Belief Context from Language to Logic

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From Logic to Language

- $\lambda o\gamma o\varsigma$ (word) $\Longrightarrow$ logic.
- Tense and aspect
- Negation
- Quantification and anaphora
- Knowledge and Belief
- Mood and modality
- Implicature

What's wrong?

- $P \rightarrow Q$ and $Q \rightarrow R$ implies $Q \rightarrow R$.
  - 'If it rained hard, it rained.'
  - 'If it rained, it didn't rain hard.'
  - Therefore, 'if it rained hard, it didn't rain hard.'
- $P \rightarrow Q$ implies $\neg Q \rightarrow \neg P$.
  - 'If she wrote a letter to Santa Claus, she didn't get an answer.'
  - Therefore, 'if she got an answer from Santa Claus, she didn't write a letter to him.'
- $P \rightarrow Q$ implies $P \wedge P' \rightarrow Q$.
  - 'If Betty had been at the party, Bill would have had a good time.'
  - Therefore, 'if Betty had been at the party and Bill had broken his leg, Bill would have had a good time.'

Opque/oblique context

- 'Anna doesn't know who the democrat candidate is.'
- 'Her favorite TV star is the democrat candidate.'
- Therefore, 'Anna doesn't know who her favorite TV star is.'

Categorial Grammar

A/B: the category becomes A, biting B.
(function $B \rightarrow A$)

John: e
Mary: e
loves: (t/e)/e

$$\dfrac{\text{John: e} \qquad \dfrac{\text{Mary: e} \quad \text{loves: (t/e)/e}}{\text{loves Mary: t/e}}}{\text{John loves Mary: t}}$$

cf. Context-Free Grammar

$$S \rightarrow NP\ VP$$
$$NP \rightarrow N$$
$$VP \rightarrow V\ N$$

Categories

(individual)		e
sentence	*S*	t
common noun	*N*	t/e
verb phrase	*VP*	t/e
intransitive verb	*IV*	t/e
transitive verb	*TV*	$(t/e)/(t/(t/e))$
noun phrase / proper noun		
	NP	$(t/(t/e))$
preposition	*Prep*	$((t/e)/(t/e))/(t/(t/e))$
(verb) adverb	*Adv*	$(t/e)/(t/e)$
sentence adverb	*Adv*	t/t
determiner	*Det*	$(t/(t/e))/(t/e)$
conjunction	*Conj*	$(t/t)/t$

Typed lambda calculus

- e – an entity – a member of U
- t – a sentence – a truth value

$\langle e, t\rangle$: a type of function, biting e and returning t (predicate)

$\langle\langle e, t\rangle, t\rangle$	from $\langle e, t\rangle$ to t
$\langle\langle e, t\rangle, \langle e, t\rangle\rangle$	from $\langle e, t\rangle$ to $\langle e, t\rangle$
$\langle\langle e, \langle e, t\rangle\rangle, \langle e, t\rangle\rangle$	from $\langle e, \langle e, t\rangle\rangle$ to $\langle e, t\rangle$
$\vdots$	

Lambda conversion

$$\lambda x[p(x)]$$

where x is an input, an argument, a parameter, a set $\{x \mid p(x)\}$, or whatsoever.

- α-conversion

$$\lambda x[p(x)] = \lambda y[p(y)]$$

- β-conversion

$$\begin{cases} \lambda x[p(x)] \colon \langle A, B\rangle \\ a \colon A \\ \lambda x[p(x)]](a) = p(a) \colon B \end{cases}$$

- η-conversion

$$\lambda x[p(x)] = p \text{ if } x \text{ does not appear free in } p$$

Ex. John loves Mary.

$$\begin{array}{lcl} \text{John} & \Rightarrow & j\colon e \\ \text{Mary} & \Rightarrow & m\colon e \\ \text{loves} & \Rightarrow & \lambda y \lambda x[love(y)(x)]\colon \langle e, \langle e, t\rangle\rangle \end{array}$$

$$\begin{array}{cl} & [\lambda y \lambda x[love(y)(x)](j)](m) \\ = & \lambda x[love(m)(x)](j) \\ = & love(m)(j) \end{array}$$

$$\dfrac{j\colon e \qquad \dfrac{\lambda y \lambda x[love(y)(x)]\colon \langle e, \langle e, t\rangle\rangle \qquad m\colon e}{\lambda x[love(m)(x)]\colon \langle e, t\rangle}}{love(m)(j)\colon t}$$

