An Analysis of Graphical Meanings in Channel Theory

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Mathematical Logic:
Development and Evolution into Various Sciences
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Today

- Illustrate “derivative meaning” of graphical representations with one, running example.
- Give an informal account of its logical origins.
- Supply other examples to show its functional significance.
- Specify what are required in formalizing the account and how channel theory meets them.

Overall goal: a proposal of the framework for formal semantics of graphical representations that helps account for their functional characteristics

Example

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

The system $\mathcal{T}$ of iconic tables, designed to express the outcomes of monthly round-robin competitions in a private table-tennis club, with the six members Jon, Bob, Gil, Ken, Ron, and Sue.

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

What information do you read off from this table?

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

What information do you read off from this table?

"Bob defeated Jon."

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

What information do you read off from this table?

"Bob defeated Jon."

"Bob defeated three players."

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

What information do you read off from this table?

"Bob defeated Jon."

"Bob defeated three players."

"Bob defeated more players than Ken did."

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	○	●	●
Ken	●	–	○	●	●	●
Ron	●	●	–	●	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

What information do you read off from this table?

"Bob defeated Jon."

"Bob defeated three players."

"Bob defeated more players than Ken did."

"Gil lost to no players."

Informational relations adopted into our reading routines

A white circle appears to the right of Bob's name, in the position below Ken"s name	means →	Bob defeated Ken
Three white circles appear to the right of Bob's name	means →	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means →	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means →	Gil lost to no players

Informational relations adopted into our reading routines

A white circle appears to the right of Bob's name, in the position below Ken"s name	means →	Bob defeated Ken
Three white circles appear to the right of Bob's name	means →	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means →	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means →	Gil lost to no players

Note: "to the right" = to the right in the direction parallel to the upper axis line.

Informational relations adopted into our reading routines

A white circle appears to the right of Bob's name, in the position below Ken"s name	means →	Bob defeated Ken
Three white circles appear to the right of Bob's name	means →	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means →	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means →	Gil lost to no players

Note: "below" = below in the direction parallel to the left axis line.

Informational relations adopted into our reading routines

A white circle appears to the right of Bob's name, in the position below Ken"s name	means →	Bob defeated Ken
Three white circles appear to the right of Bob's name	means →	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means →	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means →	Gil lost to no players

Informational relations adopted into our reading routines

A white circle appears to the right of Bob's name, in the position below Ken"s name	means	Bob defeated Ken
Three white circles appear to the right of Bob's name	means	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means	Gil lost to no players

Look similar, but.....

Informational relations adopted into our reading routines

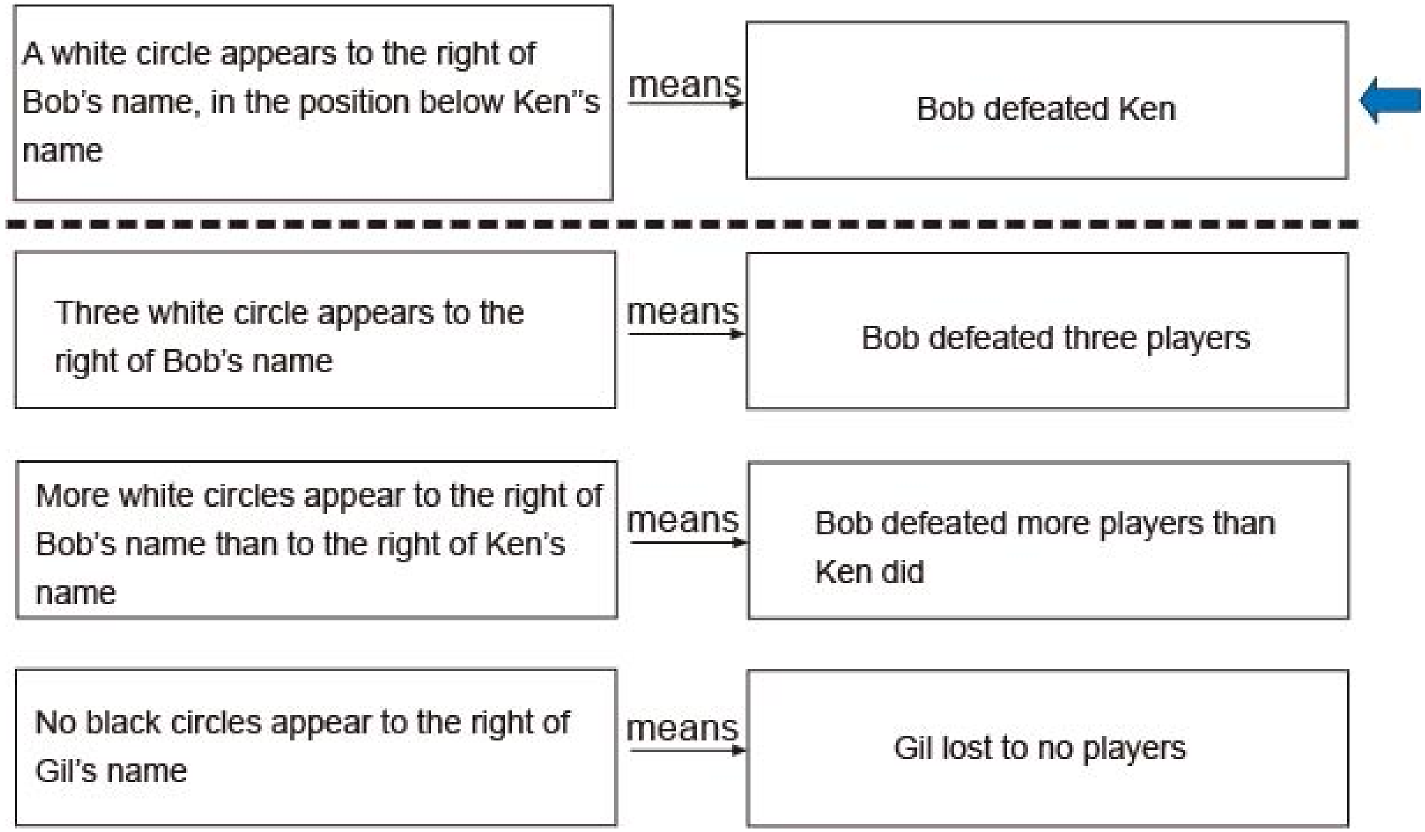
A white circle appears to the right of Bob's name, in the position below Ken"s name	means →	Bob defeated Ken
Three white circles appear to the right of Bob's name	means →	Bob defeated three players
More white circles appear to the right of Bob's name than to the right of Ken's name	means →	Bob defeated more players than Ken did
No black circles appear to the right of Gil's name	means →	Gil lost to no players

...there is an important difference.

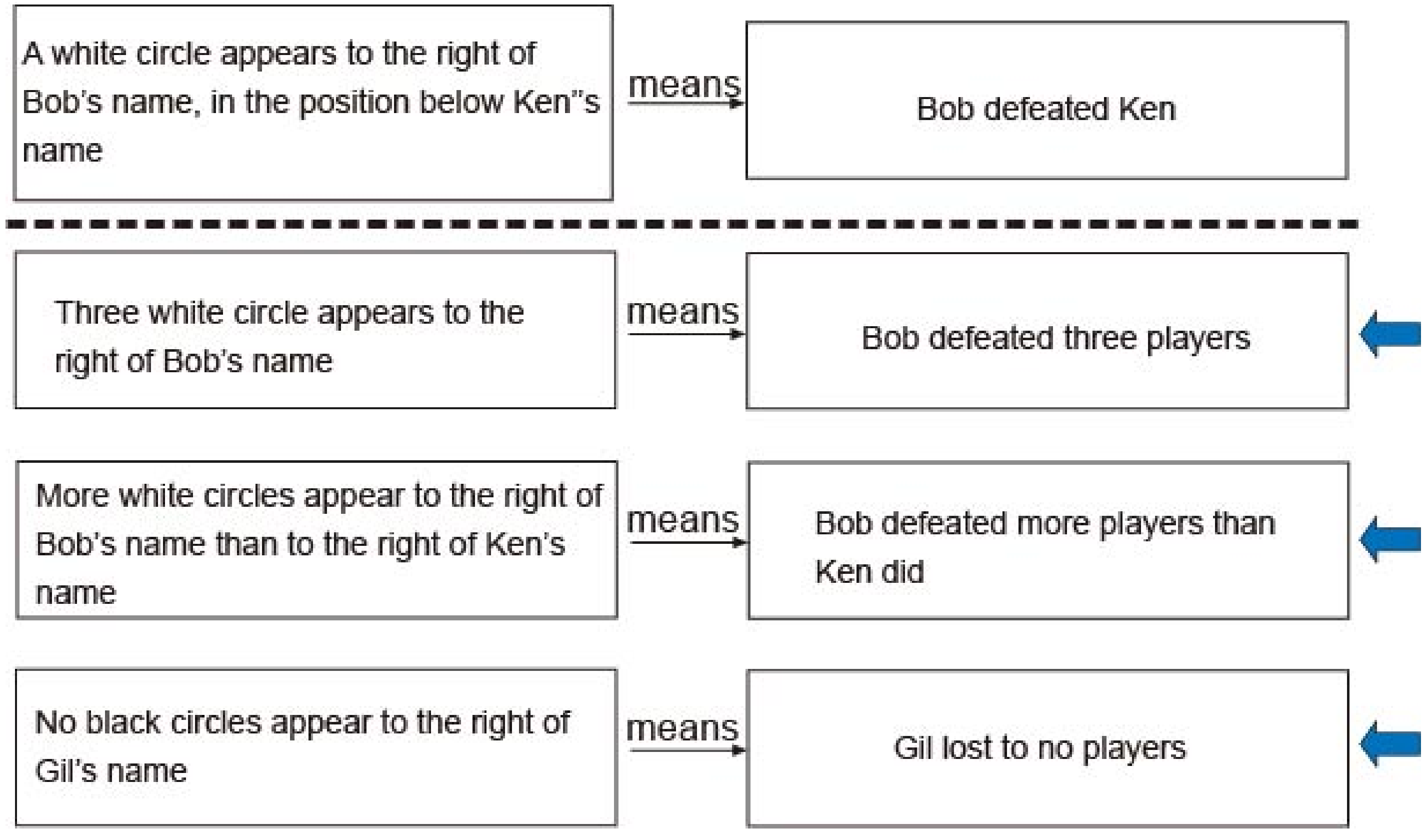
Basic semantic conventions of the system $\mathcal{T}$:

A white circle appears to the right of X's name, in the position below Y's name	means →	X defeated Y
A black circle appears to the right of X's name, in the position below Y's name	means →	X lost to Y

where $X, Y \in \{Bob, Ken, Ron, Jon, Sue, Gil\}$.



