

Equivocal 'if-then' in Legal Reasoning

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- We can combine multiple rules and by transitivity, we would realize automatic judging system.
- ... was a dream of '80s. Nowadays, none believes this idea would work.

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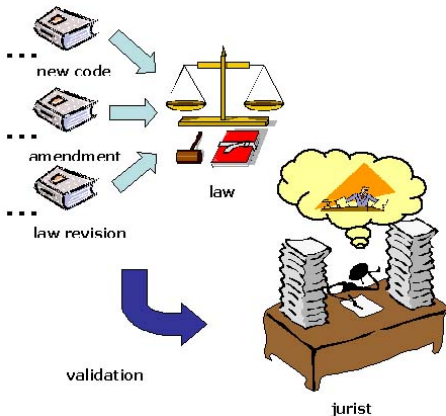
Generalized Inconsistency

- Logical inconsistency
- Conceptual opposition in ontology
- Definition loop

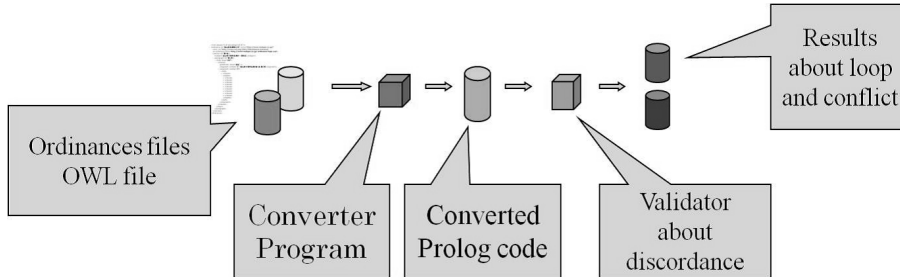
Ordinance Change in Toyama –1–

Ordinance No. 54 & 55: 'all the municipal procedure/application are available via the Internet.' Old ordinances are either

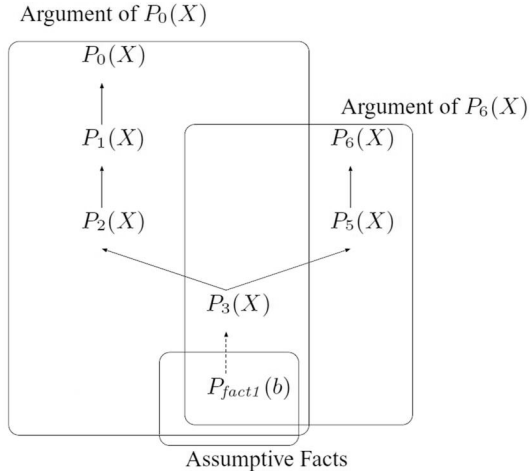
- simply overwritten,
- needed extra conditions, or
- not applicable.



Ordinance Change in Toyama -2-

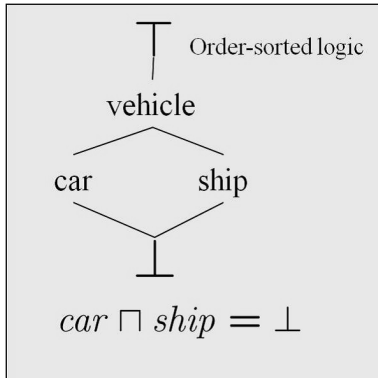


Ordinance Change in Toyama –3–



Conflict: $\forall x [P_0(x) \wedge P_6(x) \rightarrow \perp_{\mathcal{O}}]$

Ordinance Change in Toyama -4-



% Is amphibian car a common notion?