Translation

Translation function: $f_0(\mathrm{A/B}) = \langle f_0(\mathrm{B}), f_0(\mathrm{A})\rangle$. (italic: type)

$$f_0((\mathrm{t/e})/(\mathrm{t/(t/e)})) = \langle\langle\langle e, t\rangle, t\rangle, \langle e, t\rangle\rangle.$$

natural langugae	formal language
categorial grammar	typed lambda calculus
category	type

Every, some, and no

$$\begin{array}{rcl} \text{every} & \Rightarrow & \lambda Q \lambda P \forall x[Q(x) \to P(x)] \colon \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \\ \text{man} & \Rightarrow & \lambda x[man(x)] \colon \langle e,t\rangle \\ \text{walks} & \Rightarrow & \lambda x[walk(x)] \colon \langle e,t\rangle \end{array}$$

$$\dfrac{\dfrac{\begin{array}{c}\lambda Q\lambda P\forall x[Q(x)\to P(x)]\\ \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\end{array} \quad \begin{array}{c}man\\ \langle e,t\rangle\end{array}}{\begin{array}{c}\lambda P\forall x[man(x)\to P(x)]\\ \langle\langle e,t\rangle,t\rangle\end{array}} \quad \begin{array}{c}walk\\ \langle e,t\rangle\end{array}}{\begin{array}{c}\forall x[man(x)\to walk(x)]\\ t\end{array}}$$

$$\begin{array}{rcl} \text{some} & \Rightarrow & \lambda P \lambda Q \exists x[P(x) \wedge Q(x)] \colon \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \\ \text{no} & \Rightarrow & \lambda P \lambda Q \neg\exists x[P(x) \wedge Q(x)] \colon \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \end{array}$$

Translation of proper noun

Type raising

$$j\colon e \quad\longrightarrow\quad \lambda P[P(j)]\colon \langle\langle e, t\rangle, t\rangle$$

The meaning of John = The whole set of features and actions of John (λP).
This type-raising is to make the type same as that with 'every student', 'some woman, 'no person', and so on.

Ex. John walks.

$$\text{John}: \mathrm{e} \quad \Rightarrow \quad j: e$$
$$\text{walks}: \mathrm{t/e} \quad \Rightarrow \quad \lambda x[walk(x)]: \langle e, t\rangle$$

$$\frac{\lambda x[walk(x)]: \langle e, t\rangle \quad j: e}{walk(j): t}$$

$$\Downarrow$$

$$\text{John}: \mathrm{t/(t/e)} \quad \Rightarrow \quad j: \langle\langle e, t\rangle, t\rangle$$
$$\text{walks}: \mathrm{t/e} \quad \Rightarrow \quad \lambda x[walk(x)]: \langle e, t\rangle$$

$$\cfrac{\lambda P[P(j)]: \langle\langle e, t\rangle, t\rangle \quad \lambda x[walk(x)]: \langle e, t\rangle}{\cfrac{\lambda x[walk(x)](j): t}{walk(j): t}}$$

Ambiguous scope

"A problem about the environment preoccupies every serious politician."

$\exists y[\mathit{problem}(y) \land \forall x[\mathit{politician}(x) \rightarrow \mathit{preoccupies}(y, x)]]$

$\forall x[\mathit{politician}(x) \rightarrow \exists y[\mathit{problem}(y) \land \mathit{preoccupies}(y, x)]]$

Ex. A problem preoccupies every politician.

$$
\begin{array}{rcl}
\text{every} & \Rightarrow & \lambda P \lambda Q \forall x[P(x) \to Q(x)] : \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \\
\text{a} & \Rightarrow & \lambda P \lambda Q \exists x[P(x) \wedge Q(x)] : \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle \\
\text{politician} & \Rightarrow & \lambda x[pl(x)] : \langle e,t\rangle \\
\text{problem} & \Rightarrow & \lambda x[pr(x)] : \langle e,t\rangle \\
\text{preoccupies} & \Rightarrow & \lambda y \lambda x[preoc(y)(x)] : \langle e,\langle e,t\rangle\rangle
\end{array}
$$

(i) There exists a common problem.

