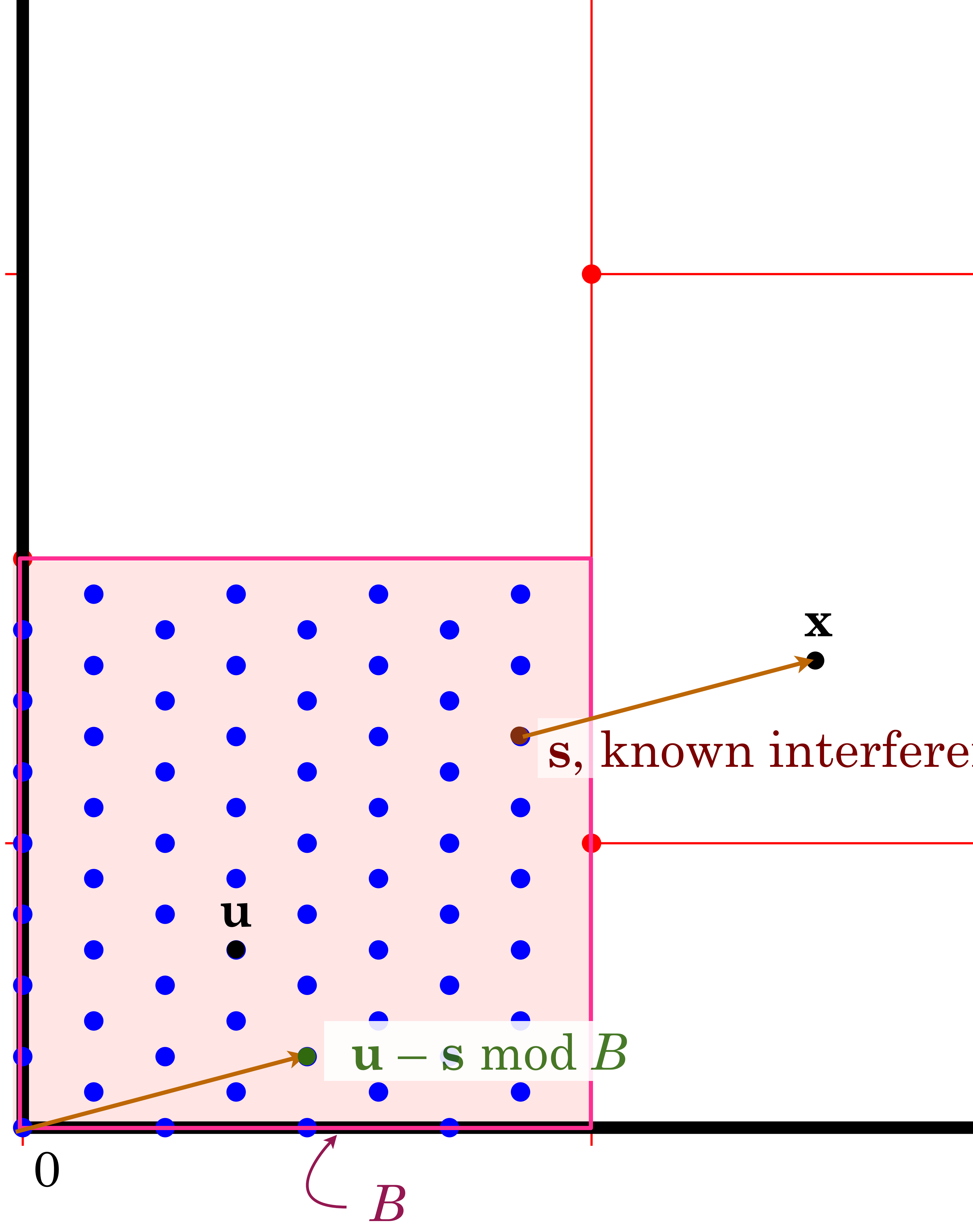
which is always positive.

Add this to $\mathbf{s}$ (encoder explicitly adds)

$$\text{"transmit" } \mathbf{x} = \mathbf{s} + (\mathbf{u} - \mathbf{s} \bmod B)$$

Decoding in absence of noise:

$$\mathbf{u} = \mathbf{x} \bmod B$$



Dirty Paper WOM: Comment

Warning

"Interference does not reduce capacity"

does not apply to WOM,

because the power constraint is different.

