Termination
Automated Reasoning and its Tools

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Ogawa Laboratory
School of Information Science
Japan Advanced Institute of Science and Technology

March 5th, 2012

JAIST Spring School 2012

Certified Termination

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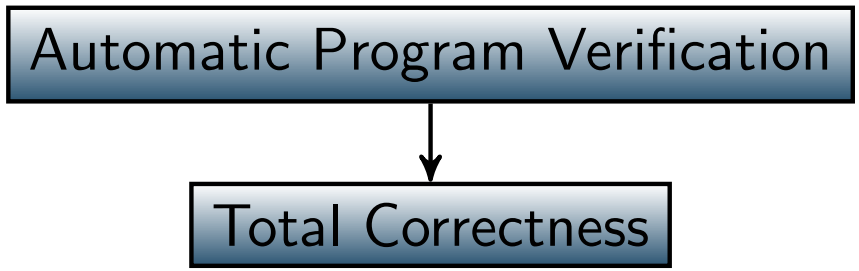
Overview

- Motivation
- Isabelle/HOL
- Certification
- Termination of HOL Functions

Motivation

Automatic Program Verification

Motivation



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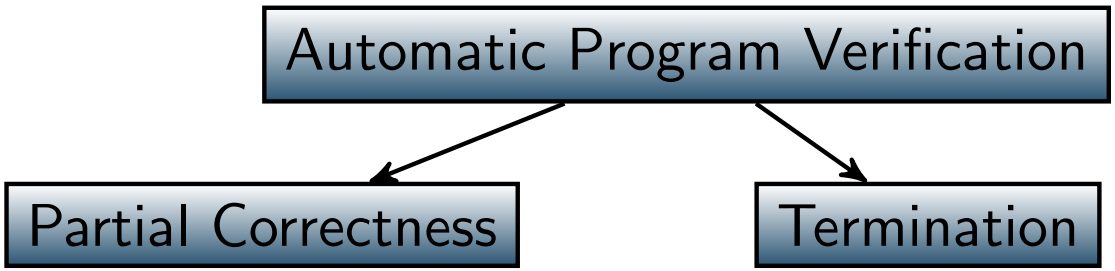
Automatic Program Verification
Partial Correctness
Termination

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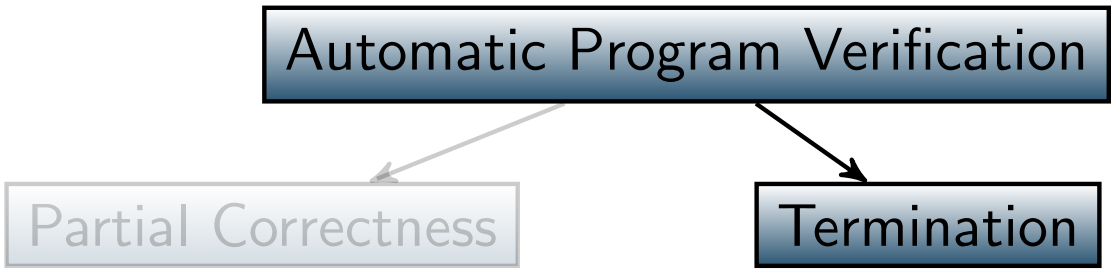
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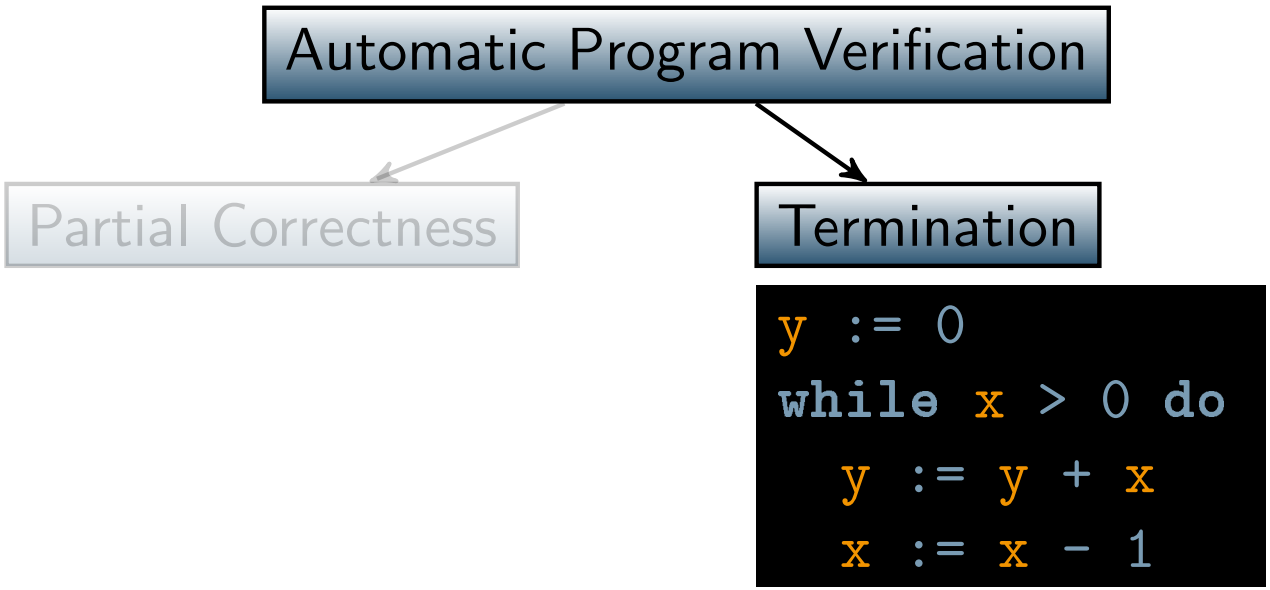
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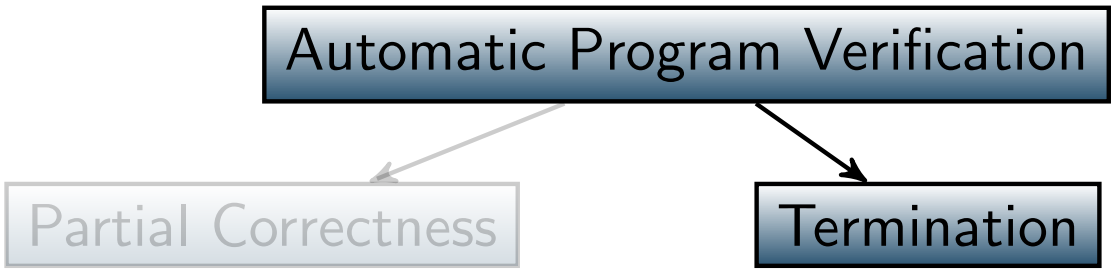
y := 0
while x > 0 do
y := y + x
x := x - 1

Motivation




```
int sum(int x, int y) {
  if (x > 0)
    return sum(x-1, y+x);
  else
    return y;
}
```

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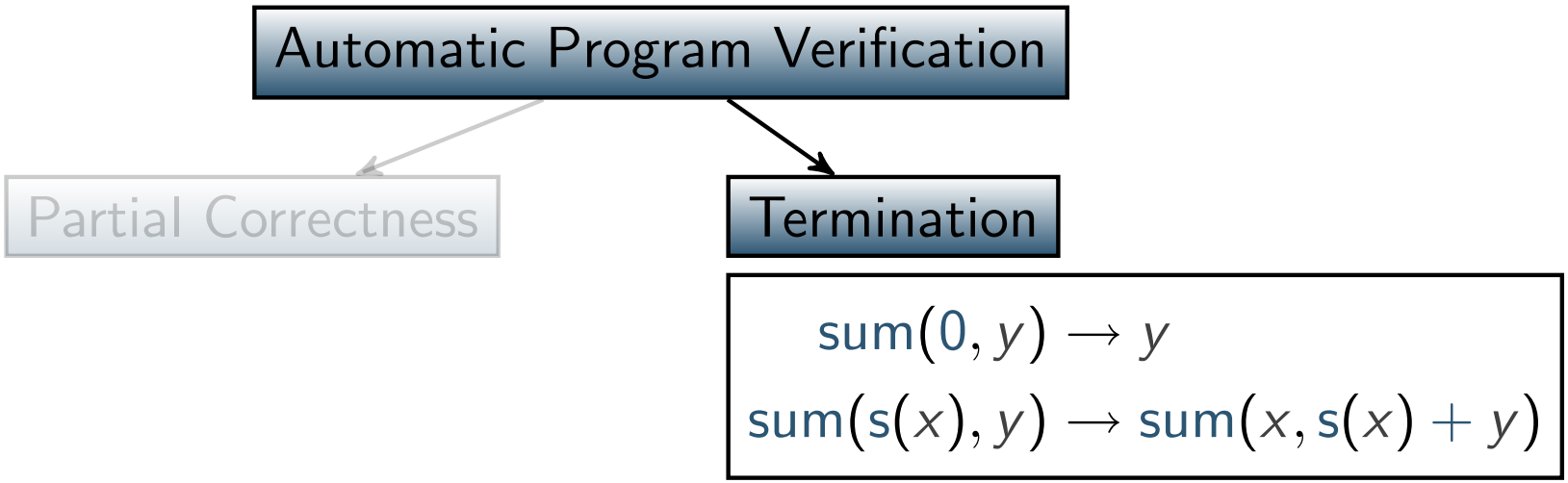
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$\mathsf{sum}(\mathsf{s}(x), y) \to \mathsf{sum}(x, \mathsf{s}(x) + y)$