$$
\dfrac{
\dfrac{
\begin{array}{c}\lambda P\lambda Q\exists y[P(y)\wedge Q(y)]\\ \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\end{array}
\quad
\begin{array}{c}pr\\ \langle e,t\rangle\end{array}
}{
\begin{array}{c}\lambda Q\exists y[pr(y)\wedge Q(y)]\\ \langle\langle e,t\rangle,t\rangle\end{array}
}
\quad
\dfrac{
\dfrac{
\dfrac{
\begin{array}{c}\lambda P\lambda Q\forall x[P(x)\to Q(x)]\\ \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\end{array}
\quad
\begin{array}{c}pl\\ \langle e,t\rangle\end{array}
}{
\begin{array}{c}\lambda Q\forall x[pl(x)\to Q(x)]\\ \langle\langle e,t\rangle,t\rangle\end{array}
}
\quad
\dfrac{
\begin{array}{c}\lambda y\lambda x[preoc(y)(x)]\\ \langle e,\langle e,t\rangle\rangle\end{array}
}{
\begin{array}{c}\lambda x[preoc(y_0)(x)]\\ \langle e,t\rangle\end{array}
}
}{
\begin{array}{c}\forall x[pl(x)\to preoc(y_0)(x)]\\ t\end{array}
}
}{
\begin{array}{c}\lambda y\forall x[pl(x)\to preoc(y)(x)]\\ \langle e,t\rangle\end{array}
}
}{
\begin{array}{c}\exists y[pr(y)\wedge\forall x[pl(x)\to preoc(y)(x)]]\\ t\end{array}
}
$$

(ii) Each politician has his own problem.

$$\dfrac{\dfrac{\begin{array}{c}\lambda P\lambda Q\forall x[P(x)\to Q(x)]\\ \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\end{array} \quad \begin{array}{c}pl\\ \langle e,t\rangle\end{array}}{\begin{array}{c}\lambda Q\forall x[pl(x)\to Q(x)]\\ \langle\langle e,t\rangle,t\rangle\end{array}} \qquad \dfrac{\dfrac{\dfrac{\begin{array}{c}\lambda P\lambda Q\exists y[P(y)\wedge Q(y)]\\ \langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle\end{array} \quad \begin{array}{c}pr\\ \langle e,t\rangle\end{array}}{\begin{array}{c}\lambda Q\exists y[pr(y)\wedge Q(y)]\\ \langle\langle e,t\rangle,t\rangle\end{array}} \quad \dfrac{\begin{array}{c}\lambda y\lambda x[preoc(y)(x)]\\ \langle e,\langle e,t\rangle\rangle\end{array}}{\begin{array}{c}\lambda y[preoc(y)(x_0)]\\ \langle e,t\rangle\end{array}}}{\begin{array}{c}\exists y[pr(y)\wedge preoc(y)(x_0)]\\ t\end{array}}}{\begin{array}{c}\lambda x\exists y[pr(y)\wedge preoc(y)(x)]\\ \langle e,t\rangle\end{array}}}{\begin{array}{c}\forall x[pl(x)\to\exists y[pr(y)\wedge preoc(y)(x)]]\\ t\end{array}}$$

de re/de dicto

$$
\begin{array}{ccc}
 & \textit{language expression} & \\
 & \downarrow & \\
\langle \textit{possible world} \rangle & \xrightarrow{\langle \textit{intension} \rangle} & \langle \textit{extension} \rangle
\end{array}
$$

$$
\begin{array}{lll}
\textit{de re} & = \textit{extensional} & = \textit{reference (Bedeutung)} \\
\textit{de dicto} & = \textit{intensional} & = \textit{sense (Sinn)}
\end{array}
$$

Intensional hat

s is an index of a possible world, and the meaning of α depends upon w.

$$\alpha\colon a \leftrightarrow {}^\wedge\alpha\colon \langle s, a\rangle \leftrightarrow {}^\vee{}^\wedge\alpha\colon a$$
$$\gamma\{x\} \equiv {}^\vee\gamma(x) \text{ (Brace convention)}$$

Translation function f_1:

$$f_1(\mathrm{t}) = t$$
$$f_1(\mathrm{e}) = \langle s, e\rangle$$
$$f_1(\mathrm{t/e}) = \langle e, t\rangle \quad \text{(by Bennet)}$$
$$f_1(\mathrm{A/B}) = \langle\langle s, f_1(\mathrm{B})\rangle, f_1(\mathrm{A})\rangle$$

Categories – revised

(individual)		e
sentence	S	t
common noun	N	$\langle e, t \rangle$
verb phrase/ intransitive verb		
	IV	$\langle e, t \rangle$
transitive verb (find, love, eat)		
	$IV/(t/IV)$	$\langle\langle s, \langle\langle s, \langle e, t \rangle\rangle, t \rangle\rangle, \langle e, t \rangle\rangle$
transitive verb (believe, assert)		
	IV/t	$\langle\langle s, t \rangle, \langle e, t \rangle\rangle$
noun phrase/ proper noun		
	t/IV	$\langle\langle s, \langle e, t \rangle\rangle, t \rangle$
determiner	Det	$\langle\langle s, \langle e, t \rangle\rangle, \langle\langle s, \langle e, t \rangle\rangle, t \rangle\rangle$