Absolute constraints on:

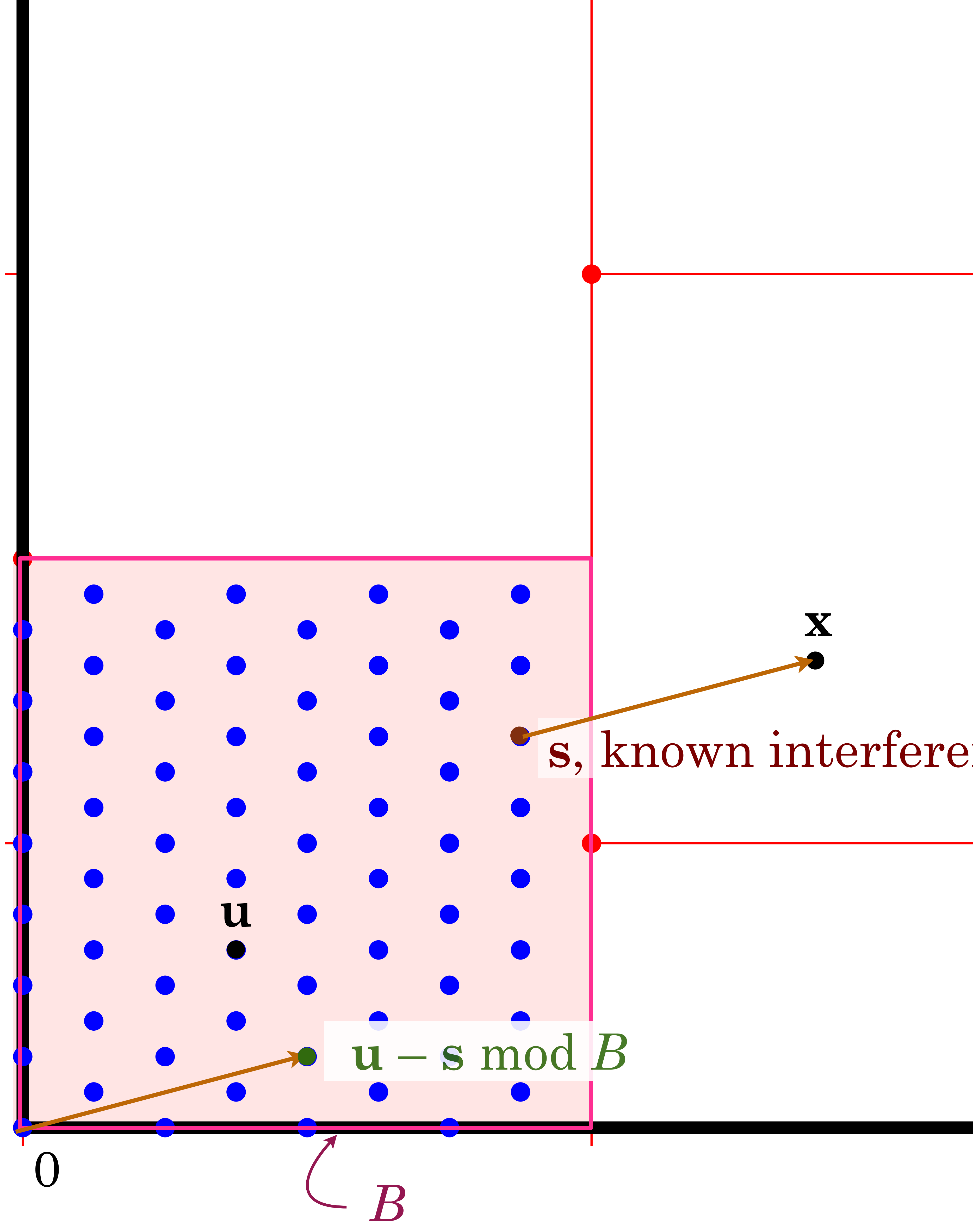
$\mathbf{s}$ and $(\mathbf{u} - \mathbf{s} \bmod B)$

due to the [0,V] restriction of flash cells.

Benefit

Transmitted codeword is positive. Apply to:

- optical communications, power-detection wireless, etc.