So: only this informational relation is implied by the system's basic semantic conventions <u>alone</u>.



The rest is something else...derivative informational relations implied by the basic semantic conventions and something else.

Point 1

A large portion of our reading practice on graphical representations consists of:

- Interpretations of derivative informational relations,
- As opposed to interpretations based on the system's basic semantic conventions.

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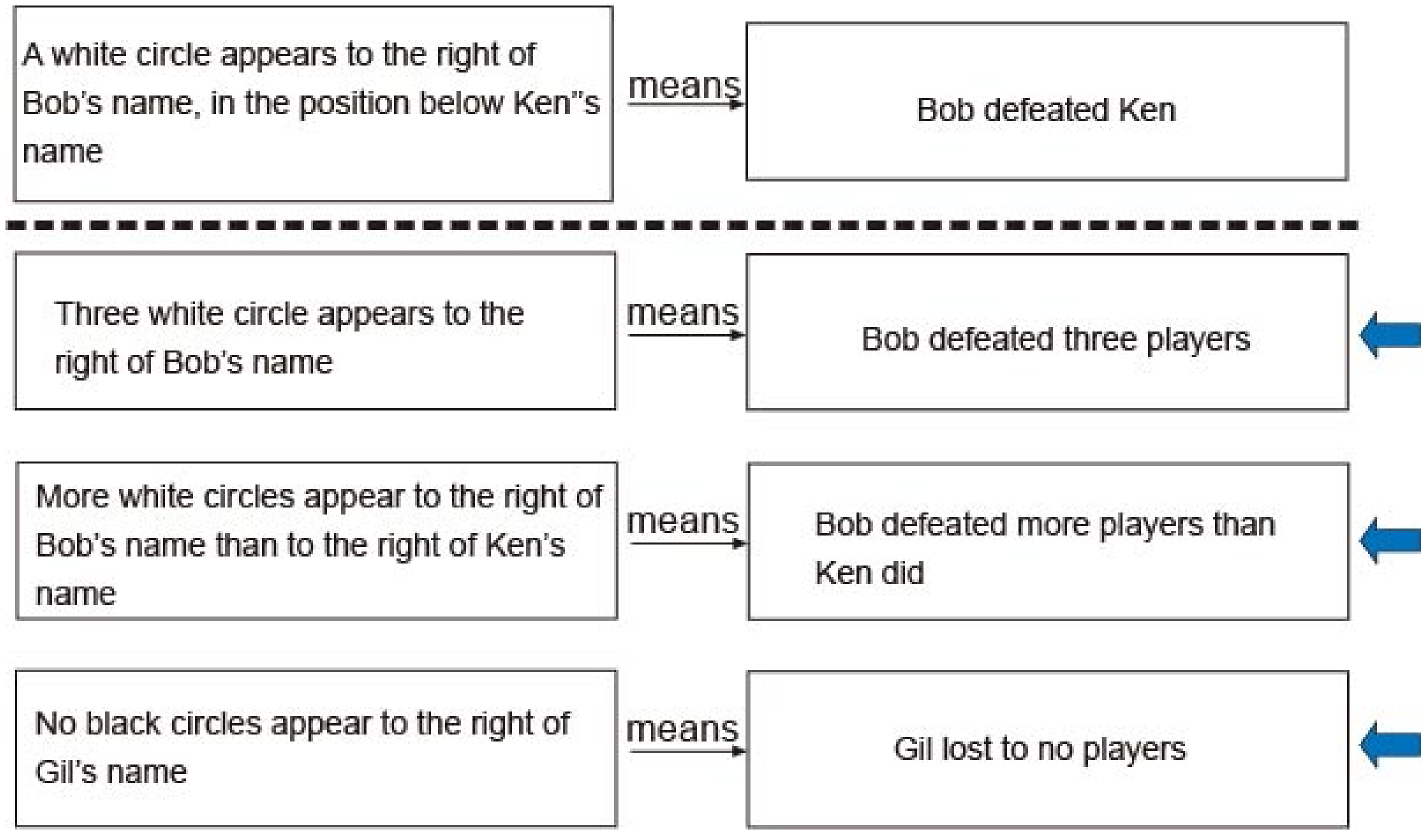
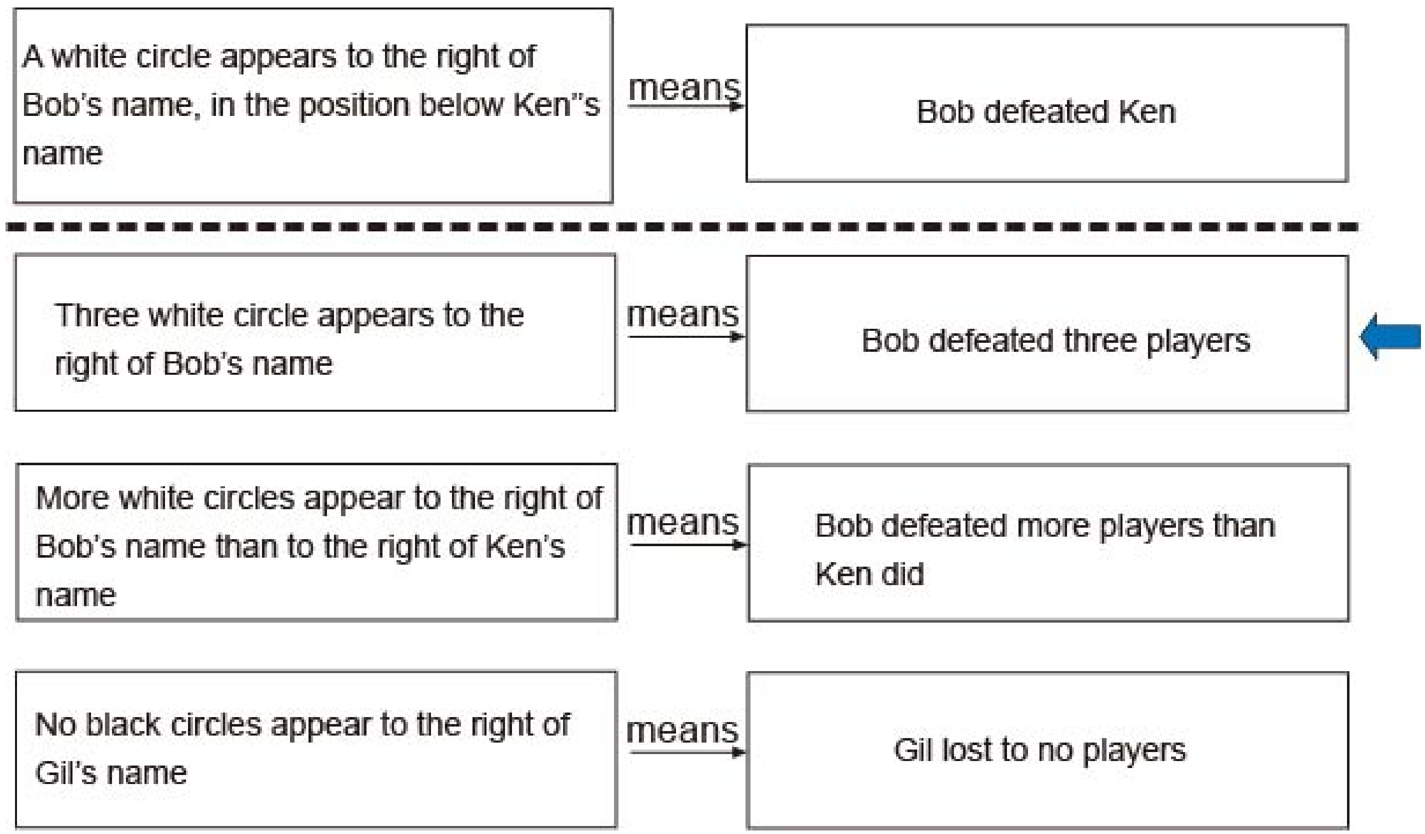
A large portion of our reading practice on graphical representations consists of:

- Interpretations of <u>derivative</u> <u>informational relations</u>,
- As opposed to interpretations based on the system's basic semantic conventions.