Ex. Anna believes a decmocrat wins.

$$\begin{array}{lcl}
\text{Anna : t/(t/e)} & \Rightarrow & \lambda P[P\{j\}]\colon \langle\langle s, \langle e, t\rangle\rangle, t\rangle \\
\text{believes : (t/e)/t} & \Rightarrow & B\colon \langle\langle s, t\rangle, \langle e, t\rangle\rangle \\
\text{a : (t/(t/e))/(t/e)} & \Rightarrow & \lambda P \lambda Q \exists x[P\{x\} \wedge Q\{x\}] \\
 & & :\ \langle\langle s, \langle e, t\rangle\rangle, \langle\langle s, \langle e, t\rangle\rangle, t\rangle\rangle \\
\text{decmocrat : t/e} & \Rightarrow & d\colon \langle e, t\rangle \\
\text{wins : t/e} & \Rightarrow & w\colon \langle e, t\rangle
\end{array}$$

(i) An imaginary decmocrat would win.

$$\dfrac{d}{\langle e,t\rangle} \quad\Rightarrow\quad \dfrac{{}^\wedge d}{\langle s,\langle e,t\rangle\rangle}$$

$$\dfrac{w}{\langle e,t\rangle} \quad\Rightarrow\quad \dfrac{{}^\wedge w}{\langle s,\langle e,t\rangle\rangle}$$

$$\dfrac{\lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}] \quad {}^\wedge d}{\langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle \quad \langle s,\langle e,t\rangle\rangle} \quad\Rightarrow\quad \lambda Q\exists x[d(x)\wedge Q\{x\}] \;\; \langle\langle s,\langle e,t\rangle\rangle,t\rangle$$

$$\lambda Q\exists x[d(x)\wedge Q\{x\}]\;\langle\langle s,\langle e,t\rangle\rangle,t\rangle \quad {}^\wedge w\;\langle s,\langle e,t\rangle\rangle \quad\Rightarrow\quad \exists x[d(x)\wedge w(x)] \;\; t$$

$$\exists x[d(x)\wedge w(x)]\;t \quad\Rightarrow\quad {}^\wedge\exists x[d(x)\wedge w(x)] \;\; \langle s,t\rangle$$

$$B\;\langle\langle s,t\rangle,\langle e,t\rangle\rangle \quad {}^\wedge\exists x[d(x)\wedge w(x)]\;\langle s,t\rangle \quad\Rightarrow\quad B({}^\wedge\exists x[d(x)\wedge w(x)]) \;\; \langle e,t\rangle$$

$$B({}^\wedge\exists x[d(x)\wedge w(x)])\;\langle e,t\rangle \quad\Rightarrow\quad {}^\wedge B({}^\wedge\exists x[d(x)\wedge w(x)]) \;\; \langle s,\langle e,t\rangle\rangle$$

$$\lambda P[P\{a\}]\;\langle\langle s,\langle e,t\rangle\rangle,t\rangle \quad {}^\wedge B({}^\wedge\exists x[d(x)\wedge w(x)])\;\langle s,\langle e,t\rangle\rangle \quad\Rightarrow\quad B({}^\wedge\exists x[d(x)\wedge w(x)])(a) \;\; t$$

(ii) The decmocrat on the TV would win.

$$\dfrac{\begin{matrix} w \\ \langle e,t\rangle \end{matrix}}{\begin{matrix} {}^\wedge w \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} \lambda P[P\{x_5\}] \\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle \end{matrix} \qquad \begin{matrix} {}^\wedge w \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}{\begin{matrix} w(x_5) \\ t \end{matrix}}$$

$$\dfrac{\begin{matrix} w(x_5) \\ t \end{matrix}}{\begin{matrix} {}^\wedge w(x_5) \\ \langle s,t\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} {}^\wedge w(x_5) \\ \langle s,t\rangle \end{matrix} \qquad \begin{matrix} B \\ \langle\langle s,t\rangle,\langle e,t\rangle\rangle \end{matrix}}{\begin{matrix} B({}^\wedge w(x_5)) \\ \langle e,t\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} B({}^\wedge w(x_5)) \\ \langle e,t\rangle \end{matrix}}{\begin{matrix} {}^\wedge B({}^\wedge w(x_5)) \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} \lambda P[P\{a\}] \\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle \end{matrix} \qquad \begin{matrix} {}^\wedge B({}^\wedge w(x_5)) \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}{\begin{matrix} B({}^\wedge w(x_5))(a) \\ t \end{matrix}}$$