Code with coset select bits

Evaluate the number of writes

- Bad result!
- The code is not "shaped"

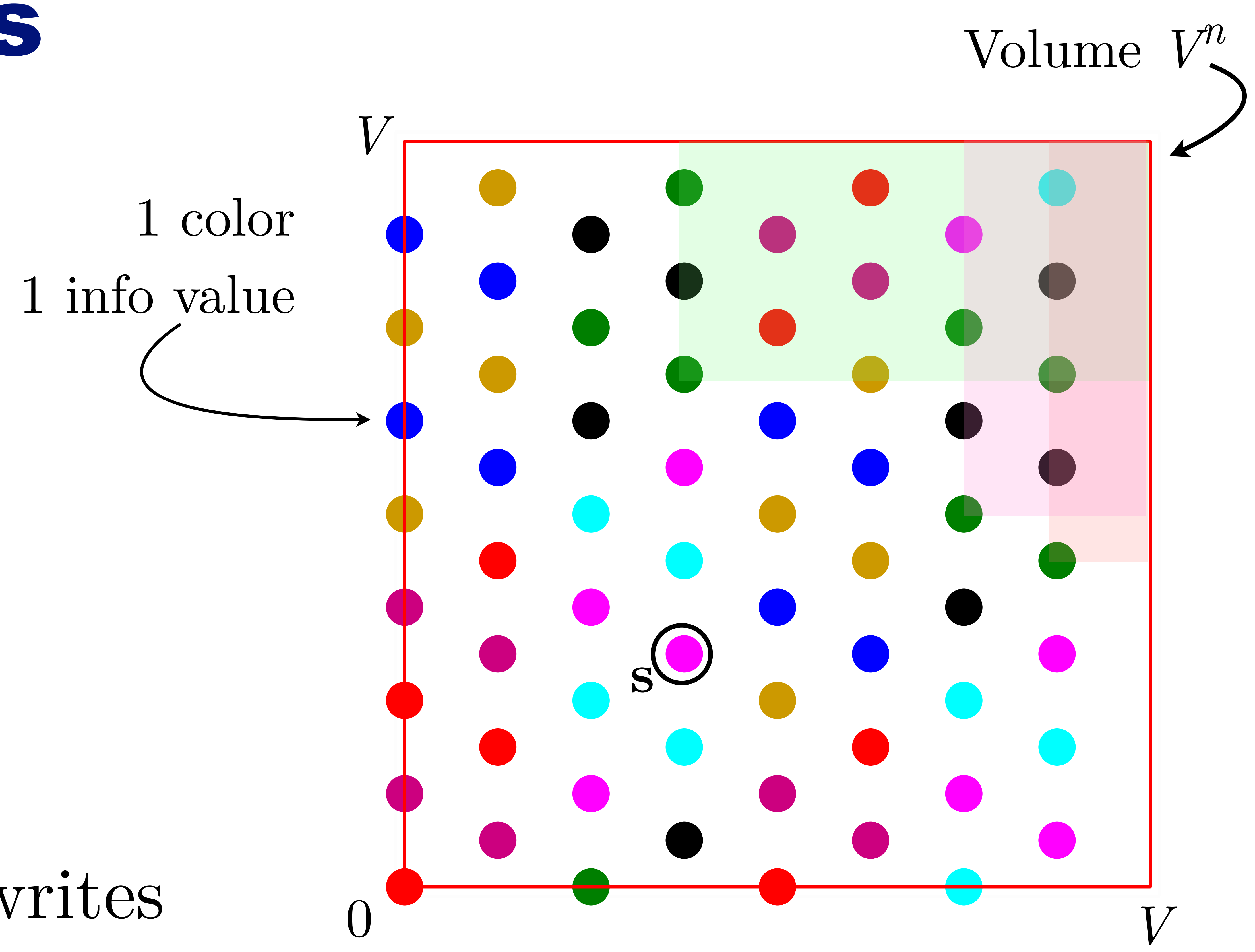
Solution: Break the base code into cosets

- Replace information bits with "coset select" bits.
- Reduces information rate/increases # of writes

