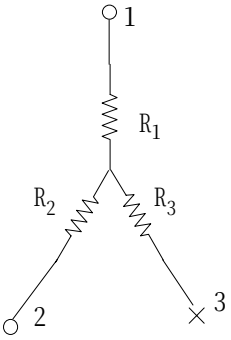
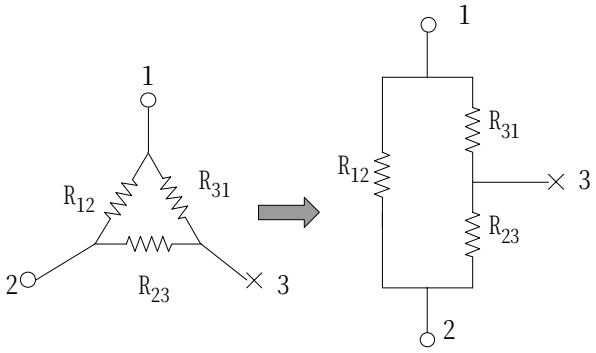
7-3 の解答例





（Δ→Y 変換）

ポイント1，2が回路に繋がっていて、ポイント3は接続が切れている場合を考える

このとき、1- 2間の抵抗は

Y 接続では　　$R_1 + R_2$ で

Δ接続では　抵抗 R_{12} と抵抗 $(R_{23} + R_{31})$ の並列だから　$\dfrac{R_{12}(R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$　となる

これらが等しいとするので　　$R_1 + R_2 = \dfrac{R_{12}R_{23} + R_{12}R_{31})}{R_{12} + R_{23} + R_{31}}$　　・・・・①

同様にして　　$R_2 + R_3 = \dfrac{R_{23}R_{31} + R_{23}R_{12}}{R_{23} + R_{31} + R_{12}}$　　・・・・②

$R_3 + R_1 = \dfrac{R_{31}R_{12} + R_{31}R_{23}}{R_{31} + R_{12} + R_{23}}$　　・・・・③　　が導かれる

①＋②＋③　　$R_1 + R_2 + R_3 = \dfrac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12} + R_{23} + R_{31}}$　　・・・・④　で

④－②　　$R_1 = \dfrac{R_{31}R_{12}}{R_{12}+R_{23}+R_{31}}$　　と、R_1 は求まる

（Y→Δ変換）

Δ→Y 変換の結果から

$$R_1R_2 = \frac{(R_{12})^2 R_{23}R_{31}}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{(R_{12}+R_{23}+R_{31})R_{12}R_{23}R_{31}}{(R_{12}+R_{23}+R_{31})^2} = \frac{R_{12}R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

という等式を用いて

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \left(\frac{R_{12}R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right) \Big/ \left(\frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}\right)$$

$$= R_{12}$$

と、R_{12} が導かれる

（Δ→Y Transformation）

We think the case when terminals 1 and 2 are connected to some circuit and terminal 3 is open

Resistance between 1 to 2 is

Y-connection $R_1 + R_2$

Δ-connection it's parallel resistance of R_{12} and $(R_{23} + R_{31})$

So it becomes $\dfrac{R_{12}(R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$

These are equal to each other so that $R_1 + R_2 = \dfrac{R_{12}R_{23} + R_{12}R_{31})}{R_{12} + R_{23} + R_{31}}$ ・・・・①

Similarly, $R_2 + R_3 = \dfrac{R_{23}R_{31} + R_{23}R_{12}}{R_{23} + R_{31} + R_{12}}$ ・・・・②

$R_3 + R_1 = \dfrac{R_{31}R_{12} + R_{31}R_{23}}{R_{31} + R_{12} + R_{23}}$ ・・・・③ are derived

①＋②＋③ $R_1 + R_2 + R_3 = \dfrac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12})}{R_{12} + R_{23} + R_{31}}$ ・・・・④

R_1 is derived as

④－② $R_1 = \dfrac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}}$

（Y→Δ transform）

From the results of Δ→Y transform

$$R_1R_2 = \frac{(R_{12})^2 R_{23}R_{31}}{(R_{12} + R_{23} + R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{(R_{12} + R_{23} + R_{31})R_{12}R_{23}R_{31}}{(R_{12} + R_{23} + R_{31})^2} = \frac{R_{12}R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}$$

From this equation, we derive R_{12} as

$$\frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \left(\frac{R_{12}R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}\right) \Big/ \left(\frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}}\right)$$

$$= R_{12}$$