Question: How exactly are these "derivative" informational relations derivable from the system's basic semantic conventions? What exactly makes them valid informational relations?



For example, how exactly are these informational relations derivable from the semantic conventions of the system $\mathcal{T}$?



Let us take this for example.

Abbreviations

$\bigcirc(X, Y)$ = the condition that a white circle appears to the right of X's name, in the position below Y's name

$\bullet(X, Y)$ = the condition that a black circle appears to the right of X's name, in the position below Y's name

$D(X, Y)$ = the condition that X defeated Y

$L(X, Y)$ = the condition that X lost to Y

where $X, Y \in \{Bob, Ken, Ron, Jon, Sue, Gil\}$.

Then the semantic conventions of the system $\mathcal{T}$ can be expressed as:

$\bigcirc(X, Y)$ means $D(X, Y)$

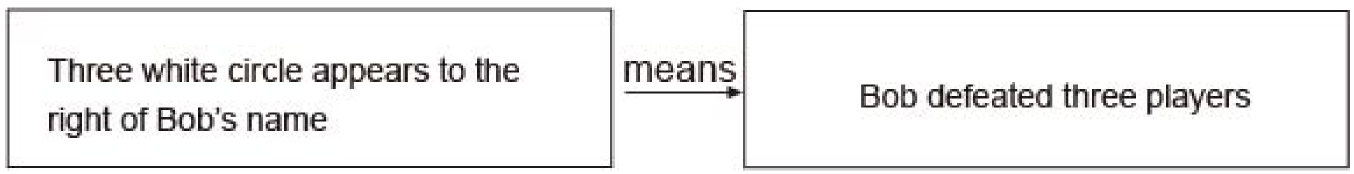
$\bullet(X, Y)$ means $L(X, Y)$

In symbol, let us write:

$\bigcirc(X, Y) \rightharpoonup_{\mathcal{T}} D(X, Y)$

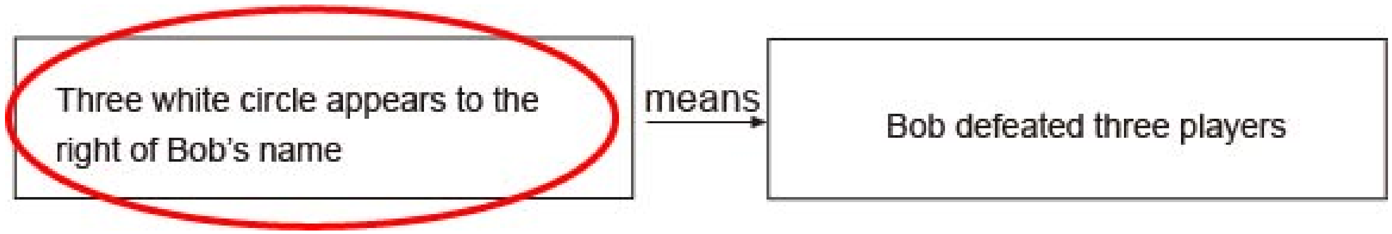
$\bullet(X, Y) \rightharpoonup_{\mathcal{T}} L(X, Y)$

Consider the derivative informational relation:



How is this derivative informational relation derivable from the basic semantic conventions of $\mathcal{T}$?

Consider the left-hand condition:



This is an abstract condition, with several different ways in which it is realized.

> Three white circle appears to the right of Bob's name

Here is one way:

Three white circle appears to the right of Bob's name

$$\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob},\mathrm{Ken}), & \bigcirc(\mathrm{Bob},\mathrm{Ron}), \\ \bigcirc(\mathrm{Bob},\mathrm{Jon}), & \bullet(\mathrm{Bob},\mathrm{Gil}), \\ \bullet(\mathrm{Bob},\mathrm{Sue}) & \end{array} \right\}$$

That is, if all conditions in this set hold, then the top condition holds.

So we write:

Three white circle appears to the right of Bob's name

$$\Uparrow$$

$$\left\{ \begin{array}{ll} \bigcirc(\text{Bob,Ken}), & \bigcirc(\text{Bob,Ron}), \\ \bigcirc(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array} \right\}$$

So we write:

Three white circle appears to the right of Bob's name

$$\Uparrow$$

$$\left\{ \begin{array}{ll} \circ(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array} \right\}$$

Note: Syntactic stipulations of the system $\mathcal{T}$ significantly contribute to the holding of this conditional constraint.

This is prohibited by syntactic stipulations:

	Bob	Ken	Jon Ron	Gil	Sue
Bob	–	○	○	●	●
Ken	●	–	○	●	●
Ron	●	●	–	●	●
Jon	●	○	○	●	●
Gil	○	○	○	–	○
Sue	○	○	○	●	–

This one too:

	Bob	Ken	Ron	Jon	Gil	Sue
Bob	–	○	○	●	●	●
Ken	●	–	○	●	●	●
Bob	●	●	–	○	●	●
Jon	●	○	○	–	●	●
Gil	○	○	○	○	–	○
Sue	○	○	○	○	●	–

Also:

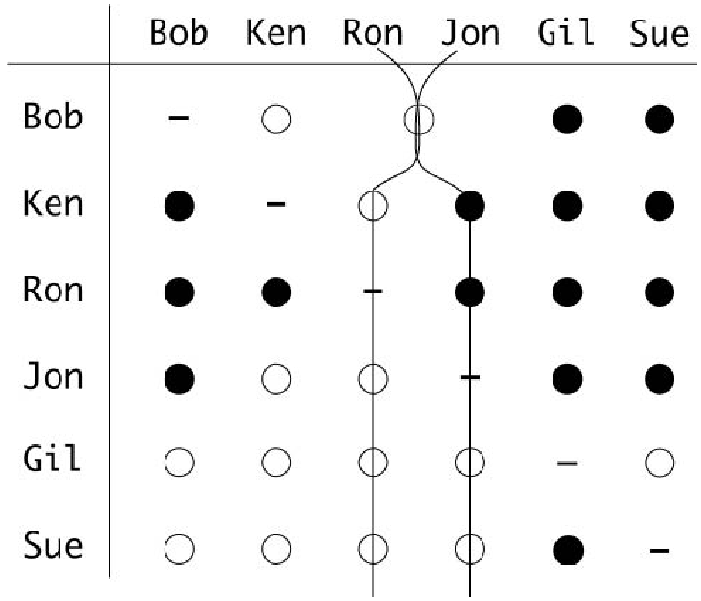
Three white circle appears to the right of Bob's name

$$\Uparrow$$

$$\left\{ \begin{array}{ll} \bigcirc(\text{Bob,Ken}), & \bigcirc(\text{Bob,Ron}), \\ \bigcirc(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array} \right\}$$

Note: spatial constraints on Euclidean planes significantly contribute.

Lines parallel to the left axis passing the center of Ron’s name should be unique:



Three white circle appears to the right of Bob's name

$$\left\{ \begin{array}{ll} \circ(\mathrm{Bob},\mathrm{Ken}), & \circ(\mathrm{Bob},\mathrm{Ron}), \\ \bullet(\mathrm{Bob},\mathrm{Jon}), & \circ(\mathrm{Bob},\mathrm{Gil}), \\ \bullet(\mathrm{Bob},\mathrm{Sue}) & \end{array} \right\}$$

Here is another, slightly different way in which three white circles appear to the right of Bob's name.

$\boxed{\text{Three white circle appears to the right of Bob's name}}$

$$\Uparrow$$

$$\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob},\mathrm{Ken}), & \bigcirc(\mathrm{Bob},\mathrm{Ron}), \\ \bullet(\mathrm{Bob},\mathrm{Jon}), & \bigcirc(\mathrm{Bob},\mathrm{Gil}), \\ \bullet(\mathrm{Bob},\mathrm{Sue}) & \end{array} \right\}$$

Again, a conditional constraint holds partly because of the syntactic formation rules in the system $\mathcal{T}$, supported by spatial constraints on Euclidean planes.

Exhaust all such ways:

Three white circle appears to the right of Bob's name

$$
\left\{
\begin{array}{lll}
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bigcirc(\text{Bob},\text{Ron}),\\ \bigcirc(\text{Bob},\text{Jon}), & \bullet(\text{Bob},\text{Gil}),\\ \bullet(\text{Bob},\text{Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bigcirc(\text{Bob},\text{Ron}),\\ \bullet(\text{Bob},\text{Jon}), & \bigcirc(\text{Bob},\text{Gil}),\\ \bullet(\text{Bob},\text{Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bigcirc(\text{Bob},\text{Ron}),\\ \bullet(\text{Bob},\text{Jon}), & \bullet(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bullet(\text{Bob},\text{Ron}),\\ \bigcirc(\text{Bob},\text{Jon}), & \bigcirc(\text{Bob},\text{Gil}),\\ \bullet(\text{Bob},\text{Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bullet(\text{Bob},\text{Ron}),\\ \bigcirc(\text{Bob},\text{Jon}), & \bullet(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll}\bigcirc(\text{Bob},\text{Ken}), & \bullet(\text{Bob},\text{Ron}),\\ \bullet(\text{Bob},\text{Jon}), & \bigcirc(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\}, \\
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\left\{\begin{array}{ll}\bullet(\text{Bob},\text{Ken}), & \bigcirc(\text{Bob},\text{Ron}),\\ \bigcirc(\text{Bob},\text{Jon}), & \bullet(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll}\bullet(\text{Bob},\text{Ken}), & \bigcirc(\text{Bob},\text{Ron}),\\ \bullet(\text{Bob},\text{Jon}), & \bigcirc(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll}\bullet(\text{Bob},\text{Ken}), & \bullet(\text{Bob},\text{Ron}),\\ \bigcirc(\text{Bob},\text{Jon}), & \bigcirc(\text{Bob},\text{Gil}),\\ \bigcirc(\text{Bob},\text{Sue}) & \end{array}\right\} & &
\end{array}
\right\}
$$