Ordinance Change in Toyama -5-

```
pv_sub(Root,任命権者(x)):-  
    pv(Root,市町村教育委員会(x)),  
    pv(Root,県費負担職員(y)),  
    pv(Root,所属(y,x)),  
    pv(Root,acceptable(富山県職員等の旅費に関する条例:第4条:第1項)).  
  
pv_sub(Root,市町村教育委員会(Var_0)):-  
    pv(Root,旅行命令権者(Var_0)).  
  
pv_sub(Root,旅行命令権者(z)):-  
    pv(Root,任命権者(z)),  
    pv(Root,acceptable(富山県職員等の旅費に関する条例:第4条:第1項)).
```

Ordinance Change in Toyama –6–

```
pv_sub(Root,要求行為(x,y,a)):-  
    usecheck(Root,use_384,pv_sub(Root,要求行為(x,y,a))),  
    pv(Root,申請者(x)),  
    pv(Root,行政庁(y)),  
%    pv(Root,拒否根拠(a)),  
    pv(Root,acceptable(富山県行政手続条例:第8条:第1項)),  
    usedcheck(Root,use_384,pv_sub(Root,要求行為(x,y,a))).
```

Ordinance Change in Toyama -7-

- Data
 - ordinance #54 (1)–(10)
 - ordinance #55 (1)–(7)
 - ordinance on administrative procedure
 - ordinance on handling fee
 - ordinance on travelling expense
 - ordinance on permit to climb mountains
- Experiment
 - CPU: Intel(R) Xeon(R) CPU X3350 @ 2.66GHz (4 cores), Memory: 3.2GB, OS: Windows XP SP3
 - preprocessor: ruby 1.8.7 (2008-06-20 patchlevel 22) [i386-cygwin]
 - verifier: SWI-Prolog (Multi-threaded, 32 bits, Version 5.6.52)
- Scale
 - 278 rules
 - hypernym–hyponym relations 281 with 562 technical terms
 - oppositions 11,602

Law Interpretation

'Vehicles are prohibited to enter the park.'

Expansion Horses are not admitted.

Reduction Baby cars are admitted.

Analogy Deer also are not admitted.

Limitation Human are admitted.

Mixture of Ordinance and Subsumption

'Vehicles (V) are prohibited (P) to enter the park.' but 'Baby cars (B) are admitted.' Is

$$K = \{V \rightarrow P, B \rightarrow A, B \rightarrow V, P \rightarrow \neg AV\}$$

consistent?

Translation to First-Order Logic

- 'Vehicles are not admitted to the park.'

$$\forall x[\mathit{vehicle}(x) \rightarrow \neg \mathit{admitted}(x)]$$

- 'Baby cars are admitted to the park.'

$$\forall x[\mathit{babycar}(x) \rightarrow \mathit{admitted}(x)]$$

- Baby car is a vehicle.

$$\forall x[\mathit{babycar}(x) \rightarrow \mathit{vehicle}(x)]$$

Problematic Mixture

- If a bird can fly, then it also can fly over the lake.

$$\forall(x : \textit{bird})[\textit{fly}(x) \rightarrow \textit{fly_over_the_lake}(x)].$$

- If a bird flew over the lake, then it flew.

$$\exists(e : \textit{event})[\textit{fly_over_the_lake}(e) \rightarrow \textit{fly}(e)]$$

Davidsonian semantics!

- 'If it rained hard, it rained.'
- 'If it rained, it didn't rain hard.'
- Therefore, 'if it rained hard, it didn't rain hard.'

Equivocal 'if-then'

- Classical Logic:

$$A \rightarrow B \Leftrightarrow \neg(A \wedge \neg B) = \neg A \vee \neg\neg B = \neg A \vee B.$$

- Non-classical logic

- Intuitionistic Logic: $A \rightarrow \neg\neg A$ but not $\neg\neg A \rightarrow A$.
- Paraconsistent Logic: $A \wedge \neg A \not\rightarrow B$.
- Relevant Logic: there should be some relevancy between A and B .

- Temporal Relation: A precedes B , $A < B$

- Conceptual Hierarchy: A is B iff $\llbracket A \rrbracket \subseteq \llbracket B \rrbracket$.

- Indicative: 'if A was the case, B was the case.'

- Counterfactual: 'if A were the case, B would be the case.'