$$\dfrac{\begin{matrix} B({}^\wedge w(x_5))(a) \\ t \end{matrix}}{\begin{matrix} \lambda z[B({}^\wedge w(z))(a)] \\ \langle e,t\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} \lambda z[B({}^\wedge w(z))(a)] \\ \langle e,t\rangle \end{matrix}}{\begin{matrix} {}^\wedge \lambda z[B({}^\wedge w(z))(a)] \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} d \\ \langle e,t\rangle \end{matrix}}{\begin{matrix} {}^\wedge d \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} \lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}] \\ \langle\langle s,\langle e,t\rangle\rangle,\langle\langle s,\langle e,t\rangle\rangle,t\rangle\rangle \end{matrix} \qquad \begin{matrix} {}^\wedge d \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}{\begin{matrix} \lambda Q\exists x[d(x)\wedge Q\{x\}] \\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle \end{matrix}}$$

$$\dfrac{\begin{matrix} \lambda Q\exists x[d(x)\wedge Q\{x\}] \\ \langle\langle s,\langle e,t\rangle\rangle,t\rangle \end{matrix} \qquad \begin{matrix} {}^\wedge \lambda z[B({}^\wedge w(z))(a)] \\ \langle s,\langle e,t\rangle\rangle \end{matrix}}{\begin{matrix} \exists x[d(x)\wedge B({}^\wedge w(x))(a)] \\ t \end{matrix}}$$

Ex. John seeks a friend.

$$\begin{array}{lcl}
\text{John} : \text{t}/(\text{t}/\text{e}) & \Rightarrow & \lambda P[P\{j\}] : \langle\langle s, \langle e, t\rangle\rangle, t\rangle \\
\text{seeks} : (\text{t}/\text{e})/(\text{t}/(\text{t}/\text{e})) & \Rightarrow & \mathit{seek} : \langle\langle s, \langle\langle s, \langle e, t\rangle\rangle, t\rangle\rangle, \langle e, t\rangle\rangle \\
\text{a} : (\text{t}/(\text{t}/\text{e}))/(\text{t}/\text{e}) & \Rightarrow & \lambda P \lambda Q \exists x[P\{x\} \wedge Q\{x\}] \\
 & & : \langle\langle s, \langle e, t\rangle\rangle, \langle\langle s, \langle e, t\rangle\rangle, t\rangle\rangle \\
\text{friend} : \text{t}/\text{e} & \Rightarrow & f : \langle e, t\rangle
\end{array}$$

- He needs someone to talk with.

$$\dfrac{\lambda P[P\{j\}] \quad \dfrac{seek \quad \dfrac{\lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}] \quad f}{\lambda Q\exists x[f(x)\wedge Q\{x\}]}}{seek(\hat{}\lambda Q\exists x[f(x)\wedge Q\{x\}])}}{seek(\hat{}\lambda Q\exists x[f(x)\wedge Q\{x\}])(j)}$$

- Where is he hiding?

$$\dfrac{\dfrac{\lambda P\lambda Q\exists x[P\{x\}\wedge Q\{x\}] \quad f}{\lambda Q\exists x[f(x)\wedge Q\{x\}]} \quad \dfrac{\lambda P[P\{j\}] \quad \dfrac{seek \quad \lambda P[P\{x_0\}]}{seek(\hat{}\lambda P[P\{x_0\}])}}{seek(\hat{}\lambda P[P\{x_0\}])(j)}}{\exists x[f(x)\wedge seek(\hat{}\lambda P[P\{x\}])(j)]}$$

- John seeks a wallet/unicorn.

Anna believes that a Democrat would win.

- $B_a\hat{}\exists x[democrat(x) \wedge win(x)]$
- $\exists x[democrat(x) \wedge B_a\hat{}win(x)]$
- $\exists x[B_a\hat{}(democrat(x) \wedge win(x))]$

Discourse Representation Theory

- partiality of information
- dynamics: the succeeding sentence affects on the preceding one.