Then:

Three white circle appears to the right of Bob's name

$$\Downarrow$$

$$\left\{ \begin{array}{lll}
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bullet(\mathrm{Bob,Gil}), \\ \bullet(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bullet(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bullet(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bullet(\mathrm{Bob,Jon}), & \bullet(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bullet(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bullet(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bullet(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bullet(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bigcirc(\mathrm{Bob,Ken}), & \bullet(\mathrm{Bob,Ron}), \\ \bullet(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} \bullet(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bullet(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bullet(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bullet(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} \bullet(\mathrm{Bob,Ken}), & \bigcirc(\mathrm{Bob,Ron}), \\ \bullet(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} \bullet(\mathrm{Bob,Ken}), & \bullet(\mathrm{Bob,Ron}), \\ \bigcirc(\mathrm{Bob,Jon}), & \bigcirc(\mathrm{Bob,Gil}), \\ \bigcirc(\mathrm{Bob,Sue}) & \end{array} \right\} & &
\end{array} \right\}$$

Note: this is again partly because of the syntactic formation rules in $\mathcal{T}$.

Three white circle appears to the right of Bob's name

$$\Downarrow$$

$$
\left\{
\begin{array}{lll}
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \bullet(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \bullet(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \bullet(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \bullet(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \circ(\text{Bob,Ken}), & \bullet(\text{Bob,Ron}), \\ \bullet(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} \bullet(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \bullet(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \bullet(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \bullet(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} \bullet(\text{Bob,Ken}), & \circ(\text{Bob,Ron}), \\ \bullet(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} \bullet(\text{Bob,Ken}), & \bullet(\text{Bob,Ron}), \\ \circ(\text{Bob,Jon}), & \circ(\text{Bob,Gil}), \\ \circ(\text{Bob,Sue}) & \end{array}\right\} & &
\end{array}
\right\}
$$

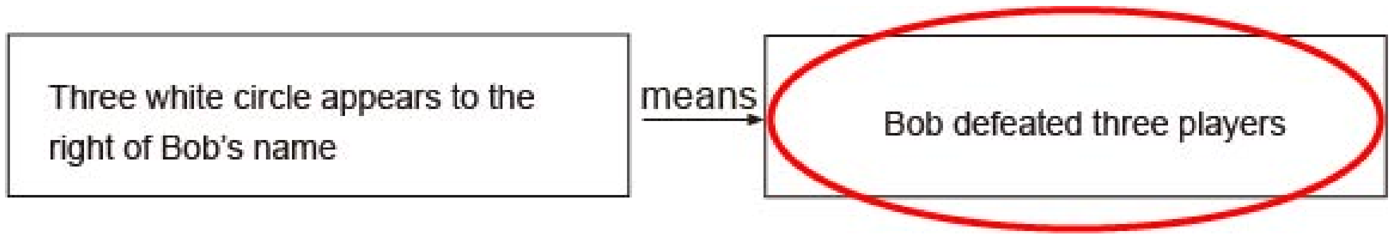
This bi-conditional constraint is called the abstraction relation.

Three white circle appears to the right of Bob's name

⧖

{
{ ○(Bob,Ken), ○(Bob,Ron), ○(Bob,Jon), ●(Bob,Gil), ●(Bob,Sue) }, { ○(Bob,Ken), ○(Bob,Ron), ●(Bob,Jon), ○(Bob,Gil), ●(Bob,Sue) }, { ○(Bob,Ken), ○(Bob,Ron), ●(Bob,Jon), ●(Bob,Gil), ○(Bob,Sue) },
{ ○(Bob,Ken), ●(Bob,Ron), ○(Bob,Jon), ○(Bob,Gil), ●(Bob,Sue) }, { ○(Bob,Ken), ●(Bob,Ron), ○(Bob,Jon), ●(Bob,Gil), ○(Bob,Sue) }, { ○(Bob,Ken), ●(Bob,Ron), ●(Bob,Jon), ○(Bob,Gil), ○(Bob,Sue) },
{ ●(Bob,Ken), ○(Bob,Ron), ○(Bob,Jon), ○(Bob,Gil), ●(Bob,Sue) }, { ●(Bob,Ken), ○(Bob,Ron), ○(Bob,Jon), ●(Bob,Gil), ○(Bob,Sue) }, { ●(Bob,Ken), ○(Bob,Ron), ●(Bob,Jon), ○(Bob,Gil), ○(Bob,Sue) },
{ ●(Bob,Ken), ●(Bob,Ron), ○(Bob,Jon), ○(Bob,Gil), ○(Bob,Sue) }
}

Now consider the right-hand condition:



This is again an abstract condition, with several different ways in which it is realized.

Bob defeated three players

Bob defeated three players

$$\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}$$

Here is one way—if all conditions in this set hold, then the top condition holds.

So we write:

$$\boxed{\text{Bob defeated three players}}$$

$$\Uparrow$$

$$\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}$$

So we write:

$$\boxed{\text{Bob defeated three players}}$$

$$\Uparrow$$

$$\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}$$

Note: this conditional constraint holds because of the game regulations in Jon’s private table-tennis club, concerning participating players and game matchings that make up round-robin competitions.

Here is another, slightly different way:

Bob defeated three players

$$\left\{ \begin{array}{ll} D(\text{Bob},\text{Ken}), & D(\text{Bob},\text{Ron}), \\ L(\text{Bob},\text{Jon}), & D(\text{Bob},\text{Gil}), \\ L(\text{Bob},\text{Sue}) & \end{array} \right\}$$

Bob defeated three players

$$\Uparrow$$

$$\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}$$

Again, a conditional constraint holds because of the game regulations in Jon's private table-tennis club.

Exhaust all such ways:

Bob defeated three players

$$
\left\{
\begin{array}{lll}
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\}, &
\left\{ \begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\}, \\
\left\{ \begin{array}{ll} L(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array} \right\} & &
\end{array}
\right\}
$$

Then again:

Bob defeated three players

$$\Downarrow$$

$$
\left\{
\begin{array}{lll}
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\} & &
\end{array}
\right\}
$$

Note: this is partly because of the game regulations in Jon's private table-tennis club.

Bob defeated three players

$$\Downarrow$$

$$\left\{\begin{array}{lll}
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\} & &
\end{array}\right\}$$

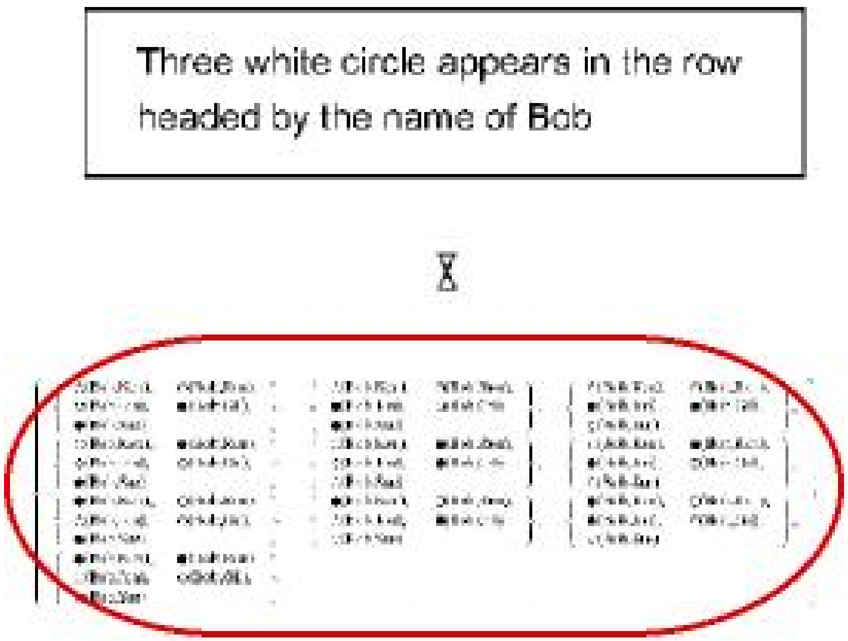
Thus we see the abstraction relation again.