- Deontic Logic: $A \rightarrow \mathfrak{D}B$ or $\mathfrak{D}(A \rightarrow B)$.

- Prerequisite-Effect structure.

Checkpoints

- Transitivity

$$\frac{A \rightarrow B \quad B \rightarrow C}{A \rightarrow C}$$

- Contraposition

$$A \rightarrow B \iff \neg B \rightarrow \neg A.$$

- Strengthening/weakening

$$\frac{\Gamma \rightarrow \Delta}{A, \Gamma \rightarrow \Delta}, \quad \frac{\Gamma \rightarrow \Delta}{\Gamma \rightarrow \Delta, A}$$

Agenda

- Legal reasoning is intuitionistic.
- Often includes counterfactuality.
- How to design time-axis.
- Prerequisite-Effect structure

- φ : there exist 20 times' consecutive 7's in π .

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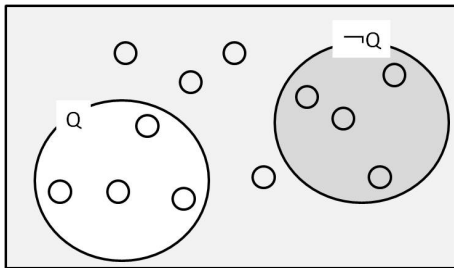
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- ψ ? We don't know yet.
- We cannot give a truth value to $\neg\varphi \vee \psi$ yet.
- Thus, $\varphi \rightarrow \psi$ does not imply $\neg\varphi \vee \psi$.

'unless' vs 'if not'

Geis (1973): 'unless' \neq 'except if'. ' P unless Q ' is stronger than ' P if not Q .'

' P unless Q ' is true if P is true in all epistemically possible worlds except those (white area) in which Q is true. ' P if not Q ' is true if P is true in all epistemically possible worlds (dark gray area) in which $\neg Q$ is true.



Intuitionistic Legal Judgement

'It is not known yet that P is being the case.' is different from $\neg P$.

- If P is negatively proved, $\neg P$.
- If $\neg P$ is not proved, P ?
% Since 'Cris is not guilty' is not proved, she is guilty.
- Assuming P , we obtain contradiction. Therefore $\neg P$.
- Assuming $\neg P$, we obtain contradiction. Therefore P (Reductio ad Absurdum)?
% If we assume 'relativity is not correct' we cannot explain why the light bends around the sun. Thus, relativity theory is correct.
- Either P or $\neg P$?

Contraposition in multiple time points

- 'If my daughter is not scolded, she does not study.'

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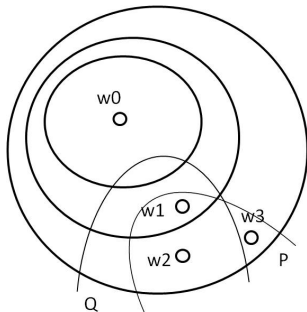
Contraposition in multiple time points

- 'If my daughter is not scolded, she does not study.'
- Therefore, 'if she studies, she is scolded.'
- Correct contraposition is: 'if she is studying, she was scolded.'

Counterfactuality

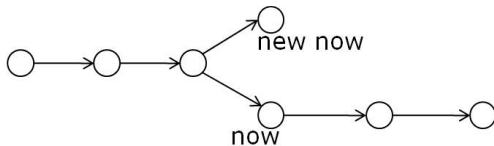
indicative $w \models A > B \iff \forall w'(wRw') \text{ if } w' \models A \text{ then } w' \models B.$

counterfactual David Lewis (1973): " $P \Box \rightarrow Q$ if and only if the most plausible $P \wedge Q$ world is nearer to the reality than the most plausible $P \wedge \neg Q$ world."



Conjunctive and Conditional

- German language family – Past-future
 - If I had more time, I would write you a longer letter.
 - Wenn ich mehr Zeit hätte, so würde ich Ihnen einen längeren Brief schreiben.
- Romance language family – conjunctive-conditional
 - Se avessi piú tempo, ti vorrei scrivere una lettera piú lunga.
imperfect conjunctive *present conditional*
- Conjunctive: to create a fresh possible world.
- Conditional: to mention matters in the fresh possible world.