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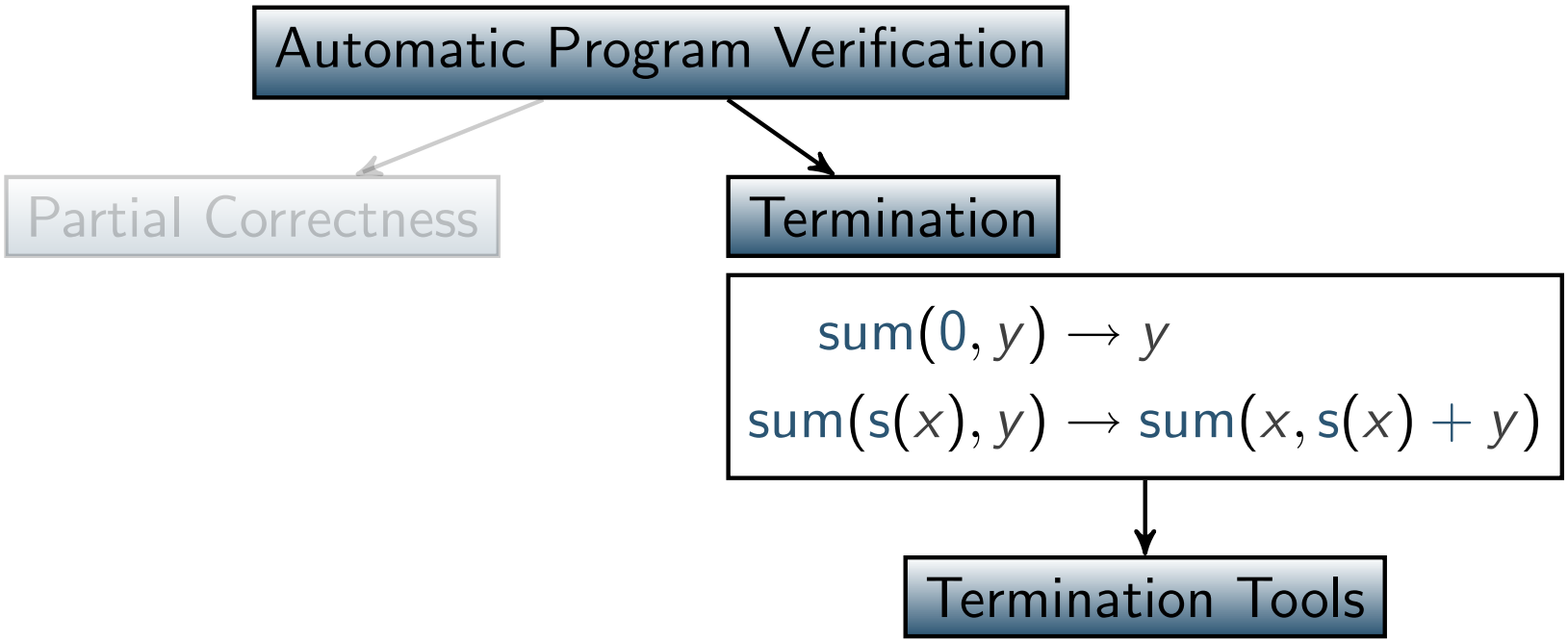
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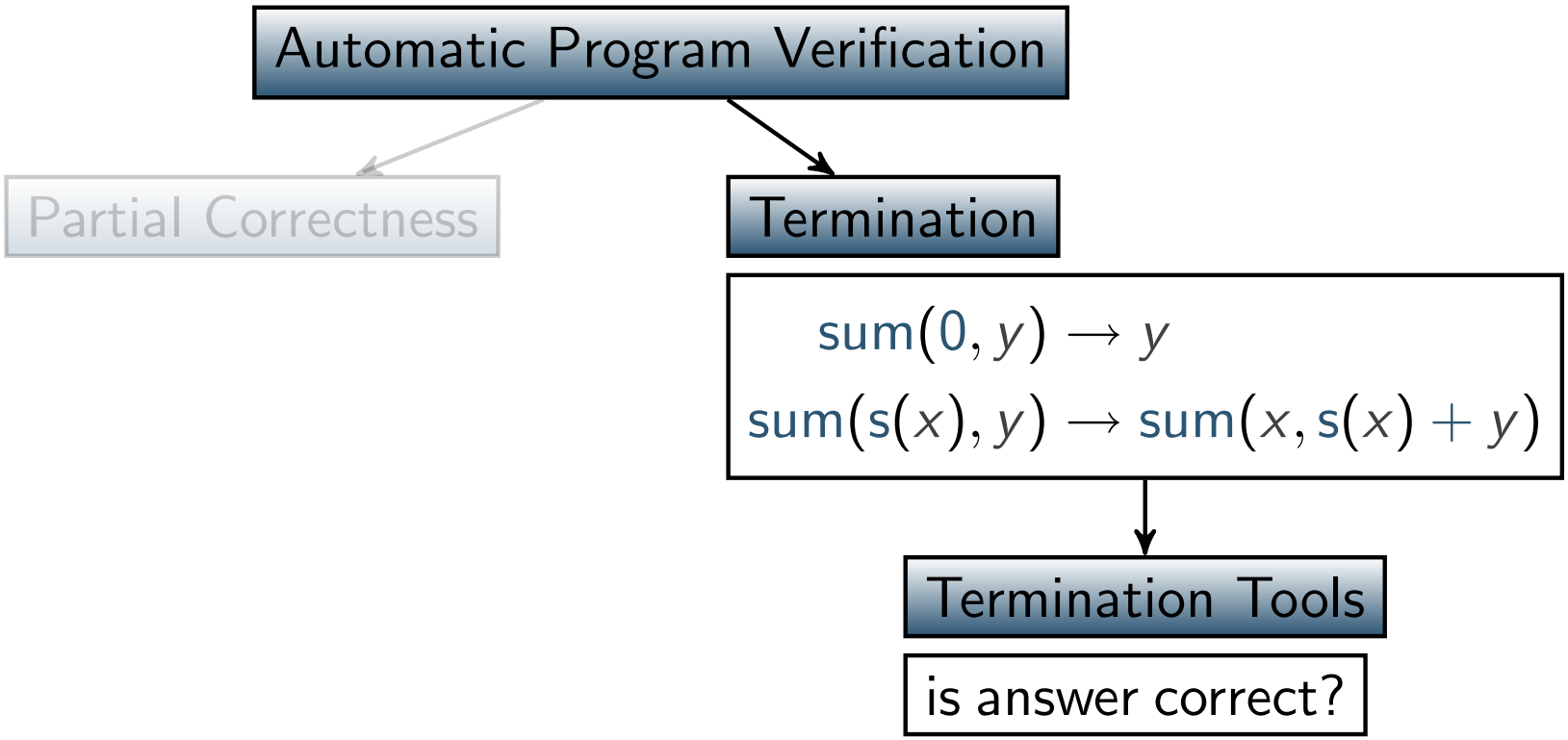
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is answer correct?