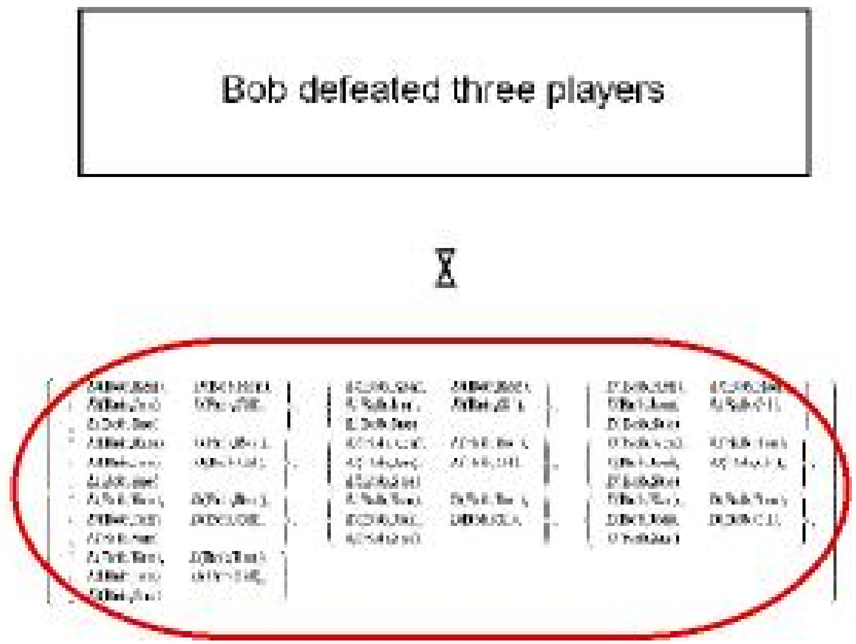
Bob defeated three players

⧖

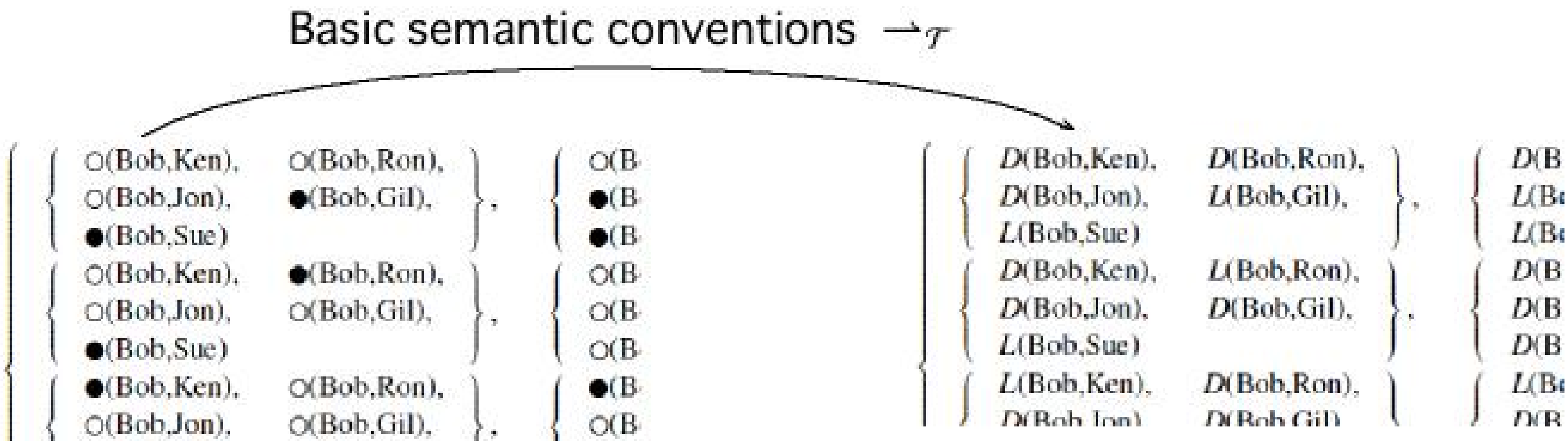
$$\left\{\begin{array}{lll}
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} D(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ L(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & L(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, &
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & D(\text{Bob,Ron}), \\ L(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\}, \\
\left\{\begin{array}{ll} L(\text{Bob,Ken}), & L(\text{Bob,Ron}), \\ D(\text{Bob,Jon}), & D(\text{Bob,Gil}), \\ D(\text{Bob,Sue}) & \end{array}\right\} & &
\end{array}\right\}$$

Finally, compare the two big collections that are abstracted over:

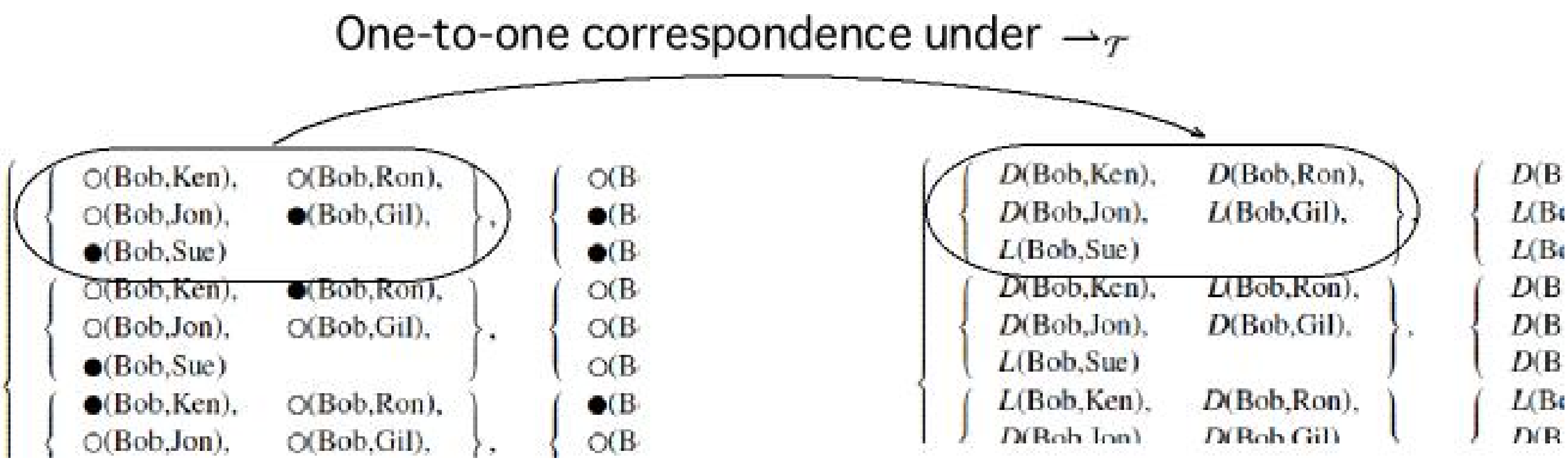




Zoom-in view of the two collections



Zoom-in view of the two collections

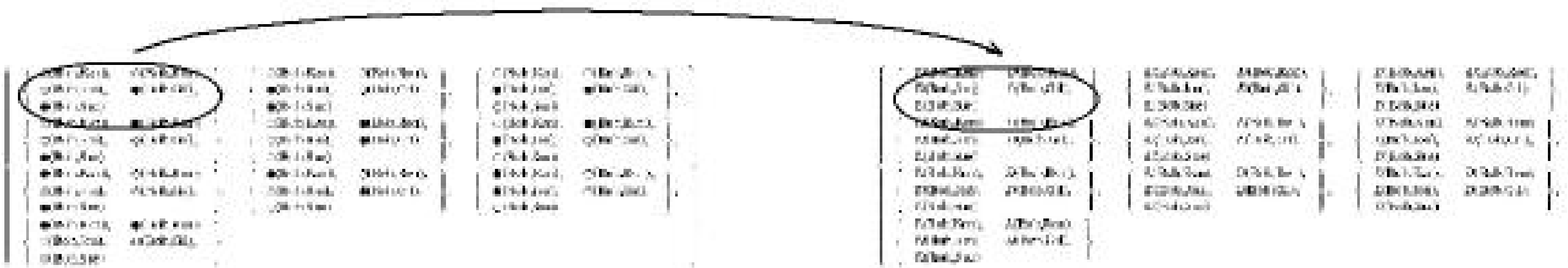


Zoom-out view of the two collections

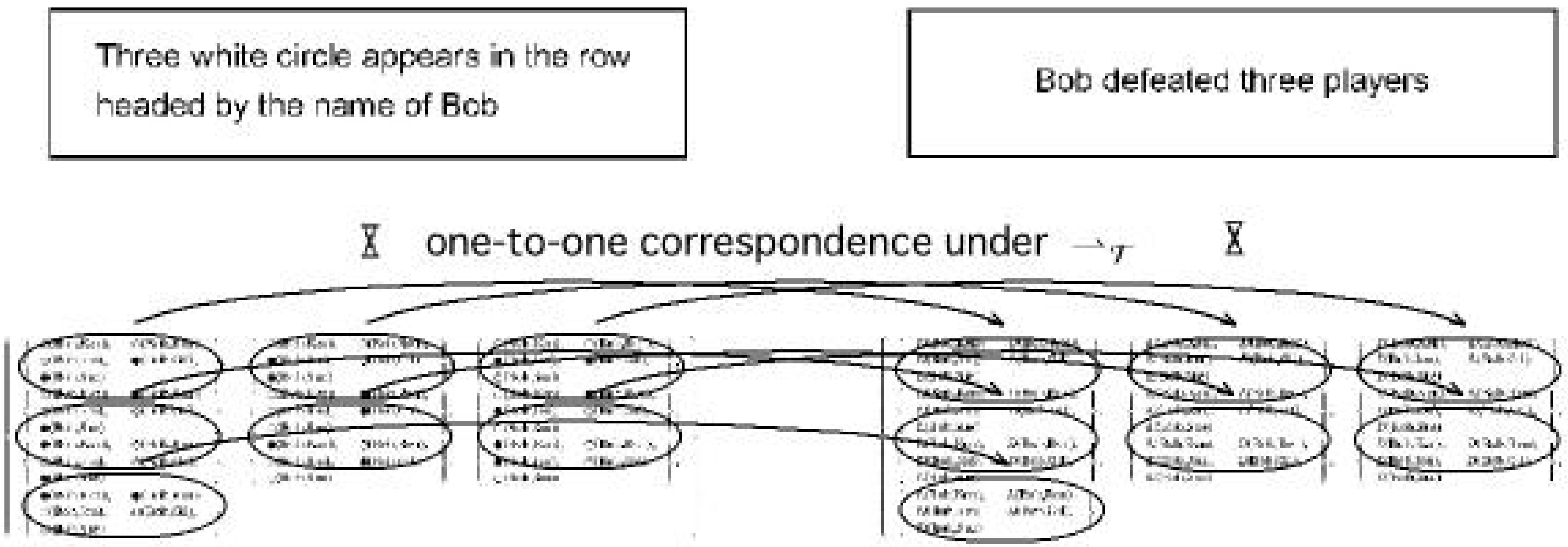
Three white circle appears in the row headed by the name of Bob