DRS (Discourse Representation Structure)

discourse referent
condition_1 condition_2 condition_3 $\vdots$

Anaphora resolution

"Jones owns Ulysses. It fascinates him."

x,y
Jones(x)
x owns y
Ulysses(y)

$\rightarrow$

x,y,u,v
Jones(x)
x owns y
Ulysses(y)
u fascinates v

$\rightarrow$

x,y,u,v
Jones(x)
x owns y
Ulysses(y)
x = v
y = u
u fascinates v

If-structure

"Jones teaches linguistics. If he owns a book on semnantics, then he uses it."

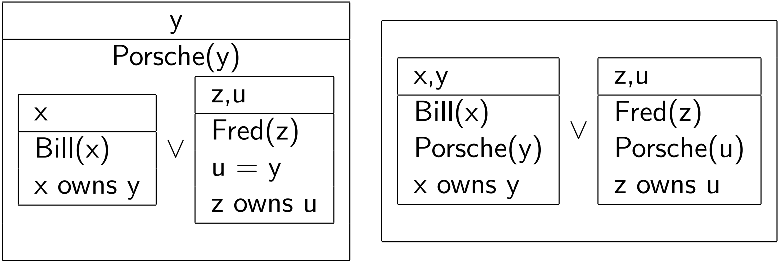
x, y		
Jones(x) linguistics(y) x teaches y		
u,z --- z=x book on semantics(u) z owns u	$\Rightarrow$	v,w --- v=x w=u v uses w

External referents 'x' can be accessible from internal structures. Also, 'u' and 'z' can be referred through '$\Rightarrow$'.

Deictic/anaphoric readings

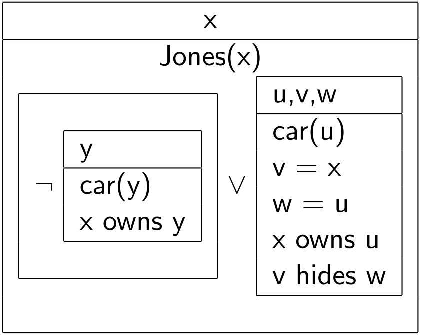
"Bill owns a Porsche or Fred owns it."

- deictic reading – refers what exists in the world.
- anaphoric reading – refers the preceding word, not necessarily an object in the world.



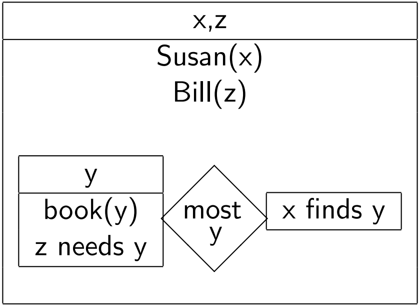
‘or’ and negation

“Either Jones doesn’t own a car or else he hides it.”



Generalized Quantifier

"Susan has found most books which Bill needs."



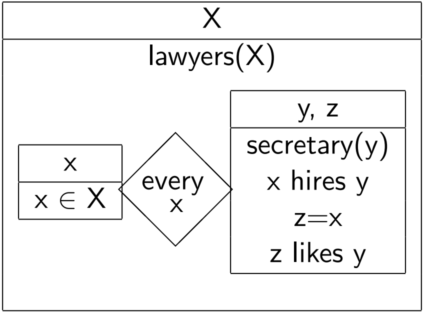
left: restrictor, to supply candidate set
mid (◊) : quantifier
right: scope

$[\![$ restrictor referent $]\!] \supseteq [\![$ scope referent $]\!]$.

'y' can be referred through ◊.

Distributive readings

"The lawyers hired a secretary they liked."
"Each lawyer employed a secretary, respectively."



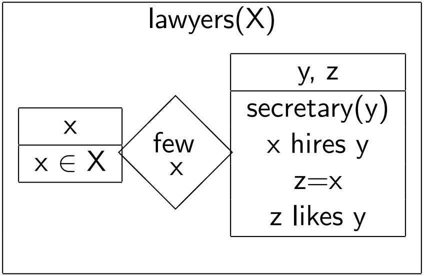
Upper case: abstraction (non-atomic object)

Collective readings

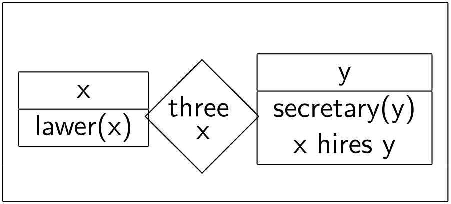
"A group of lawyers employed a common secretary."

X,y,U
lawyers(X), secretary(y), X hires y
U=X
U likes y

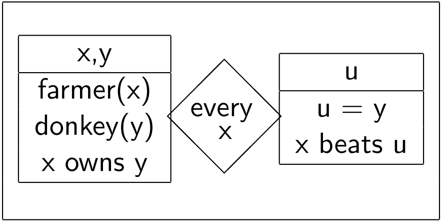
"Few lawyers hired a secretary they liked." – cannot be read collective.