Event and State

	Event	State
On time axis	Point-wise	Interval
Between intervals	Upward hereditary	Downward hereditary
Aspect	Perfective	Imperfective
View	From the outside	From the inside

Gradual change from events to states:

1. The play delighted Mary.
2. Fred was angry.
3. Alan was ill.
4. The train was standing alongside the platform.
5. The statue stood in the centre of the square.
6. Susan was a pediatrician.

[Kamp 79]

Accomplishment and Achievement

Temporal Ontology:

Preparatory phase – Culmination – Result State [Kamp 79]

In-Progress – Culmination – Holding & Resultant [Gunji]

- Telic and atelic
- Process and Progression

Imperferctive paradox:

- ‘Venus was twinkling’ implies ‘Venus has twinkled.’
- ‘John was crossing a street’ does not necessarily imply ‘John has crossed a street.’

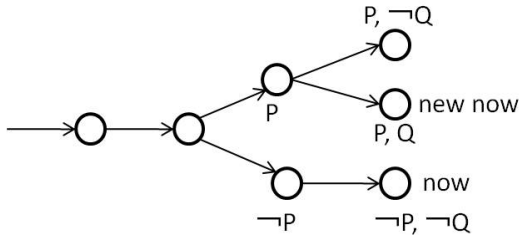
States	Activities	Accomplishments	Achievements
know, believe, have, desire, love	run, walk, swim, push a cart, drive a car	paint a picture, make a chair, deliver a sermon, draw a circle, recover from illness	recognize, spot, find, lose, reach, die

Designing Time Axis

- Priorian: *F, G, P*, and *H*.
- Precedence and Inclusion: Priorian + $\{\Box_{\downarrow}, \Box^{\uparrow}\}$ [van Benthem 2005]
- Computational Tree Logic: branching time + $\{A, X, U, S, F, G, \}$.
- Future Branching + Hereditary time

Conjunctive on Hereditary CTL

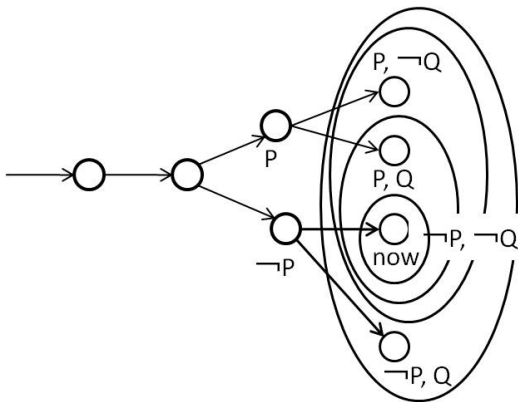
If P were the case, Q would be the case.



Conjunctive on CTL with Plausibility

If P were the case, Q would be the case.

The order of plausibility: $\neg P \wedge \neg Q > P \wedge Q > P \wedge \neg Q > \neg P \wedge Q$.



<i>Q</i> would be the case.	$\Box \rightarrow$	In all plausible possible worlds
<i>Q</i> might be the case.	$\Diamond \rightarrow$	In some plausible possible world

Rescher (1964) in the style of belief revision; M is a set of propositions in reality where

$$\neg P \wedge \neg Q \text{ and } M \cup \{P\} \vdash \perp.$$

Counterfactuality mentions such $M' (\subset M)$ that:

$$M' \cup \{P\} \text{ is consistent and } M' \cup \{P\} \vdash Q.$$

Agreed Facts

- Law: Those who damaged personal property, either with or without intent, shall have liability to compensate the detriment/deficit.
- Case: A has hurt B by careless driving.
- Judgement: A must pay B 200,000yen.
- Major premise: F : damaging other's property, G : have liability.
 $\forall x \forall y [F(x, y) \rightarrow G(x, y)]$
- conceptual subsumption: f : accident by careless driving.
 $f \sqsubseteq F$.
- Minor premise: $f(a, b)$
- Result: $g(a, b)$

Prerequisite–Effect Structure

State Change: a new effect, right, obligation and so on appears.

$$\underbrace{\varphi_1, \varphi_2, \varphi_3}_{\text{agreed facts}}, \underbrace{\varphi_4, \varphi_5}_{\text{non-liquet}} \rightarrow \psi$$

Ultimate facts Those facts which are agreed between plaintiff and defendant. No need of further argumentation.