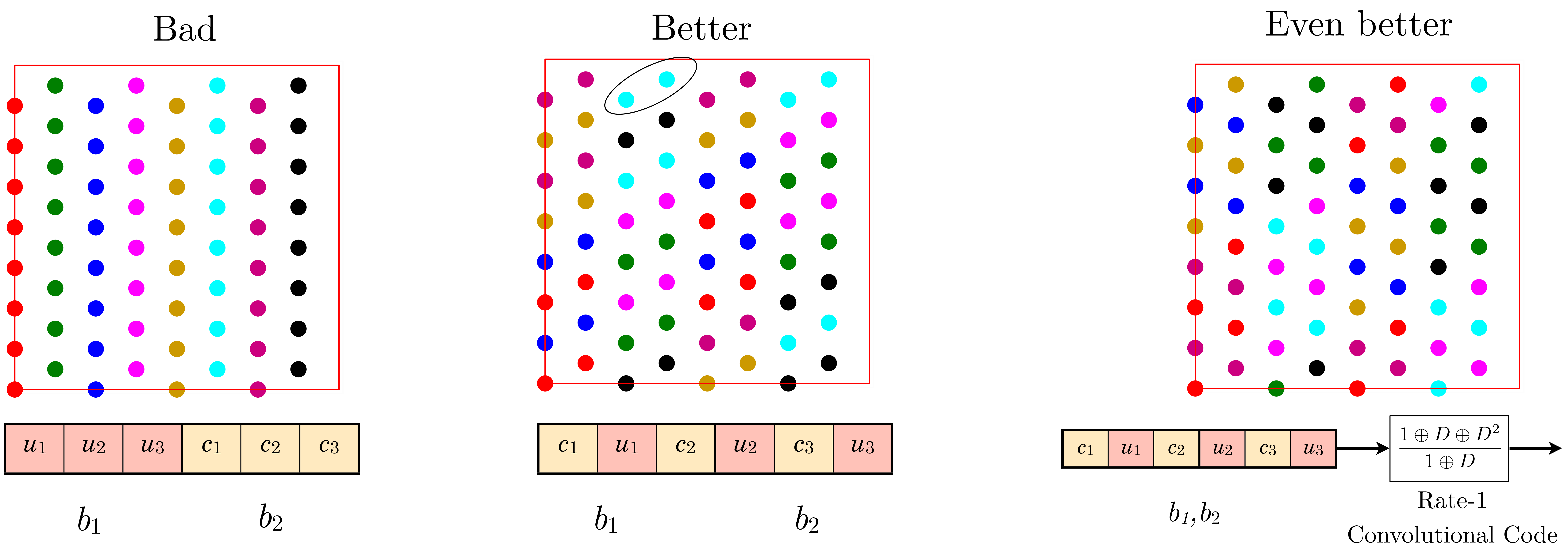
Choose the coset which maximizes future writes

$$\mathbf{x} = \arg\max_{\mathbf{c}} \left|\left| V - \mathbf{x}(\mathbf{c}) \right|\right|$$