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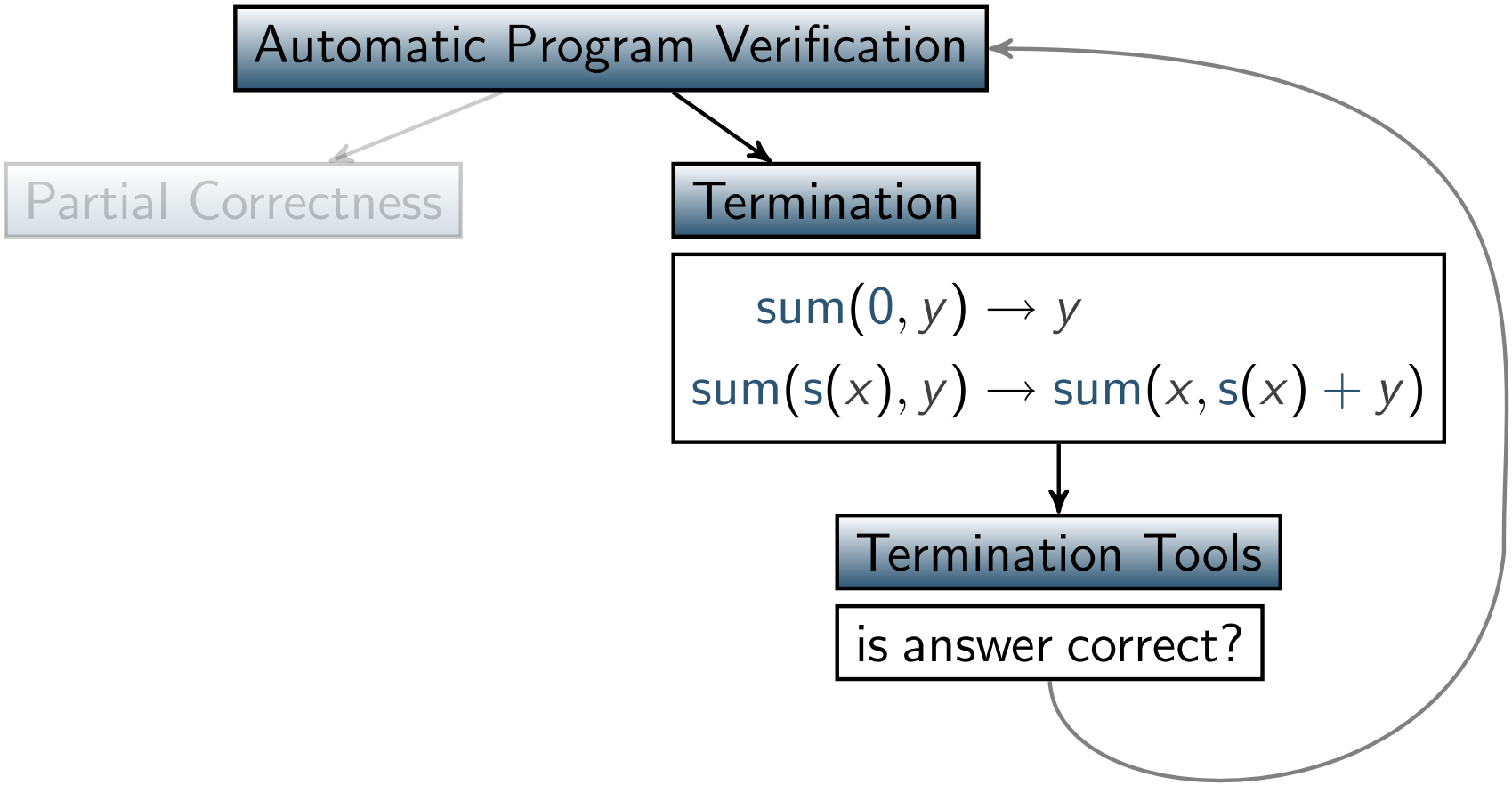
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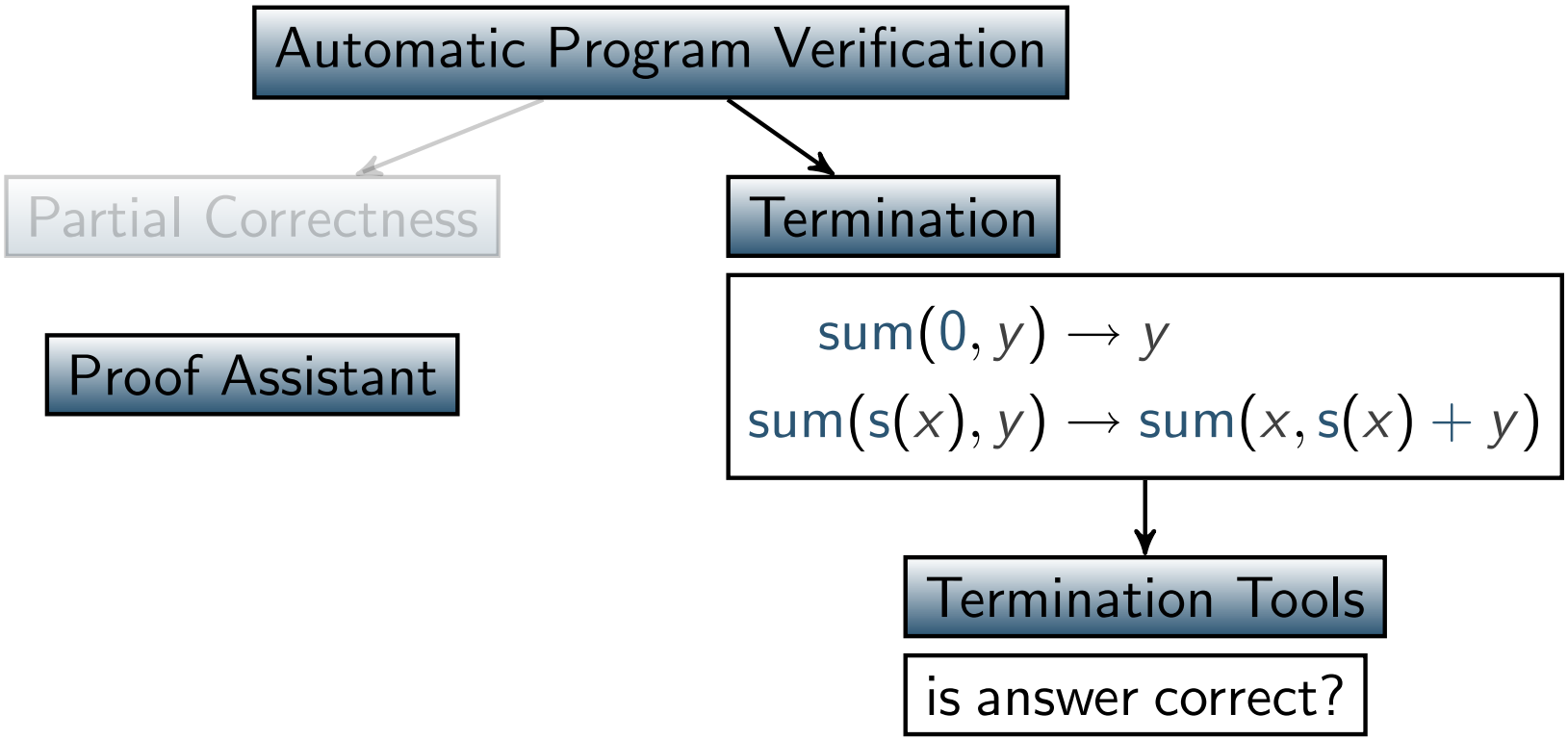
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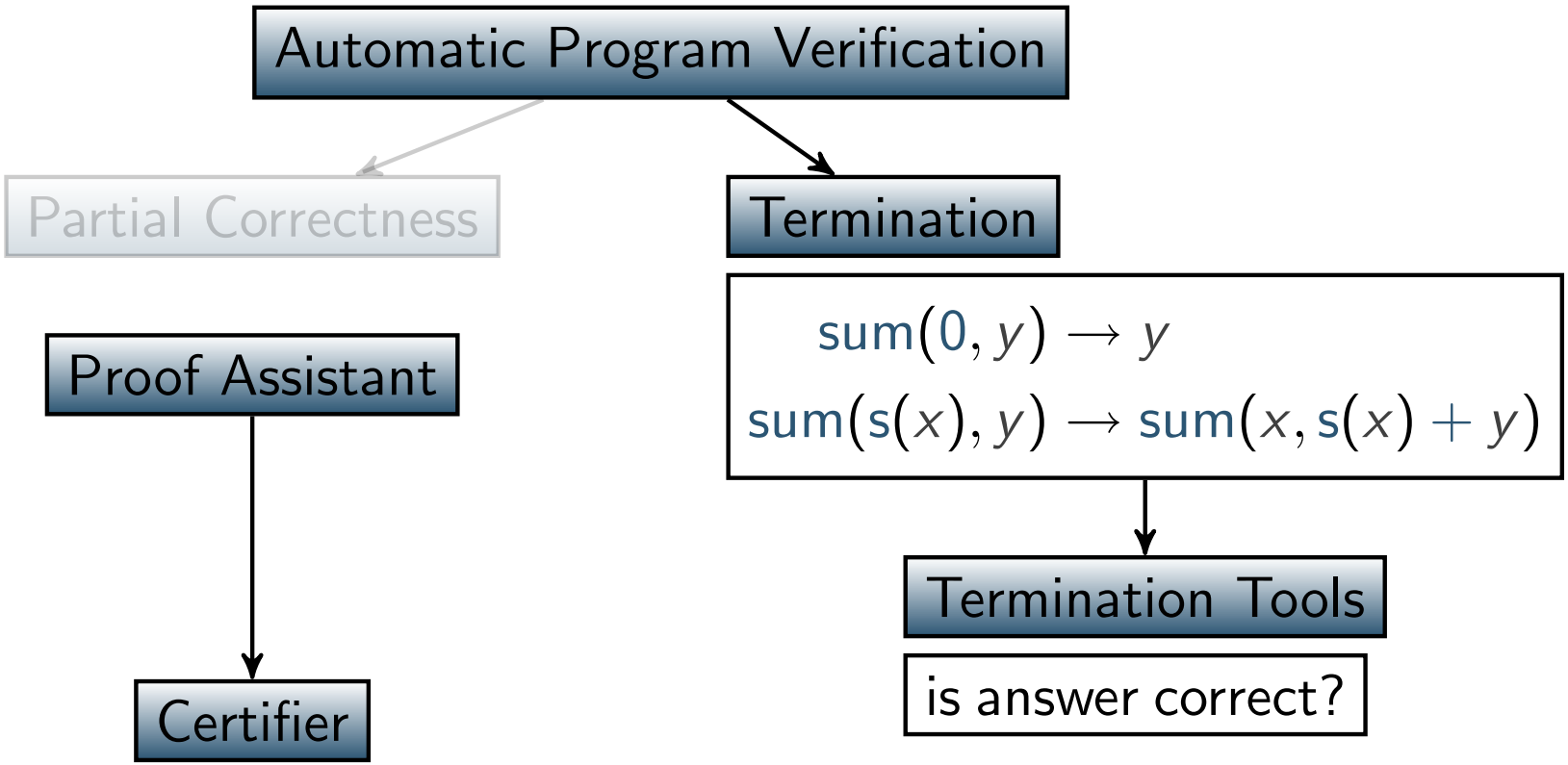
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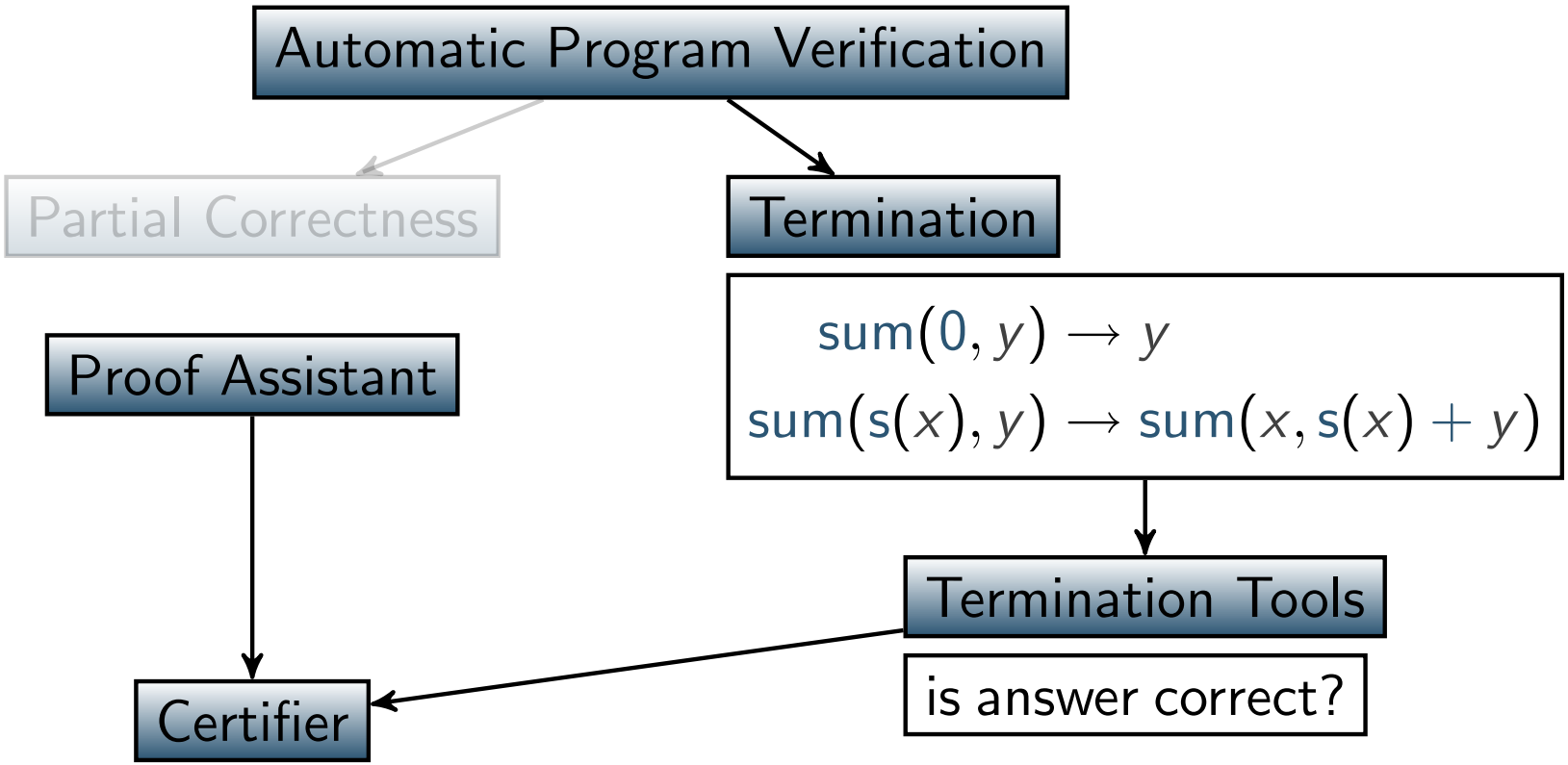
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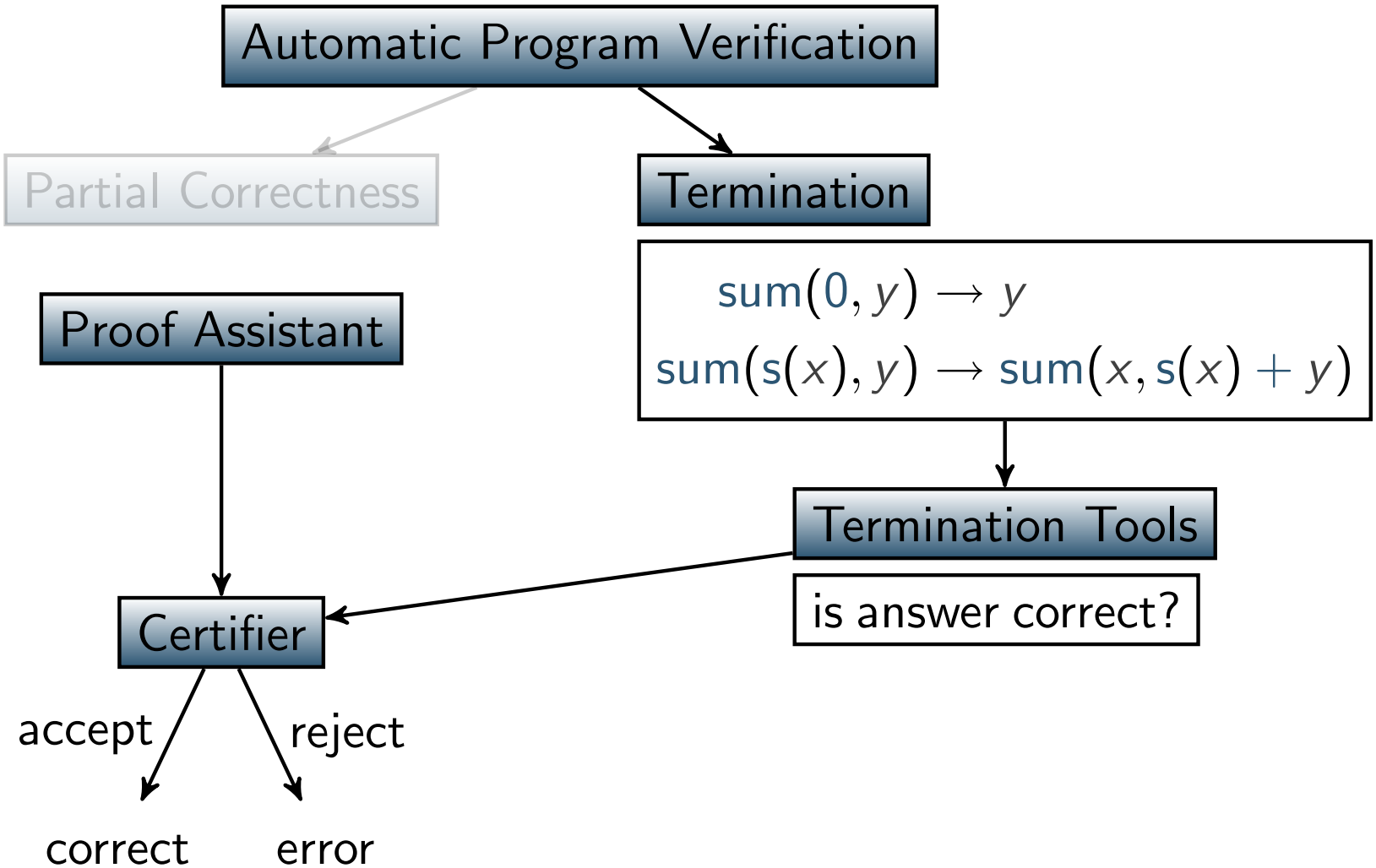
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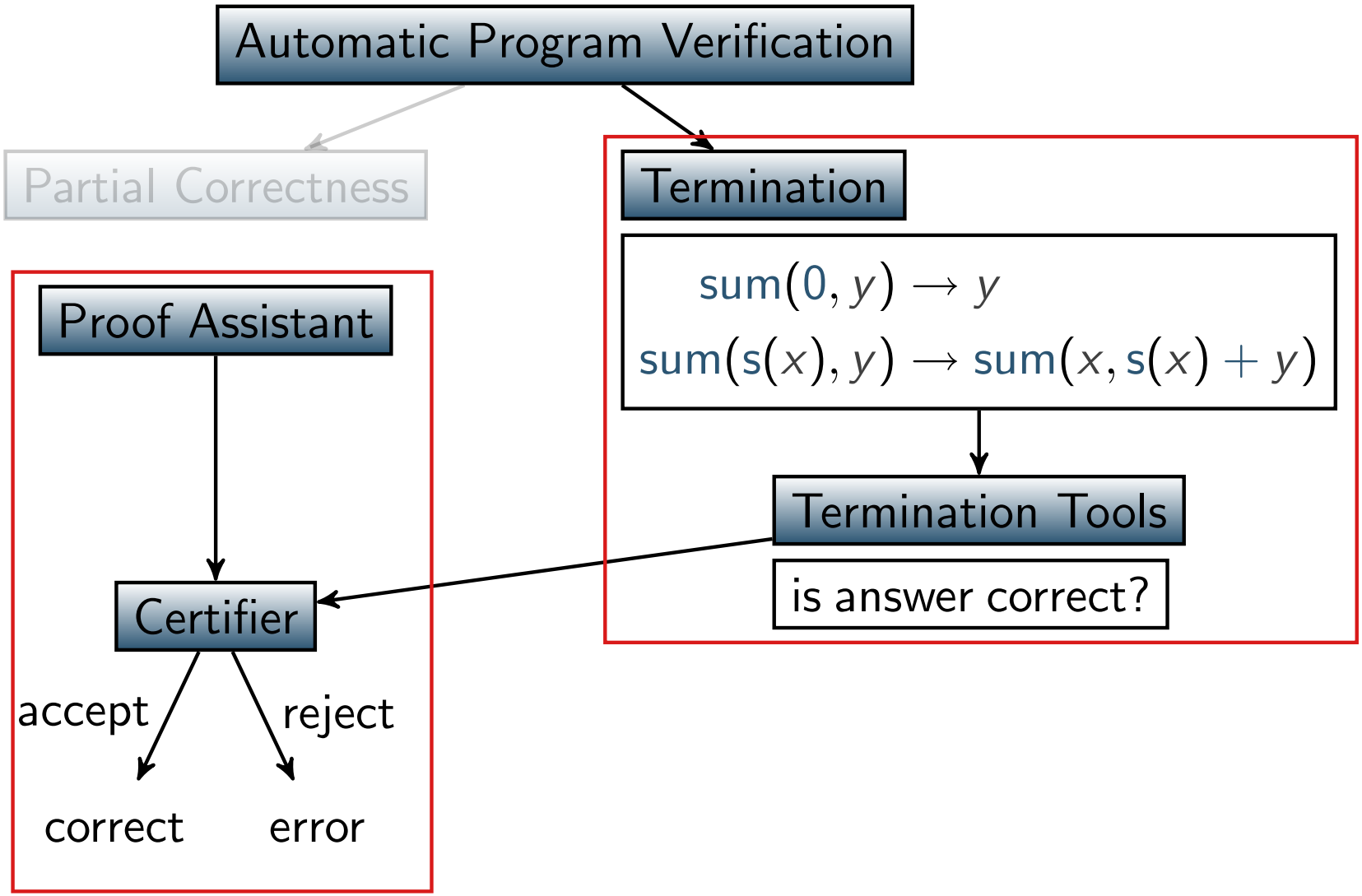
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Introduction to Isabelle