Bob defeated three players

⧗ one-to-one correspondence under $\rightarrow_T$ ⧗



Zoom-out view of the two collections

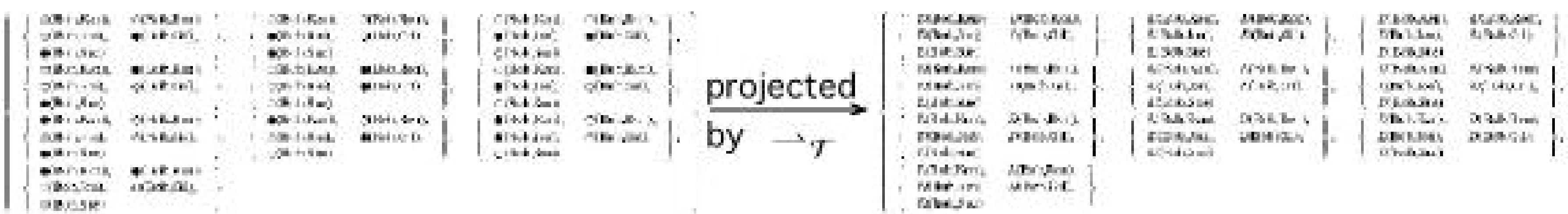


The left-hand collection is projected to the right-hand collection by the system's semantic conventions $\rightharpoonup_{\mathcal{T}}$.

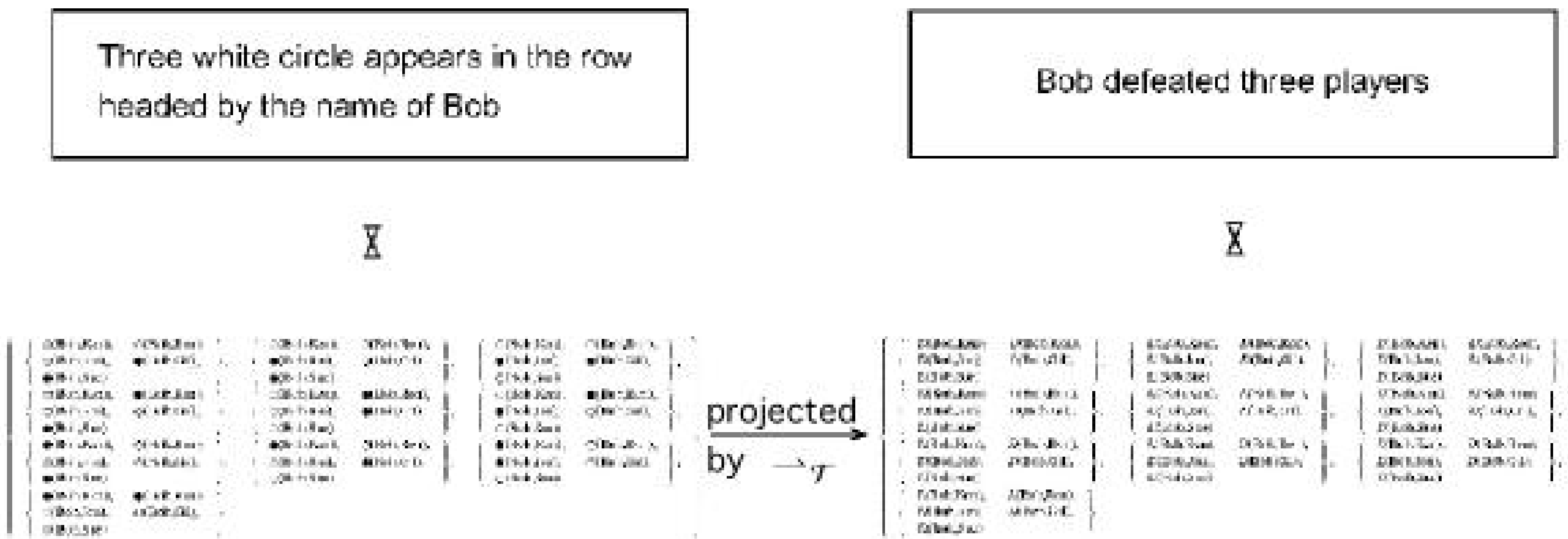
So we write:

Three white circle appears in the row headed by the name of Bob

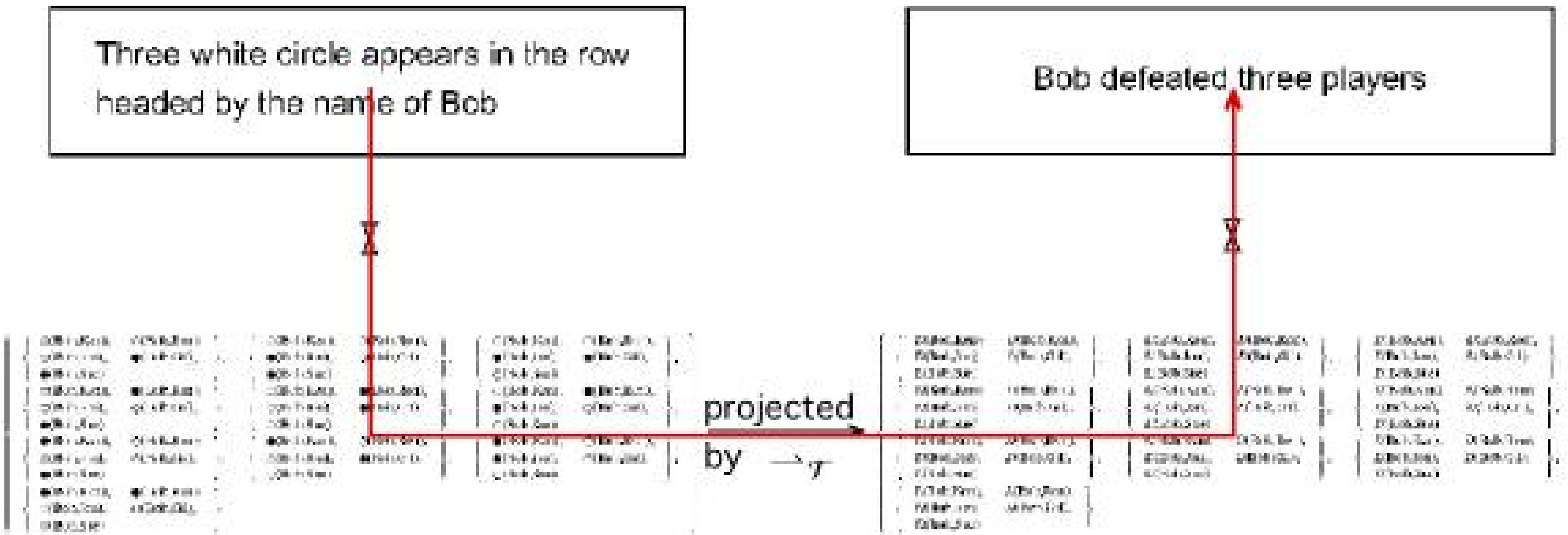
Bob defeated three players



Overall, we see the parallel abstraction relation holds, mediated by the system's semantic conventions $\rightharpoonup_{\mathcal{T}}$.



This accounts for the informational relation from the top-left condition to the top-right condition:



Point 2

Question: How exactly are these “derivative” informational relations derivable from the system’s basic semantic conventions?

Answer: Two separate systems of constraints contribute to the derivation:

- The source logic: the system of constraints governing the formation of graphical structures,

- The target logic: the system of constraints governing the represented objects.