- Volume is approximation of number of remaining points



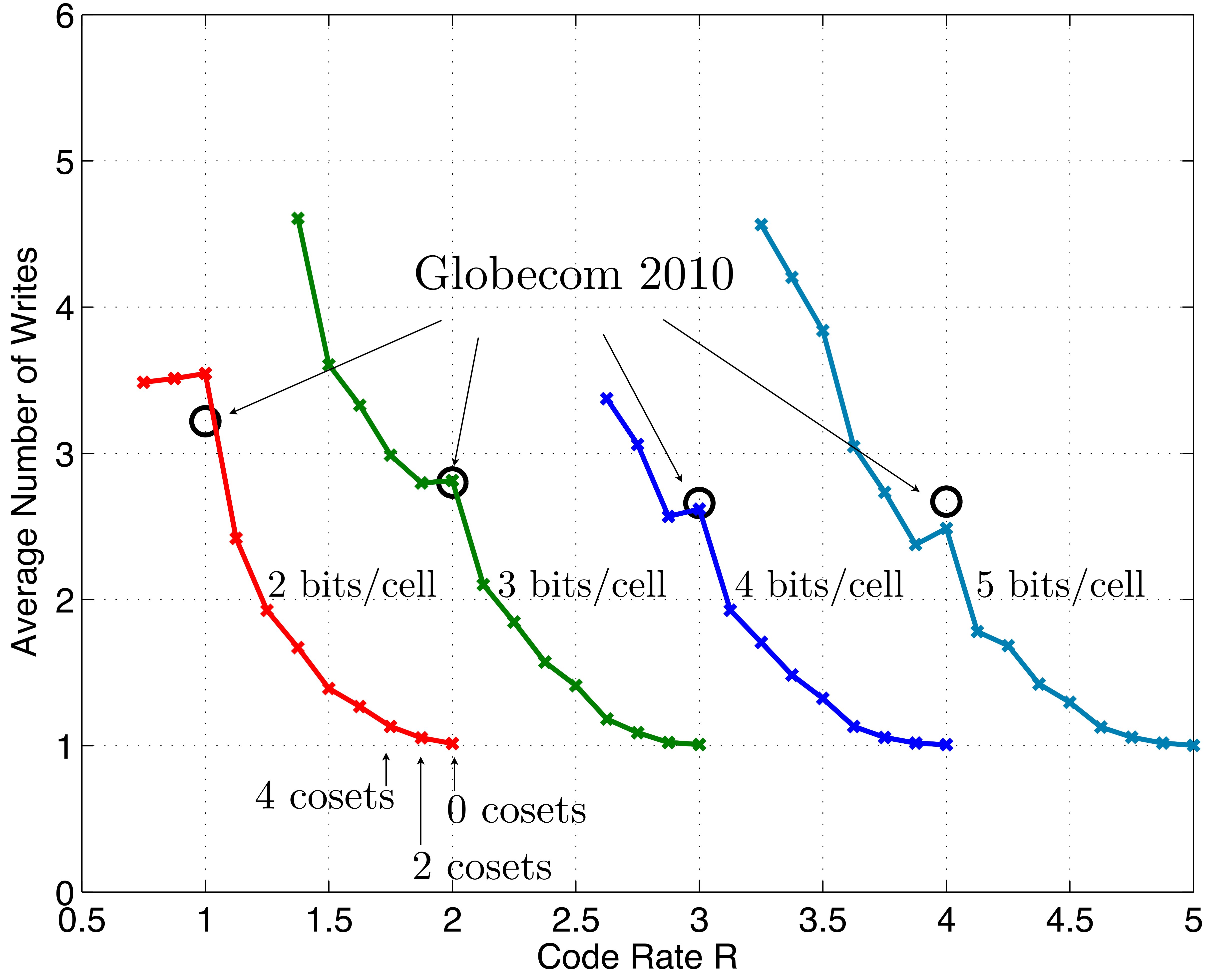
Mapping from information to lattice points



u_1	information bit
c_1	coset select bit

- Mapping should be invertible
  - Rate-1 convolutional codes improved average number of writes