Non liquet Still unknown. Those which need to be proved by either one of two sides.

Note that ‘ \rightarrow ’ in the general description of the rule does not concern temporal matters. Only ψ will be effective when all the φ_i are proved.

General Law and Law Application

- General rule

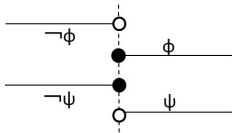
$$\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5 \rightarrow \psi$$

- Law application to Prerequisite–Effect

$$\{\textit{agreed facts}, \dots\}, \{\textit{non - liquet}\} \rightsquigarrow \psi$$

Temporal Structure of Prerequisite–Effect

$$\left\{ \begin{array}{l} \forall w' (\geq w) w' \models \varphi \\ \forall w'' (< w) w'' \not\models \varphi \\ \forall w' (> w) w' \models \psi \\ \forall w'' (\leq w) w'' \not\models \psi \end{array} \right.$$



As a result, we could define ‘immediately after’ operator.
[Sano and Tojo 2011]

Combining Multiple Implications

Intuitionistic implication	\rightarrow	KT4 (=S4) McKinsey–Tarski
Conceptual Subsumption	\sqsubseteq	classical
Prerequisite–Effect	\rightsquigarrow	Dedekind state change
Counterfactual	\boxrightarrow	K4 with concentric plausibility
Temporal Order	$<$	Priorian minimal tense logic
Deonticity	$\rightarrow \mathcal{D}$	KD
... is possible?		

One sentence!?

厚生労働大臣は、連続する三保険年度中の各保険年度において次の各号のいずれかに該当する事業であって当該連続する三保険年度中の最後の保険年度に属する三月三十一日（以下この項において「基準日」という。）において労災保険に係る保険関係が成立した後三年以上経過したものについての当該連続する三保険年度の間における労災保険法の規定による業務災害に関する保険給付（労災保険法第十六条の六第一項第二号の場合に支給される遺族補償一時金、特定の業務に長期間従事することにより発生する疾病であって厚生労働省令で定めるものにかかった者（厚生労働省令で定める事業の種類ごとに、当該事業における就労期間等を考慮して厚生労働省令で定めるものに限る。）に係る保険給付（以下この項及び第二十条第一項において「特定疾病にかかったものに係る保険給付」という。）及び労災保険法第三十六条第一項の規定により保険給付を受けることができることとされた者（以下「第三種特別加入者」という。）に係る保険給付を除く。）の額（年金たる保険給付その他厚生労働省令で定める保険給付については、その額は、厚生労働省令で定めるところにより算定するものとする。第二十条第一項において同じ。）に労災保険法第二十九条第一項第二号に掲げる事業として支給が行われた給付金のうち業務災害に係るもので厚生労働省令で定めるものの額（一時金として支給された給付金以外のものについては、その額は、厚生労働省令で定めるところにより算定するものとする。）を加えた額と一般保険料の額（第一項第一号の事業については、前項の規定による労災保険率（その率がこの項の規定により引き上げまたは引き下げられたときは、その引き上げまたは引き下げられた率）に応ずる部分の額）から非業務災害率（労災保険法の適用を受けるすべての事業の過去三年間の通勤災害に係る災害率及び二次健康診断等給付に要した費用の額その他の事情を考慮して厚生労働大臣の定める率をいう。（労働保険の保険料の徴収等に関する法律 12 条 3 項）

Toward Law Verification