Point 2

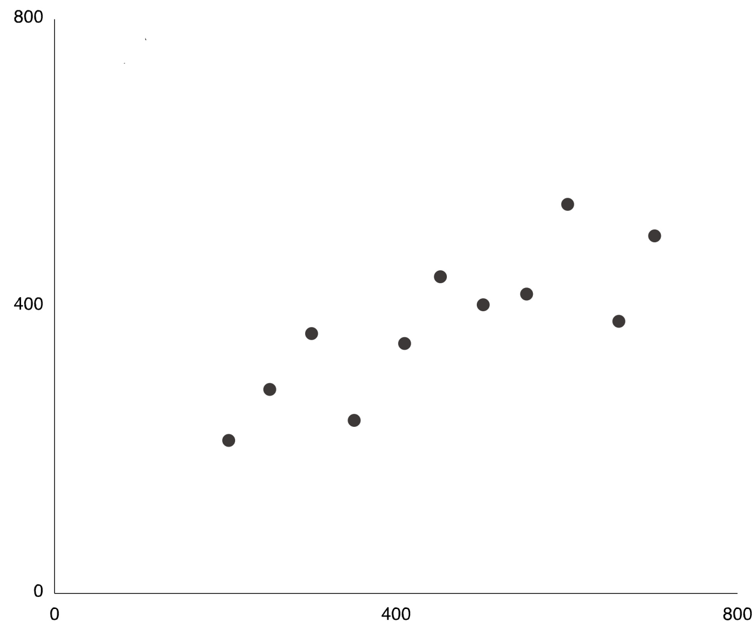
$$\begin{array}{ccc} \alpha' & & \alpha \\ \bowtie_{\mathcal{T}_S} & & \bowtie_{\mathcal{T}_T} \\ \mathcal{G} & \xrightarrow[\text{by } \rightharpoonup_{\mathcal{L}}]{\text{projected}} & \mathcal{D} \end{array}$$

Point 3

More specifically, those derivative informational relations are the result of one system of constraints is aligned to the other system to make the parallel abstraction relation between the relevant pairs of conditions.

$$
\begin{tikzcd}
\alpha' & & \alpha \\
\mathcal{G} \arrow[u, phantom, "\bowtie_{\mathcal{L}_S}"] \arrow[rr, "\text{projected}"', "\text{by } \rightharpoonup_{\mathcal{L}}"] & & \mathcal{D} \arrow[u, phantom, "\bowtie_{\mathcal{L}_T}"]
\end{tikzcd}
$$

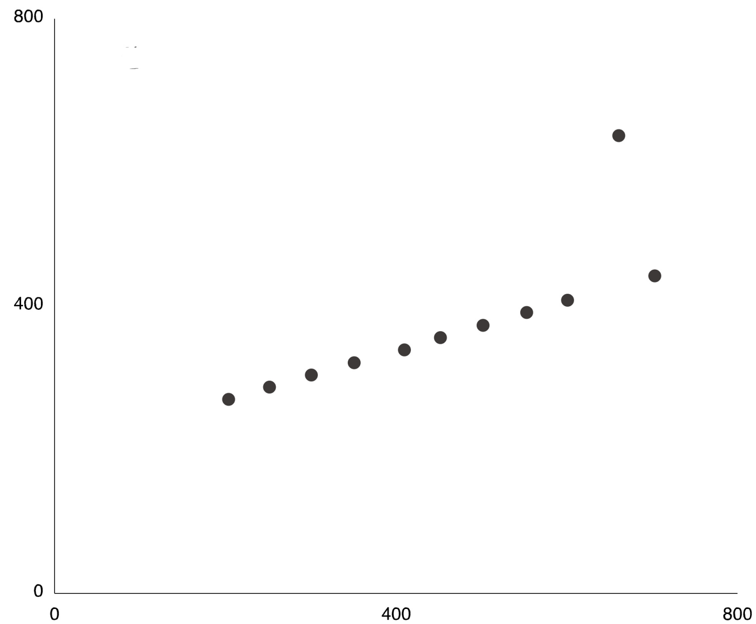
Other examples: scatter plots



Basic meaning: individual data values indicated by individual dots.

Derivative meaning: correlation of two variables indicated by the "cloud" of dots (Tufte 1994, Kosslyn 1994).

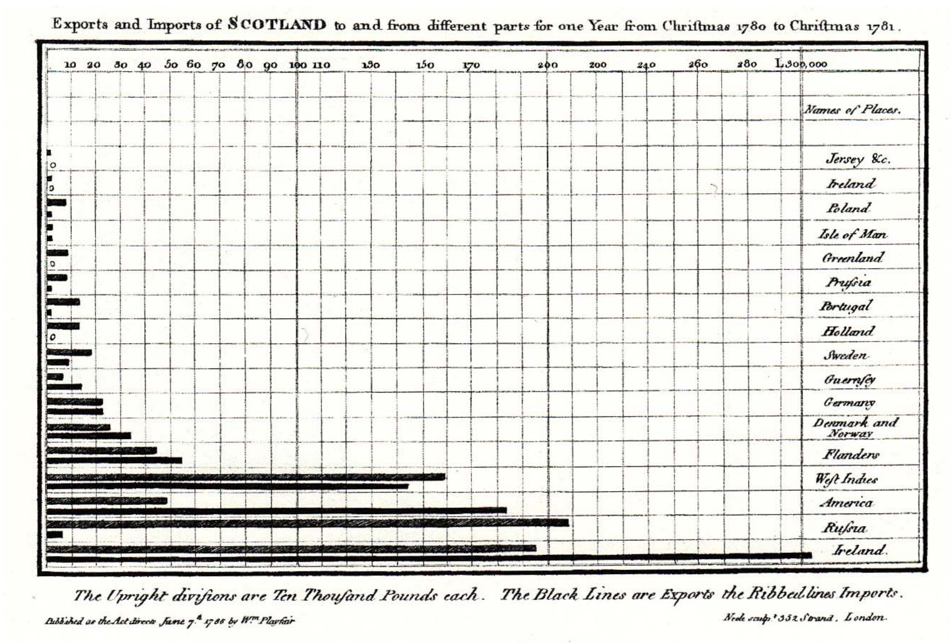
Other examples: scatter plots



Basic meaning: individual data values indicated by individual dots.

Derivative meaning: existence of an outlier indicated by the relation of a dot to the group of other dots (Tufte 1994).

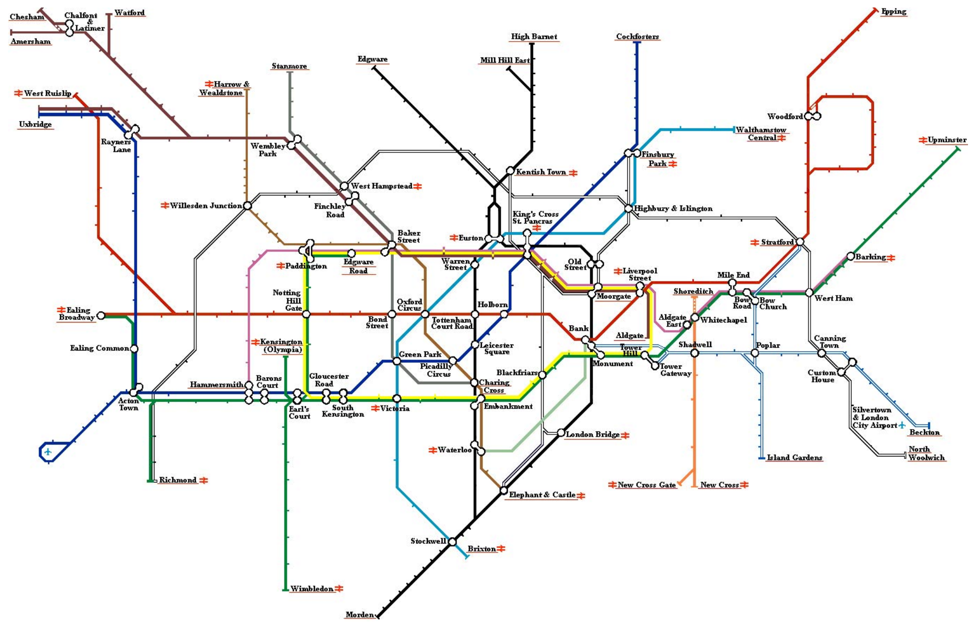
Other examples: scatter plots



Basic meaning: individual data values indicated by individual bars.

Derivative meaning: data trend indicated by the descending stair case (Pinker 1994).

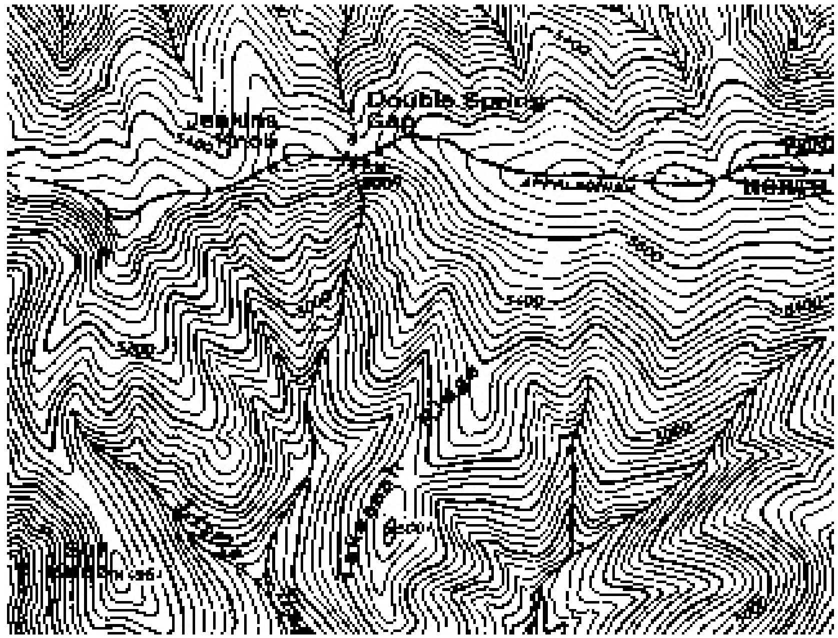
Other examples: route maps



Basic meaning: pairwise connections of stations indicated by edges of the graph.

Derivative meaning: concentration of connections to a certain station indicated by an edge concentration.

Other examples: geographical maps



Basic meaning: sea levels of individual points of the region indicated by individual points in contour lines.

Derivative meaning: existence of a valley indicated by a pattern formed by multiple contour lines.