  - Recursive codes further improved

Base Code Only — Average Number of Writes



E8 lattice best-known lattice in 8 dim.

Evaluate:

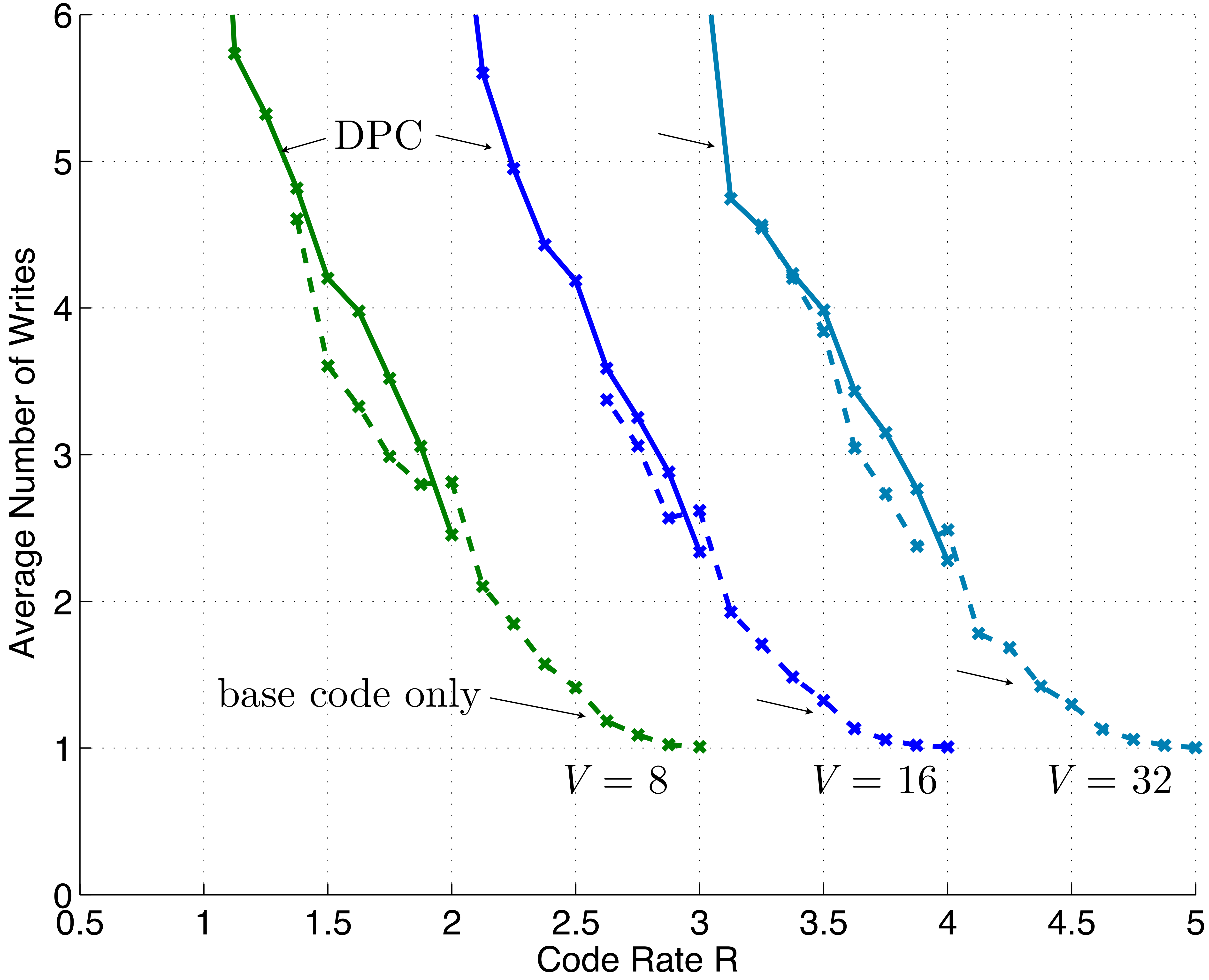
- 2, 3, 4 and 5 bits/cell
- 0,2,4,8,16,... cosets