A Bit of History

A Bit of History

1972	Stanford LCF	*compromise between fully automatic theorem proving (difficult) and single step proof checking (tedious) [Robin Milner]*

A Bi

Scott's Logic for Computable Functions

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1972	Stanford LCF	*compromise between fully automatic theo-*
		Meta Language (e.g., Standard ML, OCaml)
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1986	Isabelle	*generic proof assistant based on intuitionistic HOL and higher-order unification*

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Higher-Order Logic

System Architecture

Standard ML implementation language

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Isabelle/Pure generic proof assistant

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Isabelle/HOL Higher-Order Logic

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Proof General Emacs interface

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Isabelle/jEdit jEdit based interface

Isabelle/Scala connects ML to JVM

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Isabelle/HOL Higher-Order Logic

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Isabelle/Pure

Basic Objects

simply-typed λ-terms with equality modulo α (renaming of bound variables), β (computation), and η (extensionality)

Isabelle/Pure

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Basic Constructs

Syntax	Example	Description

Isabelle/Pure

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terms depending on terms

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$\equiv$ :: $\alpha \Rightarrow \alpha \Rightarrow$ **prop**	$P\ x \equiv Q\ x$	*equality*

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proofs depending on terms

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$\Longrightarrow$ :: **prop** $\Rightarrow$ **prop** $\Rightarrow$ **prop**	$A \Longrightarrow B$	*implication*

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Basic Inference Rules

$$\frac{\text{if } \phi \text{ implies } \psi}{\phi \Longrightarrow \psi} \qquad \frac{\phi \Longrightarrow \psi \quad \phi}{\psi} \qquad \frac{\phi[x]}{\bigwedge x.\ \phi[x]}\star \qquad \frac{\bigwedge x.\ \phi[x]}{\phi[t]}$$

$$\frac{[\![\psi_1; \ldots; \psi_m]\!] \Longrightarrow \psi \qquad [\![\phi_1; \ldots; \phi_n]\!] \Longrightarrow \phi}{[\![\phi_1\sigma; \ldots; \phi_{i-1}\sigma; \psi_1\sigma; \ldots; \psi_m\sigma; \phi_{i+1}\sigma; \ldots; \phi_n\sigma]\!] \Longrightarrow \phi\sigma}(\psi\sigma = \phi_i\sigma)$$

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x not free in assumptions

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Isabelle/HOL

Axioms

refl:	$t = t$
subst:	$\llbracket s = t; P\ s \rrbracket \Longrightarrow P\ t$
ext:	$(\bigwedge x.\ f\ x = g\ x) \Longrightarrow f = g$
the-eq-trivial:	$(\varepsilon x.\ x = a) = a$
impl:	$(P \Longrightarrow Q) \Longrightarrow P \longrightarrow Q$
mp:	$\llbracket P \longrightarrow Q; P \rrbracket \Longrightarrow Q$
iff:	$\llbracket P \longrightarrow Q; Q \longrightarrow P \rrbracket \Longrightarrow P = Q$
True-or-False:	$P = \texttt{True} \vee P = \texttt{False}$
eq-reflection:	$x = y \longrightarrow x \equiv y$

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Definitions

$$
\begin{aligned}
\mathtt{True} &\equiv (\lambda x.\ x) = (\lambda x.\ x) \\
\forall x.\ P\ x &\equiv P = (\lambda x.\ \mathtt{True}) \\
\exists x.\ P\ x &\equiv \forall Q.\ (\forall x.\ P\ x \longrightarrow Q) \longrightarrow Q \\
\mathtt{False} &\equiv \forall P.\ P \\
\neg P &\equiv P \longrightarrow \mathtt{False} \\
P \wedge Q &\equiv \forall R.\ (P \longrightarrow Q \longrightarrow R) \longrightarrow R \\
P \vee Q &\equiv \forall R.\ (P \longrightarrow R) \longrightarrow (Q \longrightarrow R) \longrightarrow R
\end{aligned}
$$

Higher-Order Logic

- HOL = Functional Programming + Logic
- data types (**`datatype`**)
- recursive functions (**`fun`**)
- logical operators ($\wedge$, $\vee$, $\longrightarrow$, $\forall$, $\exists$, ...)