"Three lawyers hired a new secretary."



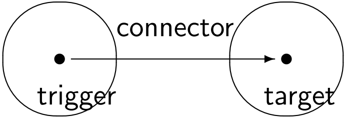
"Every farmer who owns a donkey beats it."



"Three students drank three bottles of beer."

Mental Space

We can refer ‘target’ by ‘trigger’ if they are mapped by ‘connector.’



Metonymy

- "Plato is on the top shelf."
- "The mushroom omelet left without paying the bill."
- "Plato is on the top shelf. *It* is bound in leather."
- "Plato is on the top shelf. *He* is a very interesting author."
  — Open connector: both target and trigger can be a pronoun.
- *"The mushroom omelet was too spicy. It left without paying."
  — Closed connector: only the target can be a pronoun.
- "Norman Mailer likes to read himself."
- *"Norman Mailer likes to read itself."
- 'Norman Mailer is not, in itself, a great dissertation topic."

Mother/daughter space

Although $\subset$ is defined in the mother space and the daughter space, $a \in M,\ M \subset N$ does not necessarily imply $a \in N$.
"Max believes$_M$ that in Len's picture$_M$, the flowers are yellow."

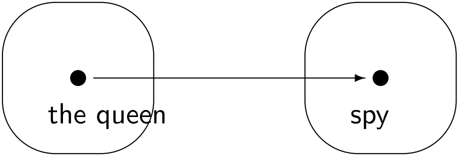
$$R(\text{speaker's real world}) \supset M' \supset M$$

mother	daughter	connector
R	"he thinks"	from 'real world' to 'belief'
R	"in Len's picture"	from the model to the picture
R	"in that movie"	from the actor to the movie world

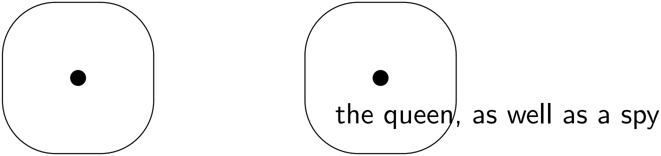
Opaque/transparent reading

"Oedipus believes that the queen of Thebes is a spy."

- transparent reading

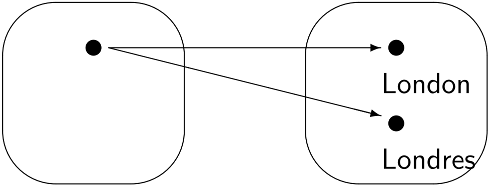


- opaque reading

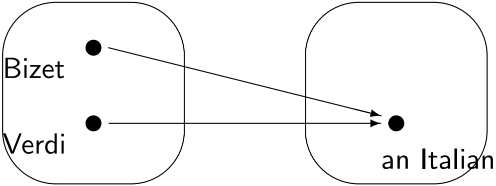


Pierre's Londres, Vizet and Verdi

"Pierre believes that Londres is pretty but London is ugly."



"If Bizet and Verdi had been compatriots, Bizet would have been Italian/ Verdi would have been French."



Role and attribute

"George thinks the winner will go to Hong Kong."

space introduction (M)	*"George thinks"*
attribute (P)	*"will go to Hong Kong."*
role (r)	*the winner*
connector (F)	*maps from R to M*

- P(r): George doesn't know who is the winner. Only he knows the winner will go to HK.
- P(r(M)): Someone George knows is the winner in George's mind, and he will go to HK.
- P(F(r(R))): George doesn't know if 'he' is the winner, though 'he' is actually the winner.

Role and attribute – cont'd

"In 1961, the president was a baby."

space introduction (M)	*"In 1961"*
attribute (P)	*"was a baby."*
role (r)	*the president*
connector (F)	*maps from R to M*

- P(r): In 1961, a president was elected from babies.
- P(r(M)): In 1961, a certain baby was the president.
- P(F(r(R))): The current president was a baby in 1961.