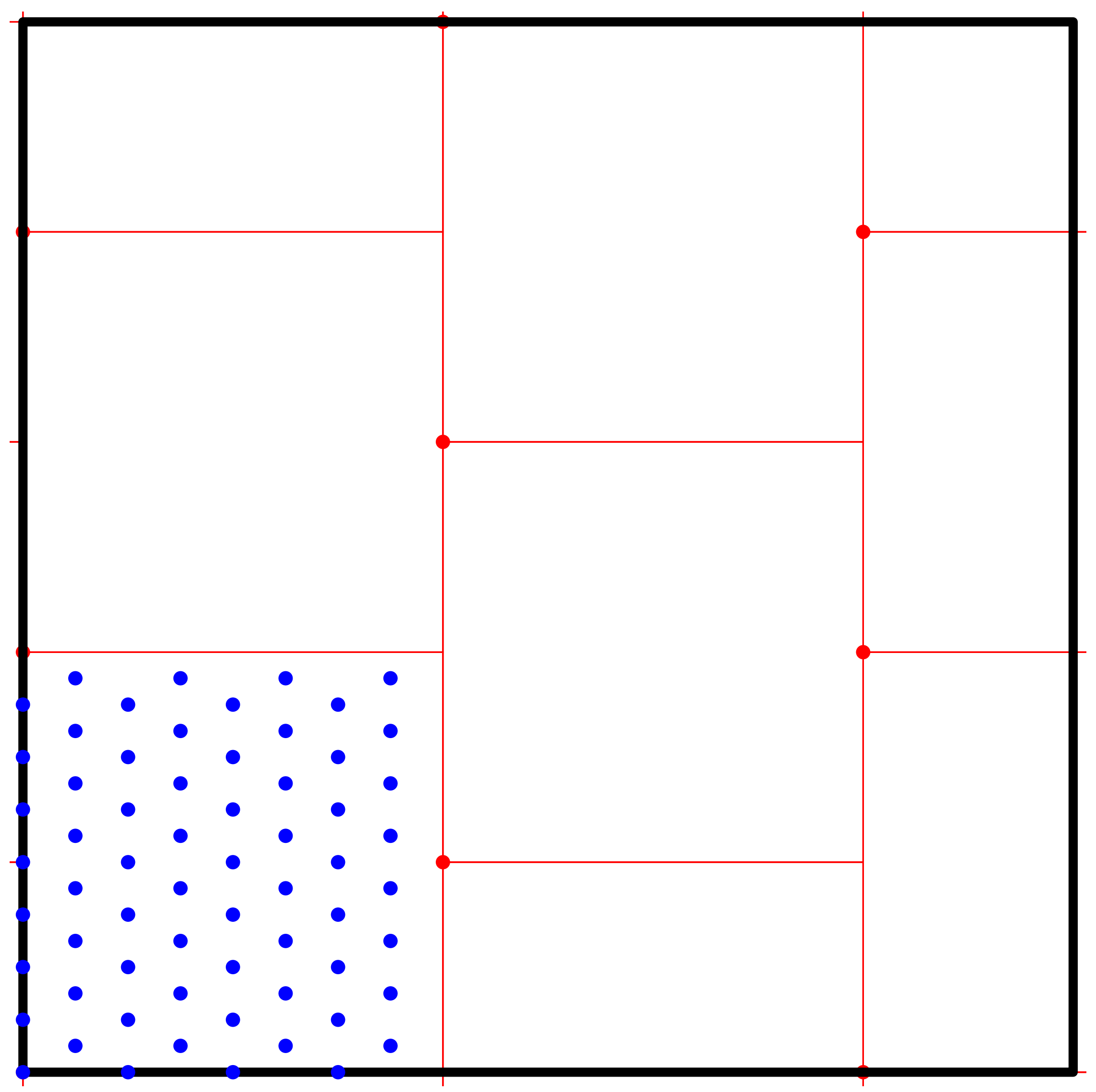
Increasing # cosets, increases average number of writes