Previous research on the phenomenon

Psychologists and researchers of graphical designs have noted the phenomenon under various names:

- “Higher-level” information (Bertin 1977, Wainer 1992)

- “Global” reading (Kinnear & Wood 1987, Gilhooly et al. 1988, Lowe 1989; 1994, Lohse 1993, Guthrie et al. 1993, Ratwani et al. 2003), “Macro” reading (Tufte 1994), Pattern Perception (Vleveland 1994), “Direct translation” Pinker (1990)

They also agree upon their functional significance.

Yet their scope and logical origins have not been well understood.

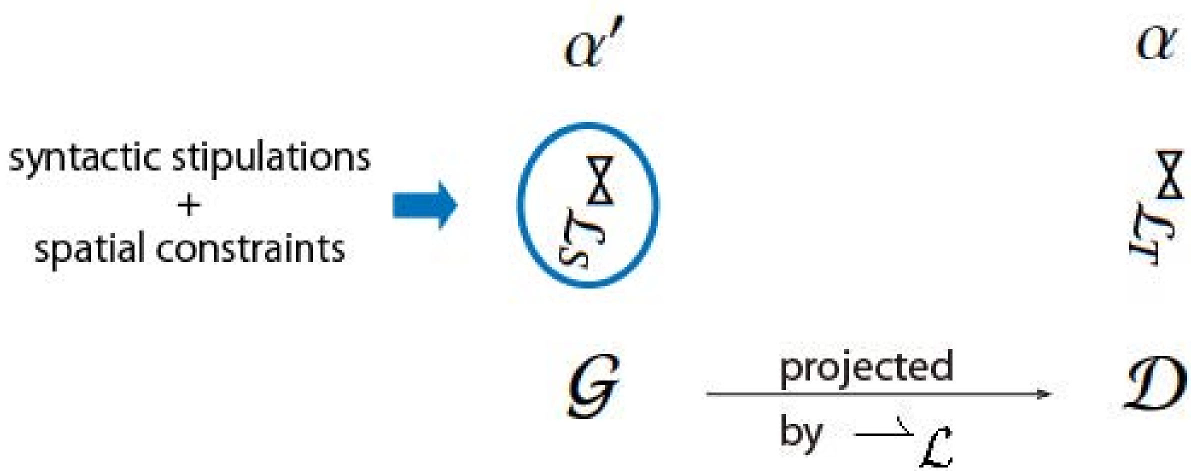
Our finding

Syntactic stipulations are designed to build parallel abstractions (hence derivative meanings) by exploiting spatial constraints on Euclidean planes.

$$
\begin{tikzcd}
\alpha' & & \alpha \\
\mathcal{G} \arrow[u, phantom, "\bowtie_{\mathcal{ST}}"] \arrow[rr, "\text{projected}", "\text{by } \rightharpoonup_{\mathcal{L}}"'] & & \mathcal{D} \arrow[u, phantom, "\bowtie_{\mathcal{LT}}"]
\end{tikzcd}
$$

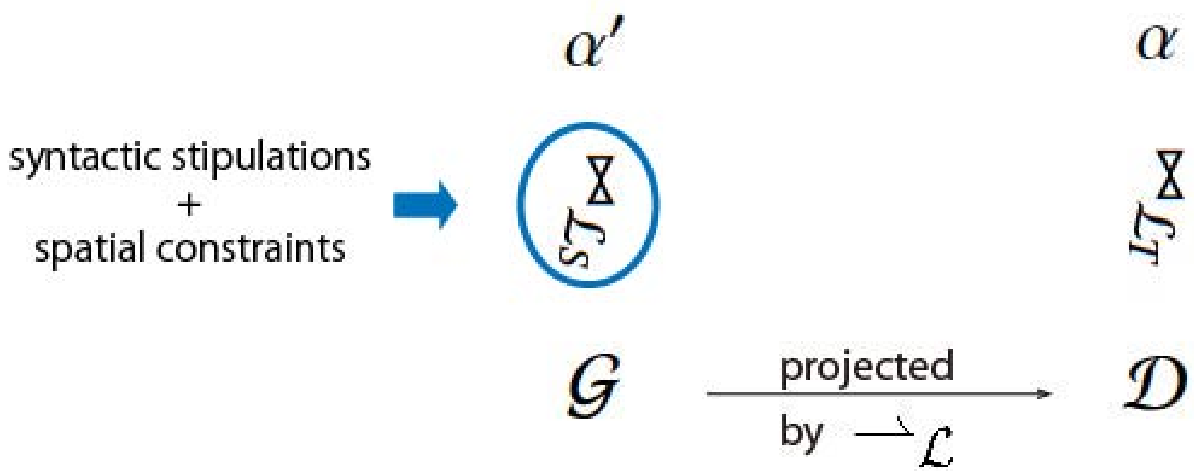
Our finding

Syntactic stipulations are designed to build parallel abstractions (hence derivative meanings) by exploiting spatial constraints on Euclidean planes.



Our finding

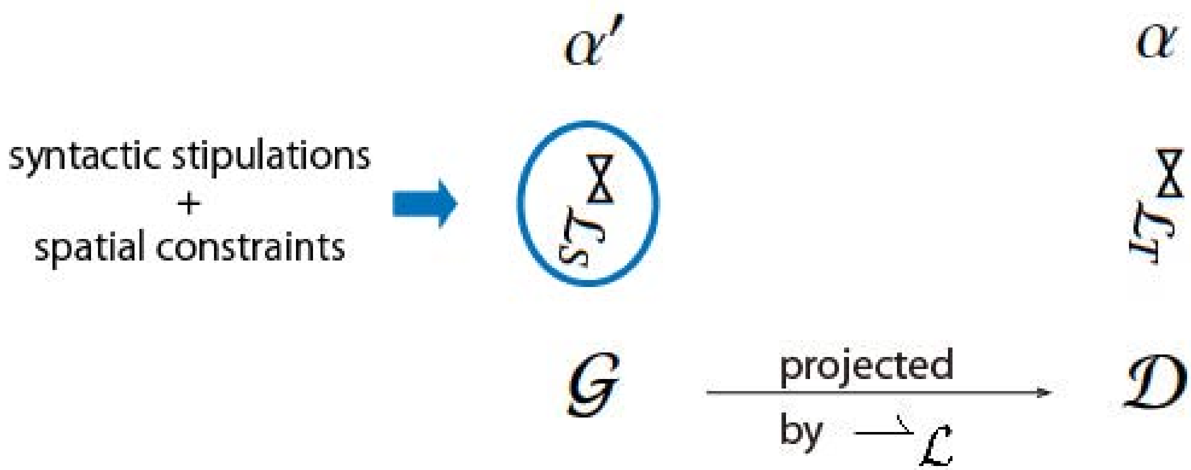
This character defines an interesting class of semantical systems that correspond to what we loosely call graphical representation systems.



→ Refinement of the notion of “graphical”

Contrast

Sentential notational systems have little such syntactic design (perhaps because they have evolved from auditory communication systems).

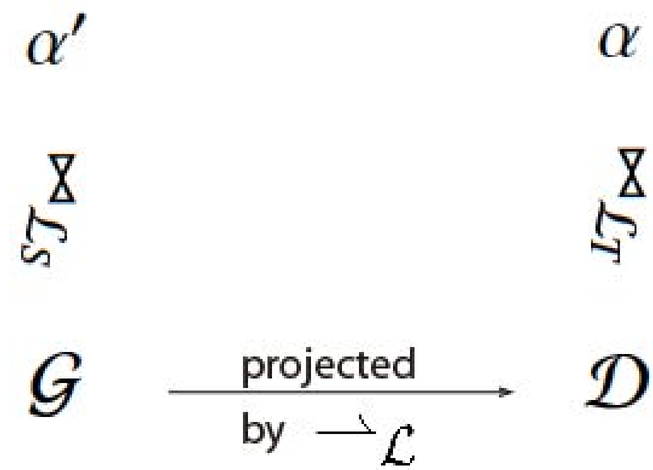


→ Semantic frameworks designed for sentential notational systems do not handle parallel abstractions or meaning derivations.

→ Need a different semantic framework for graphical notational systems.

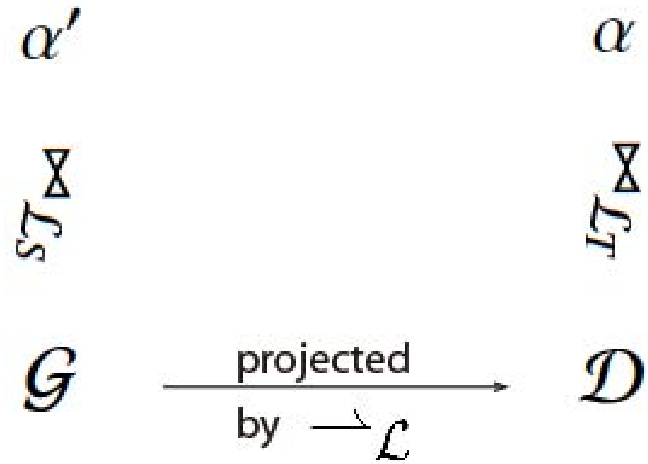
Approach with channel theory

Step 1: Define separate "local logics" on the presentational domain (left: $\mathcal{L}_S$) and the represented domain (right: $\mathcal{L}_T$) to model the abstraction relation in each domain.



Approach with channel theory

Step 2: Define another logic (center: $\mathcal{L}$) to model the informational relation between the two domains, where derivations of new informational relations from basic semantic conventions are modeled.



$\rightarrow$ Utility of the distributed logic model in channel theory.