Example - Append in Isabelle/HOL

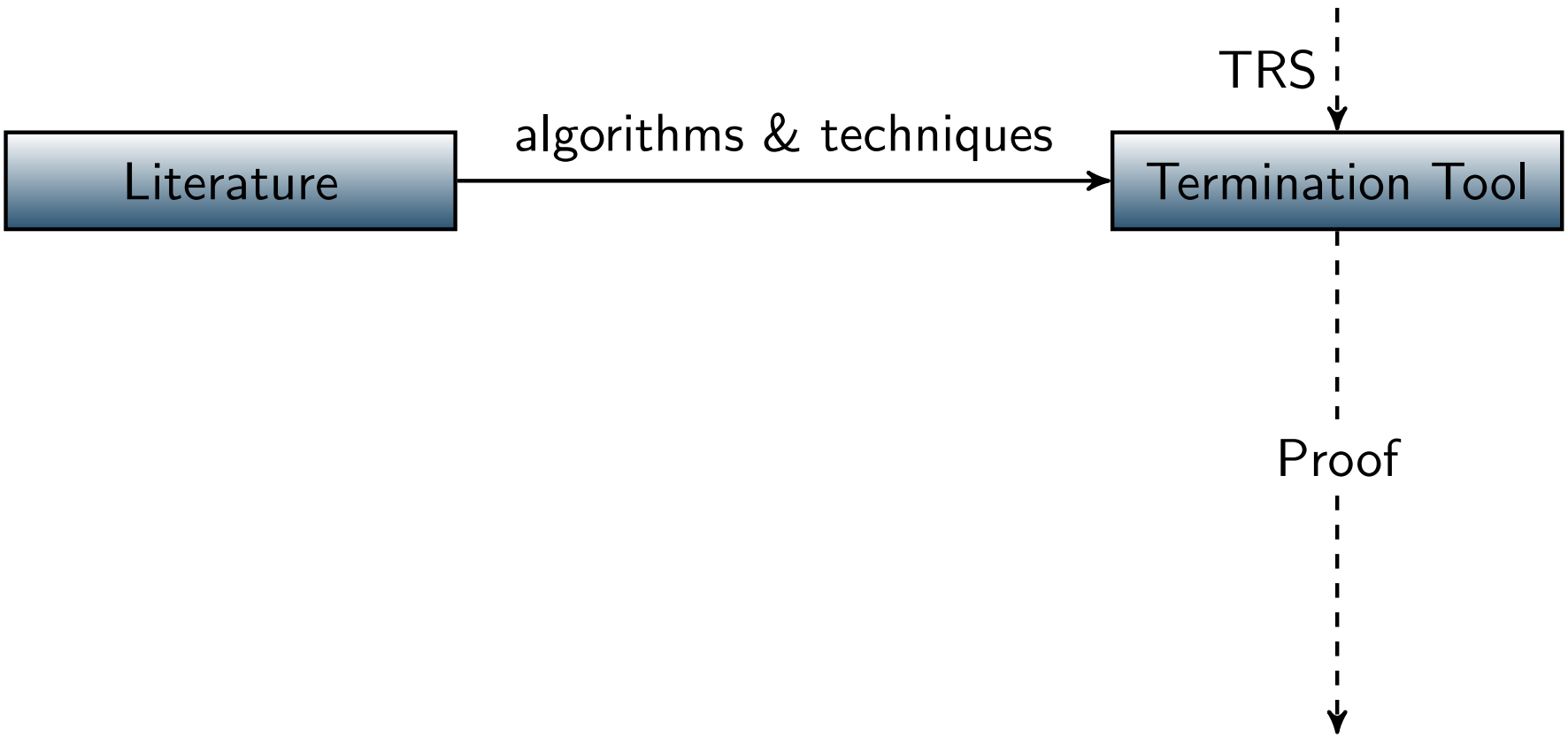
```
fun
  append :: "'a list => 'a list => 'a list"
    (infixr "@" 65)
where
  "[] @ ys = ys"
| "(x#xs) @ ys = x # (xs @ ys)"
```

Demo - Functional Programming in HOL

Certification of Termination Proofs

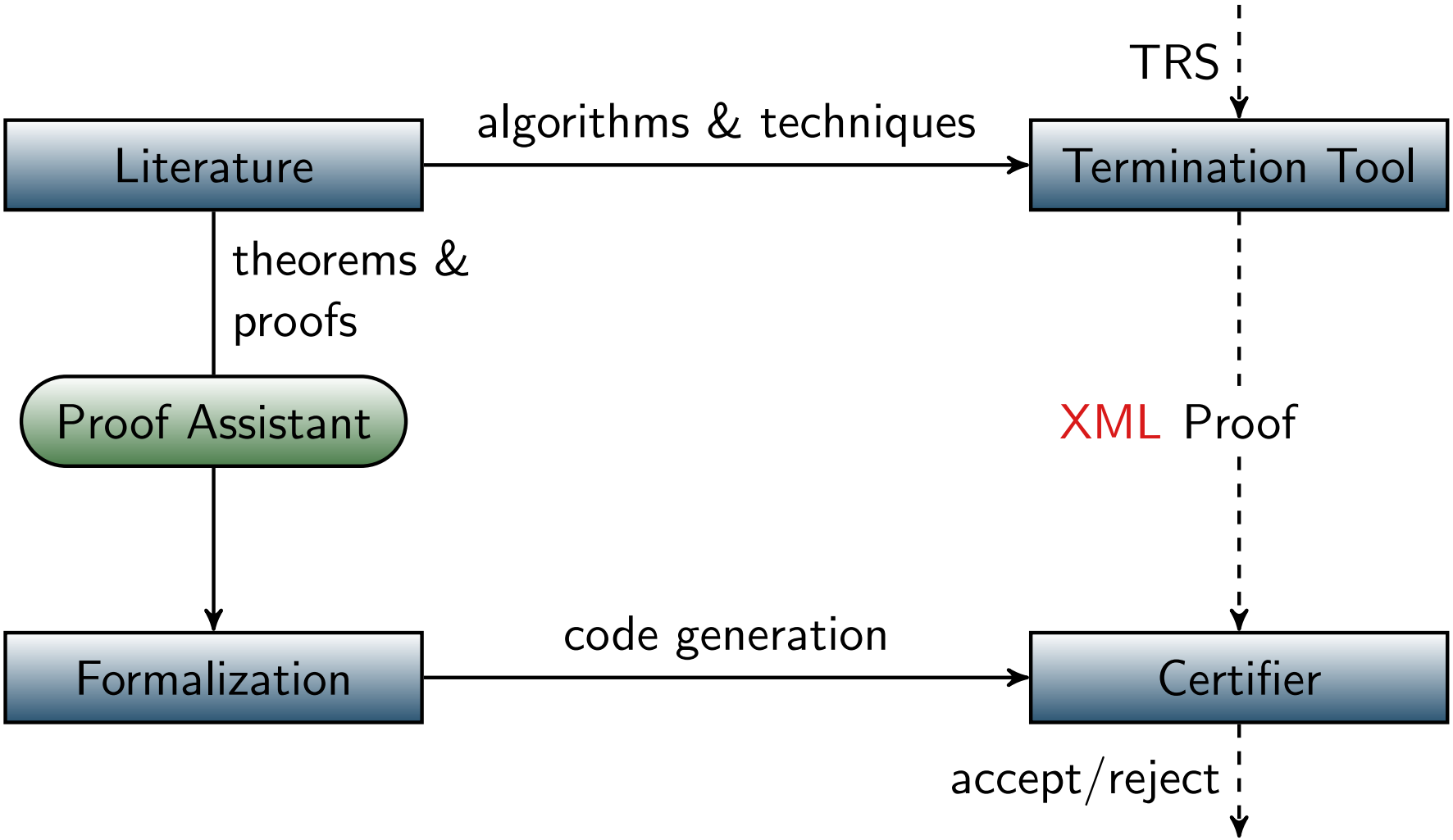
Automatic Termination Analysis

- (automatically) provide evidence of (non)termination



Automatic Termination Analysis

- (automatically) provide evidence of (non)termination
- (automatically) certify correctness of evidence



Example - Append as TRS

$$\mathsf{append}(\mathsf{nil}, ys) \longrightarrow ys$$
$$\mathsf{append}(\mathsf{cons}(x, xs), ys) \longrightarrow \mathsf{cons}(x, \mathsf{append}(xs, ys))$$

Demo - Termination Tools and Certification

An Isabelle Formalization of Rewriting

IsaFoR

- `http://cl-informatik.uibk.ac.at/software/ceta`
- abstract rewriting, relative rewriting, term rewriting, termination, the dependency pair framework, ...

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- *Certification Problem Format*
- common XML format
- output of termination tools
- input for certifiers

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Demo - Code Generation

Termination of Isabelle/HOL Functions

Example - Why Termination Matters

```
fun f :: "nat => nat" where
  "f x = Suc (f x)"
```

Proving Termination of Isabelle/HOL Functions

Built-In Automation

- primitive recursion (syntactic)
- lexicographic orders
- size-change principle

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Alternatives . . .