Compared to Globecom 2010 code:

- Is linear
- Has roughly equal performance
- Adaptable code rates

Dirty Paper WOM Code



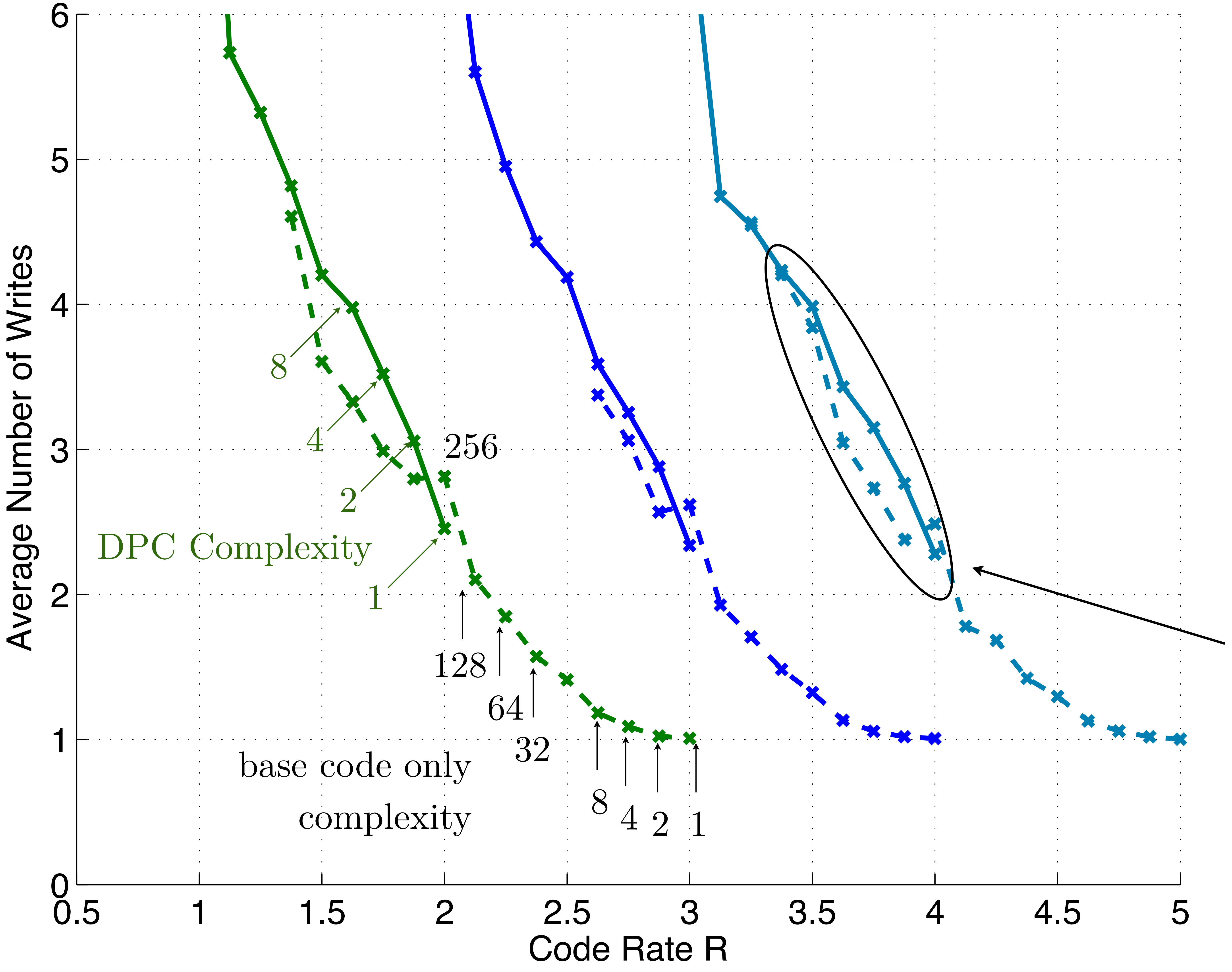


“Dirty Paper” inspired system

- Base code-only has $V = M$
- DPC has $V = 2M$

Two systems have similar average number of writes

Complexity Comparison



Choose coset to maximize:

$$\mathbf{x} = \arg\max_{\mathbf{c}} \left\| V - \mathbf{x}(\mathbf{c}) \right\|$$

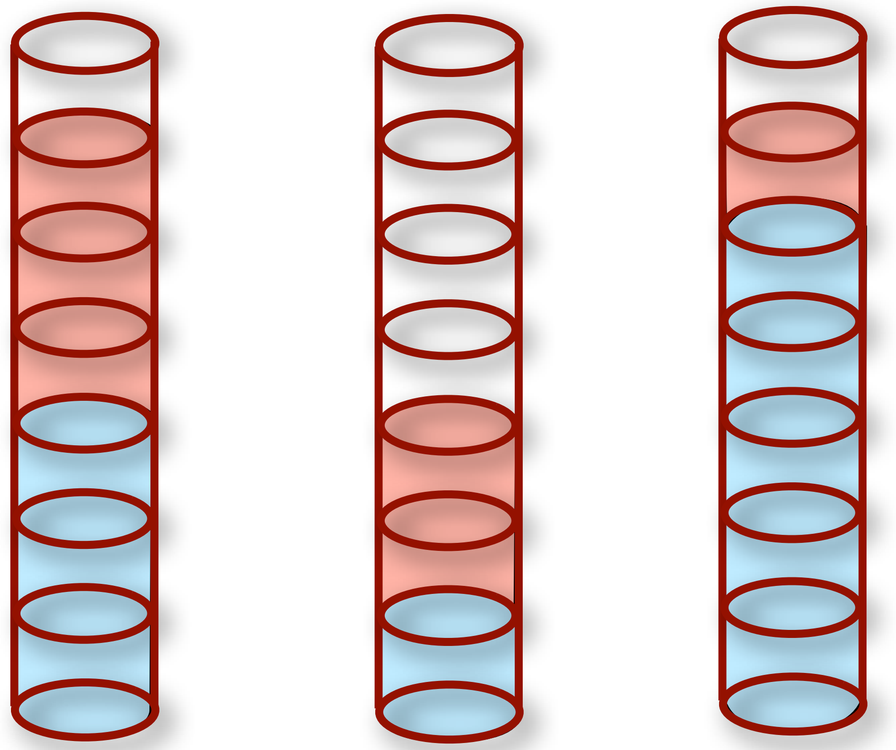
Searching over all cosets is source of complexity.

Complexity $\sim 2^C$,

$C =$ number of coset bits

Cannot achieve highest rates, but similar average number of writes, but the DPC system has much lower complexity

Discussion



1. Showed a connection between dirty paper coding and WOM codes

➢ "known interference does not reduce capacity" does not apply

But, lattice strategies do apply.

Interference and positive valued codewords: WOM, optical, power detection

2. Lattice scheme based on dirty paper WOM:

➢ Added "coset select bits" to improve the average number of writes

➢ Main problem was mapping information to lattice points

Dirty paper WOM has lower complexity than base code.