Role and attribute – cont'd

A role can be a label/a name used in metonymy while an attribute is not. But, in translation into first-oder logic, both of them become predicates.

cf. Anna believes that a Democrat would win.

attribute (P)	*'will win.'*
role (r)	*'Democrat'*

$$M \Vdash \exists x[democrat(x) \land win(x)])$$
$$\Rightarrow M \Vdash \exists(x : democrat)[win(x)])$$
$$\Rightarrow M \Vdash \exists x[win(democrat(x))])$$

Hob-Nob sentence

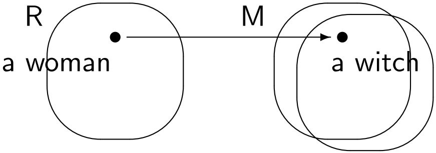
"Hob thinks$_{M_1}$ a witch has killed Bob's horse, and Nob believes$_{M_2}$ that she killed Cob's pig." (Geach 1972)

- transparent reading: a certain person in R killed the pic and the horse.
- there exists a certain person in M_1, and the correspondent person exists in M_2
- a person whose role is 'witch' exists in M_1, and a person of the same role exists also in M_2.

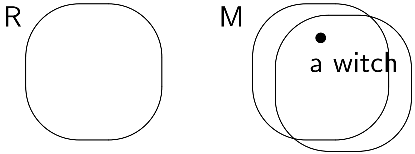
"Everybody believes that a witch killed the horse."

- whether 'witch' is in belief or not.
- whether is 'every' collective or distributive.
- whether is 'witch' a role or a person, while the role is common in every mental space.

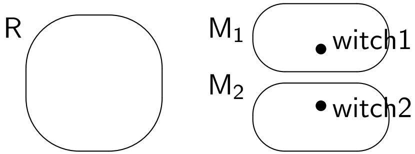
(a) A woman exists, and everyone believes that she is a witch. The person who killed the horse is she.



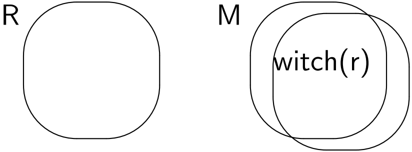
(b) Everyone believes that there exists a witch, whose image and features are common among them. The witch killed horses.



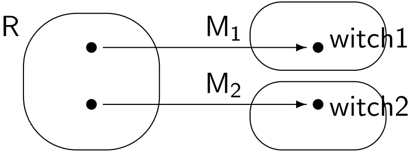
(c) Hob believes that a witch is a cute girl with short hair. He believes that the girl killed the horse. Nob believes that a witch is an ugly old woman. She committed the crime.



(d) Everyone believes in witch; but none has seen her, and it is an abstract existence. Everyone just regards that the ominous incident is caused by a witch.



(e) There are two women called Hilda and Brünhilde. Hob believes that Hilda killed the horse, and Nob believes that Brünhilde did it.



Non-monotonic Logic

- Default Logic & Extension [R. Reiter (1980). A logic for default reasoning. Artificial Intelligence, 13:81-132]
- Predicate Completion & Circumscription [J. McCarthy (1980). Circumscription - A form of non-monotonic reasoning. Artificial Intelligence, 13:27-39.]
- Belief Revision [P. Gardenfors (1988). Knowledge in Flux. The MIT Press.]
- Defeasible reasoning [H. Prakken(1997). Logical Tools for Modelling Legal Argument. Kluwer Academic Press.]

Knowledge and Belief

$K_a\varphi$	agent a knows φ
$B_a\varphi$	agent a believes φ

Logic of K

1. $K_a\varphi \rightarrow \varphi$ (T)
2. $K_a\varphi \rightarrow K_aK_a\varphi$ (4)
3. $\neg K_a\varphi \rightarrow K_a\neg K_a\varphi$ (5)

Logic of B

1. $B_a\varphi \rightarrow \neg B_a\neg\varphi$ (D)
2. $B_a\varphi \rightarrow B_aB_a\varphi$ (4)
3. $\neg B_a\varphi \rightarrow B_a\neg B_a\varphi$ (5)

BDI logic

- $B_a \forall x[bordeaux(x) \rightarrow mellow(x)]$
- $B_a B_b[bordeaux(c) \wedge mellow(c)]$
- $B_a \forall j[snob(j) \rightarrow B_j \forall y[bordeaux(y) \rightarrow mellow(y)]]$
- $B_a \forall j D_j \neg I_a president(a)$
- $B_a \forall j \neg D_j I_a president(a)$
- $B_a \exists x[bordeaux(x) \wedge mellow(x)]$
- $\exists x[bordeaux(x) \wedge B_a mellow(x)]$
- $\exists x B_a[bordeaux(x) \wedge mellow(x)]$

Future

- Noam Chomsky: Generative Grammar
- Statistical method: machine learning via the Internet
- Exodus from the Galapaos Islands
- Logic of Name, Label, and Individual constants