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Termination Techniques

transformations (semantic labeling, root-labeling, uncurrying, . . .), interpretations (polynomial, matrix, arctic, . . .), orders (Knuth-Bendix, lexicographic, multiset, RPO, . . .), advanced methods (dependency pairs, dependency graph, usable rules, . . .),

Two Worlds

Termination of Isabelle/HOL Functions

- input: defining equations E_f for function f of type `'a => 'b`
- output: call-relation $\mathcal{C}_f$ of type `('a × 'a) set`
- goal: show well-foundedness of $\mathcal{C}_f$

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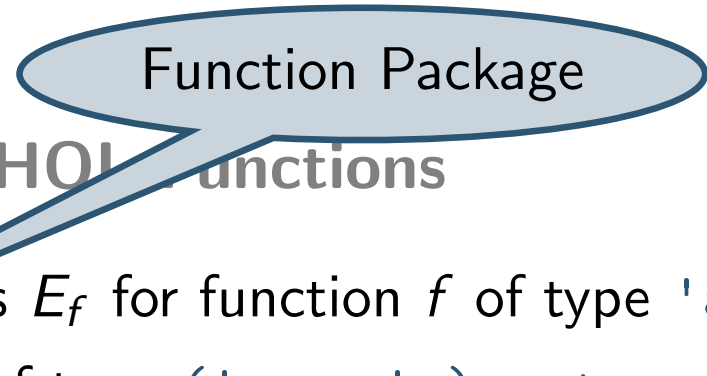

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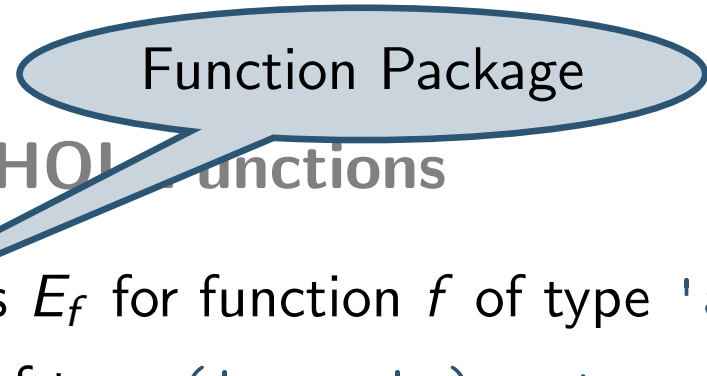

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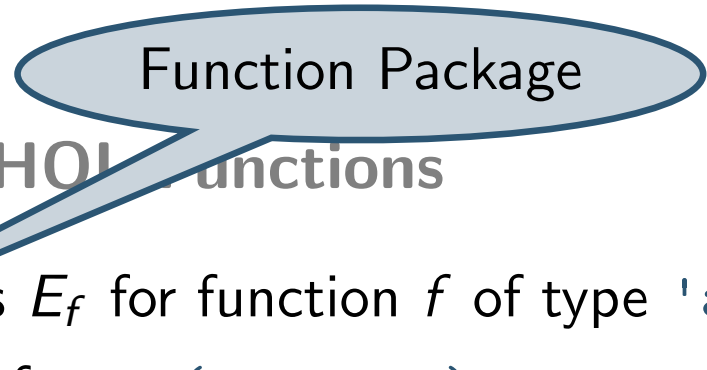
Termination of TRSs

- input: TRS $\mathcal{R}$
- goal: show well-foundedness of rewrite relation $\rightarrow_{\mathcal{R}}$

Two Worlds



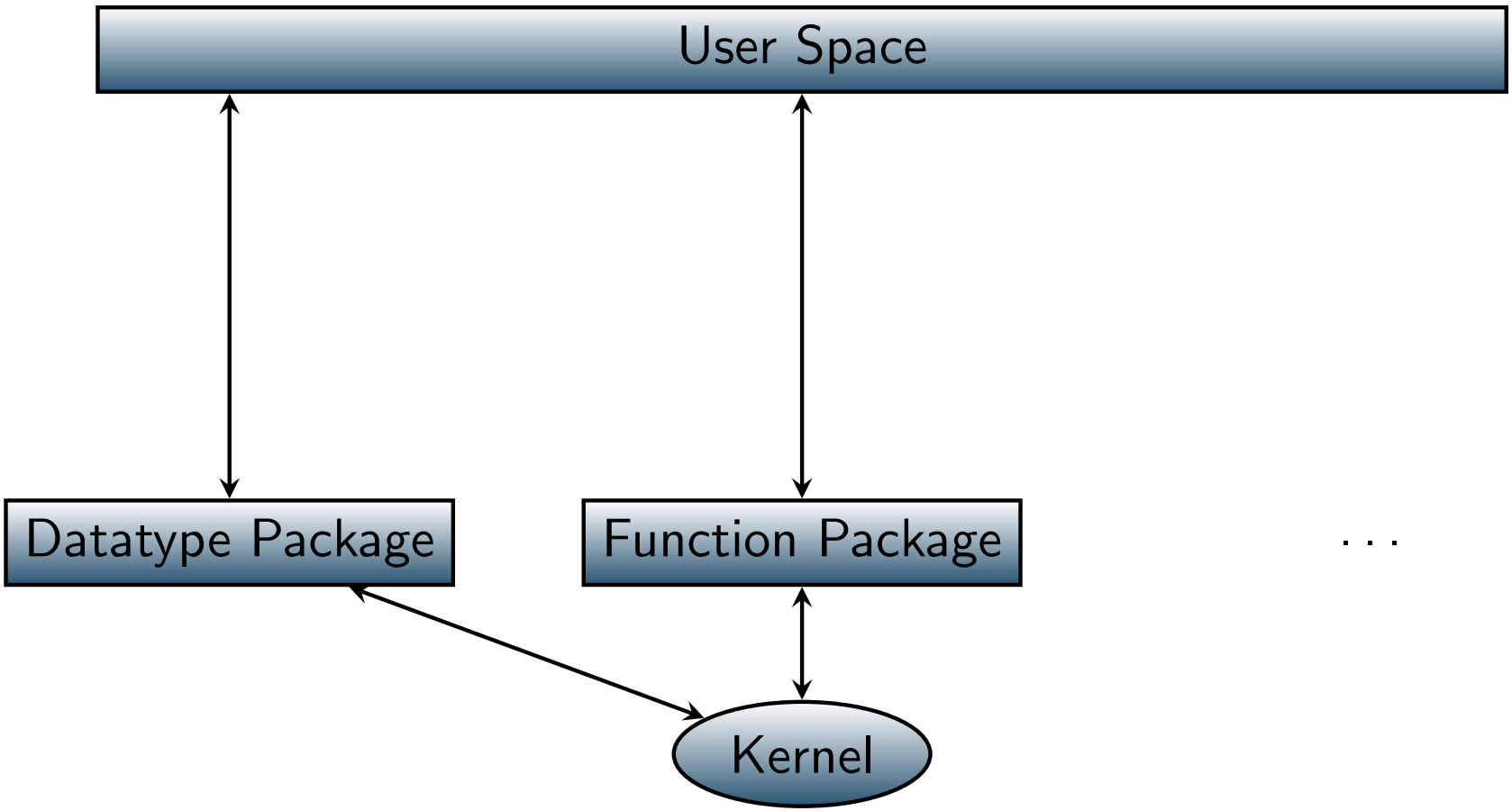
Termination of Isabelle/HOL Functions

- input: defining equations E_f for function f of type `'a => 'b`
- output: call-relation $\mathcal{C}_f$ of type `('a × 'a) set`
- goal: show well-foundedness of $\mathcal{C}_f$

Termination of TRSs

- input: TRS $\mathcal{R}$
- goal: show well-foundedness of rewrite relation $\rightarrow_{\mathcal{R}}$

The Big Picture



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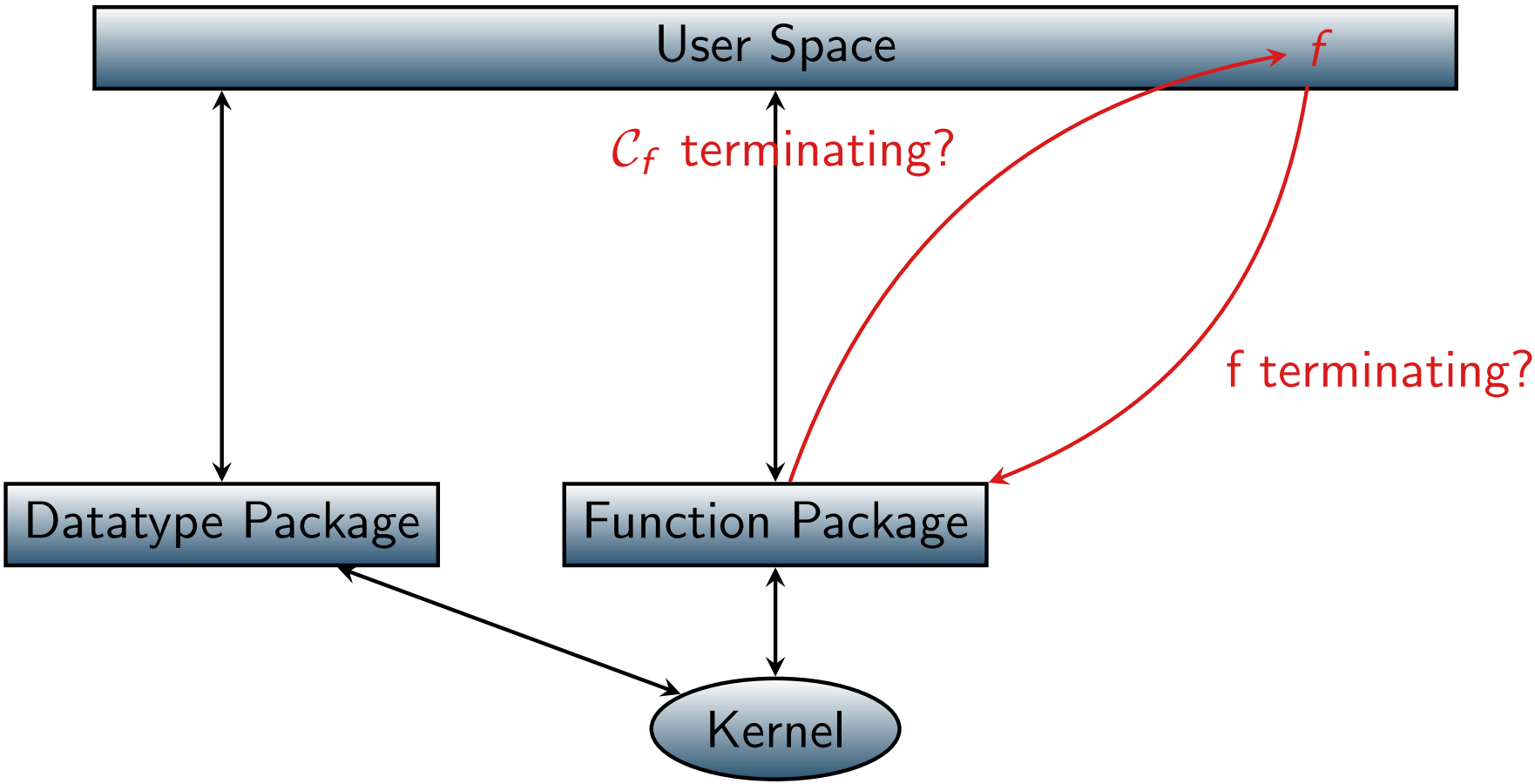

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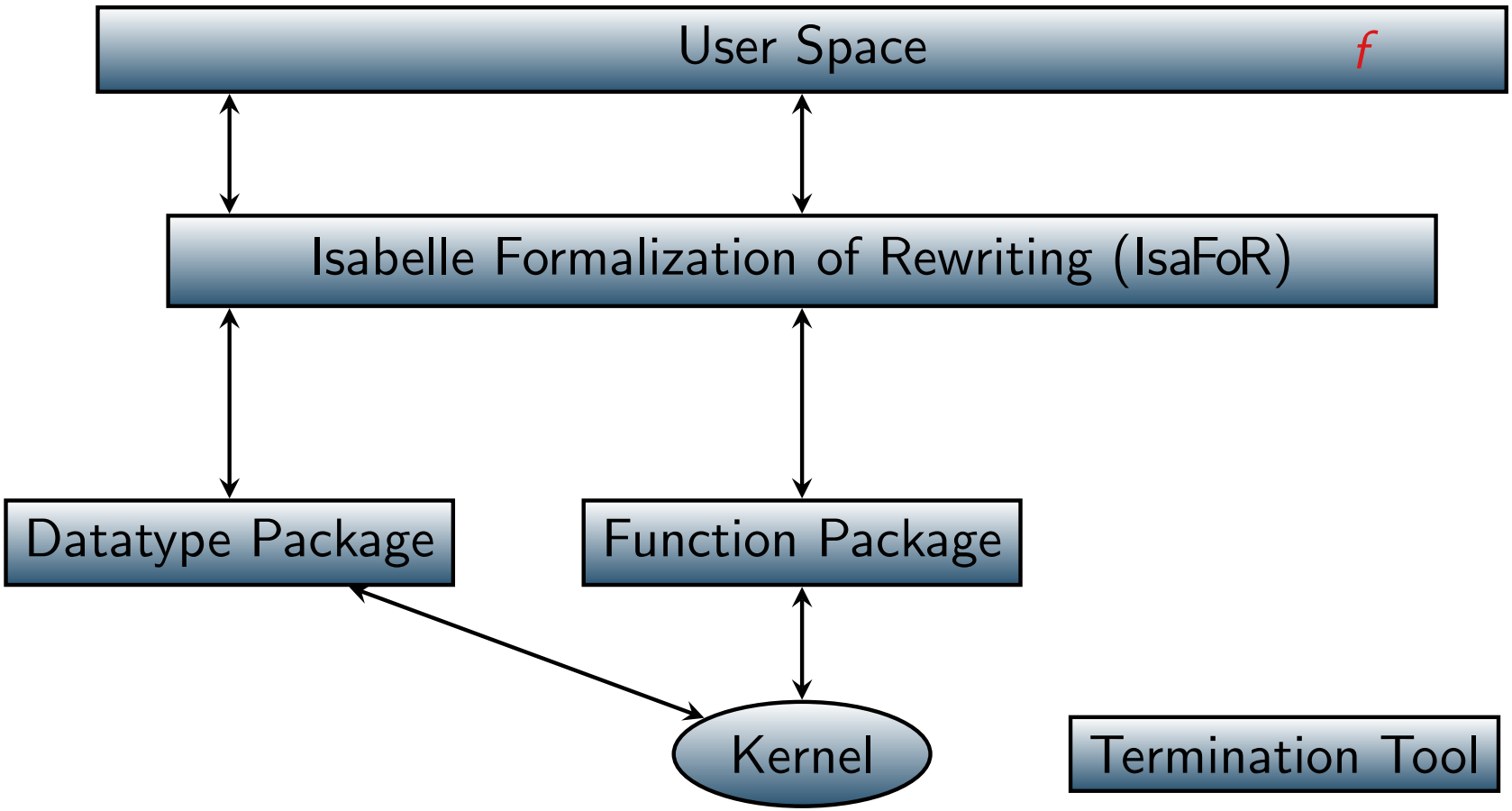

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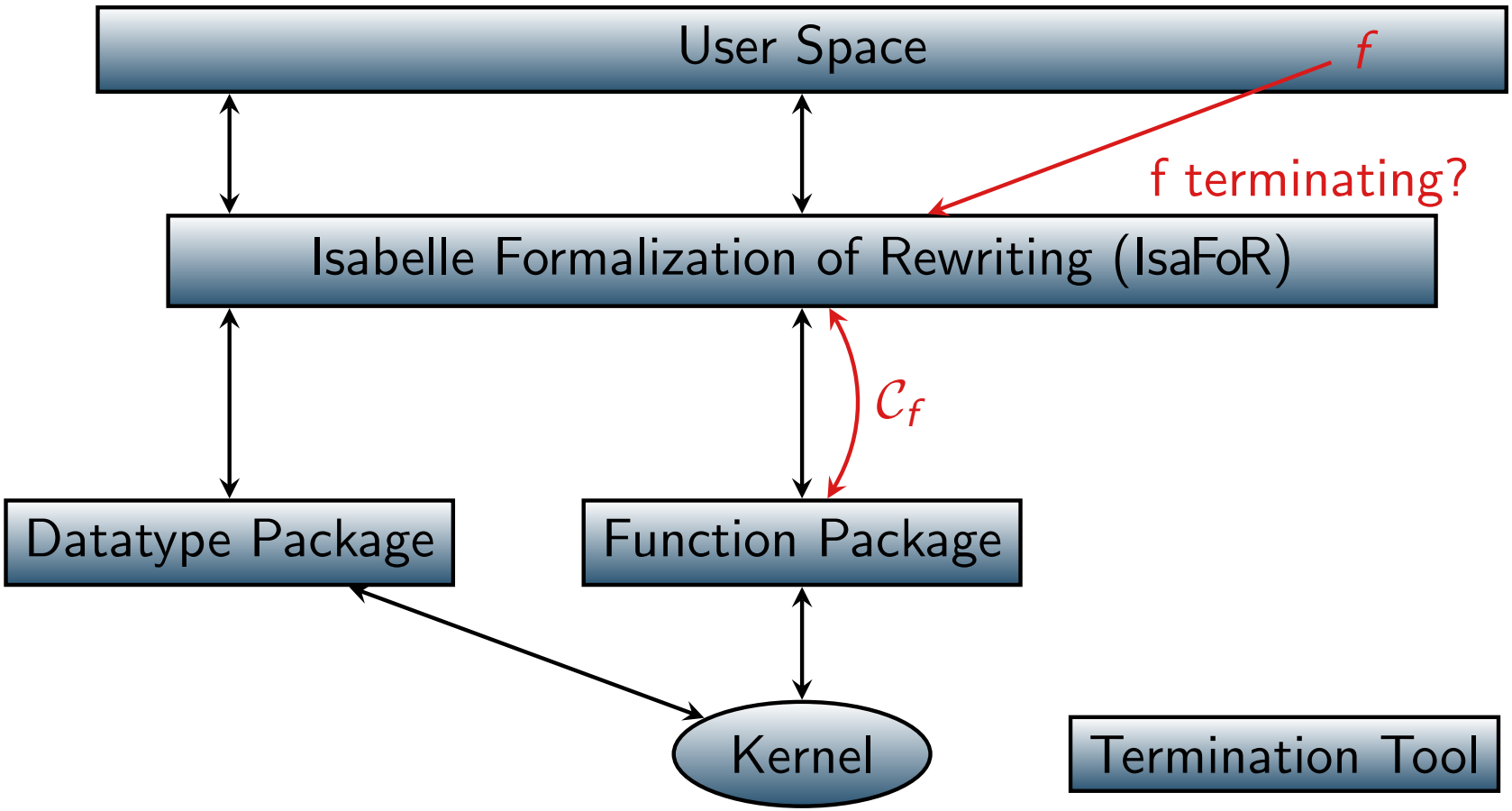

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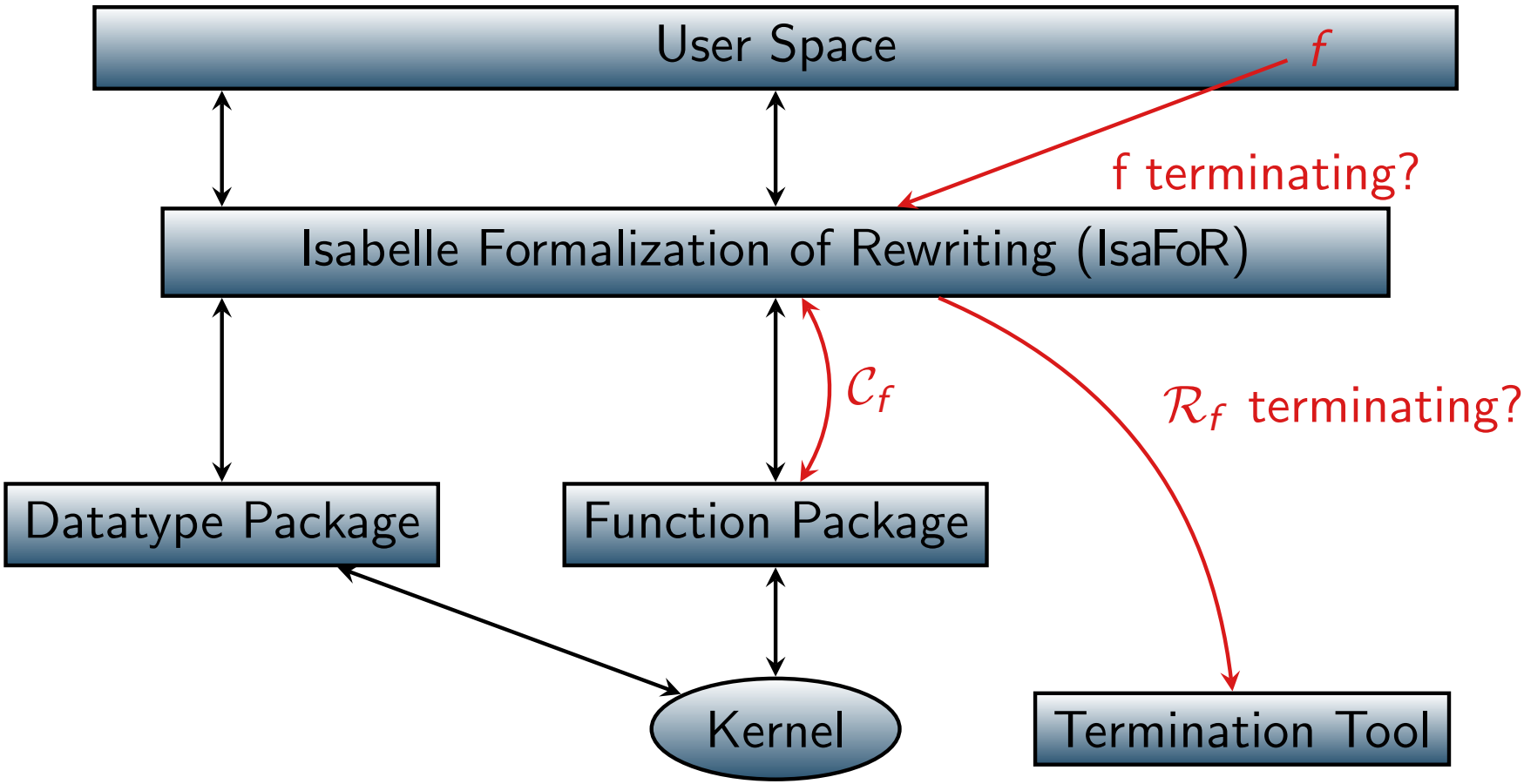
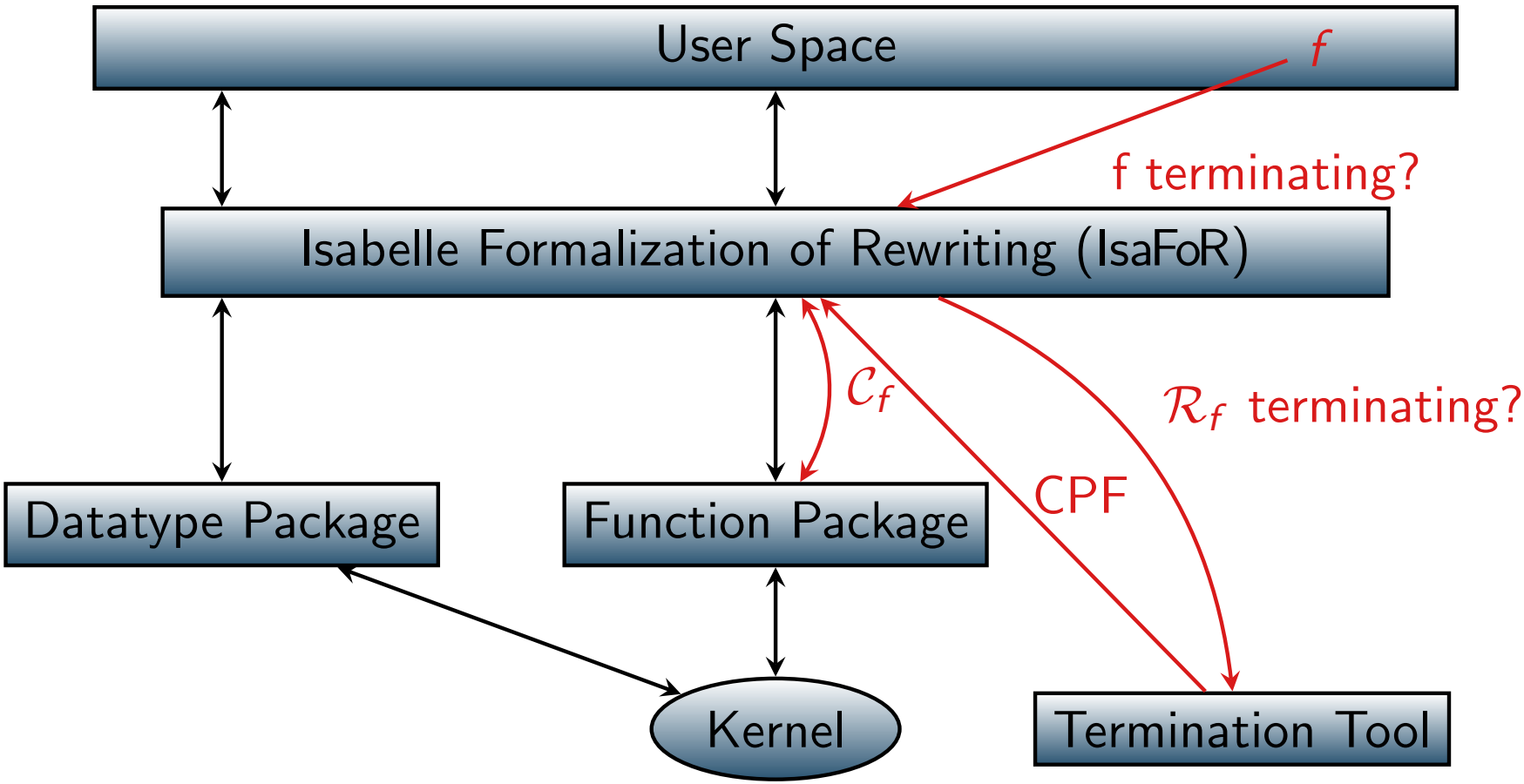
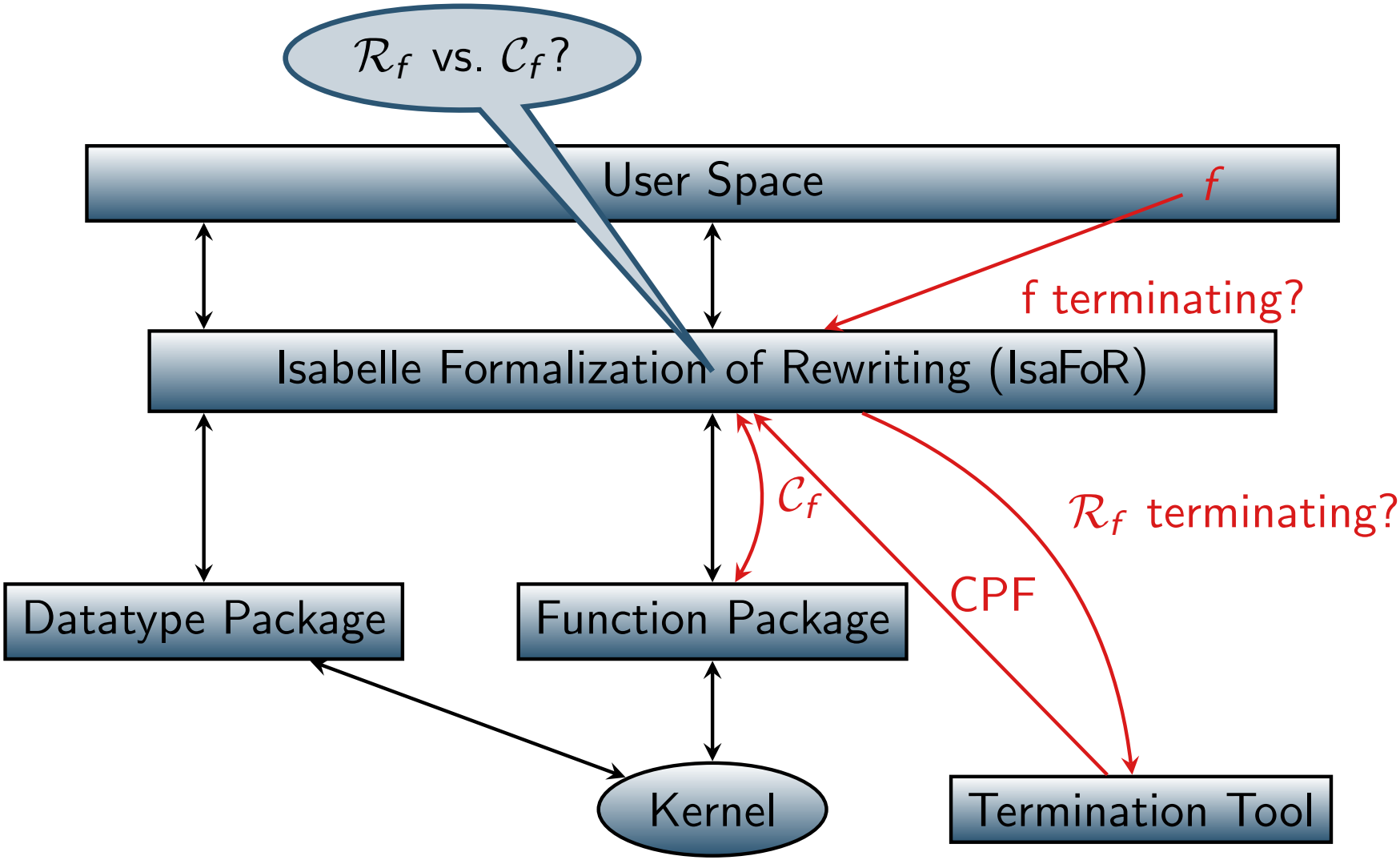


The Big Picture



The Big Picture



Necessary Glue

- import CPF certificate into Isabelle (using IsaFoR)
- generate TRS $\mathcal{R}_f$ corresponding to definition of function f
- relate termination of $\rightarrow_{\mathcal{R}_f}$ to well-foundedness of $\mathcal{C}_f$?

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- internal type

```
term = Var string | Fun string (term list)
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- encoding Isabelle/HOL expressions

$$\text{ENC}(f\ \vec{e}_n) = \texttt{Fun f}\ [\text{ENC}(e_1), \ldots, \text{ENC}(e_n)]$$
$$\text{ENC}(x) = \texttt{Var}\ x$$

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$$\textsc{enc}(x) = \texttt{Var}\ x$$

- rewrite rules for equations $l_1 = r_1, \ldots, l_k = r_k$

$$\begin{aligned}\textsc{rules}(f) = \{\ & \textsc{enc}(l_1) \to \textsc{enc}(r_1),\\ & \vdots\\ & \textsc{enc}(l_k) \to \textsc{enc}(r_k)\}\end{aligned}$$

Example - An Odd Even/Odd Function

```
function evenodd :: "bool => nat => bool" where
  "evenodd True 0 = False"
| "evenodd False x = (¬ (evenodd True x))"
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Goal

```
1. wf ?R
2. ⋀x. ((True,x),(False,x)) ∈ ?R
3. ⋀x. ((False,x),(True,Suc x)) ∈ ?R
```

Demo - Termination by External Tool

Main Goal

- embedding *emb* of type `'a => term`
- prove that $\mathcal{C}_f$ is contained in

$$\{(x, y) \mid \texttt{Fun f [}\mathit{emb}\ x\texttt{]}\ (\rightarrow_{\mathcal{R}_f} \cup \rhd)^{+}\ \texttt{Fun f [}\mathit{emb}\ y\texttt{]}\}$$

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Simulation Lemmas

- n-ary function f
- lemma:

$$\texttt{Fun f [}emb\ x_1, \ldots, emb\ x_n\texttt{]} \rightarrow^*_{\mathcal{R}_f} emb\ (f\ \vec{x}_n)$$

Restrictions

Supported

- variables, function applications
- case-expressions (let, if)

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Not Supported

- no data type constructors with functional arguments
- no "lambdas"
- no function variables
- no overlapping patterns
- no incomplete patterns
- no mutual recursion

Summary

- underlying principles of LCF-style proof assistants
- basics of HOL
- prove termination of HOL functions by external tools
- free user from tedious termination proofs (open problem since certification of termination proofs started)

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