Simulation of Common Language Acquisition by Evolutionary Dynamics

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Abstract

Creole, an emergent new language, is one of the main topics in various fields concerning language origin and language change, such as sociolinguistics, cognitive science, and so on. Thus far, we have proposed a formalization of language dynamics so that creoles may emerge, in which the change of language is represented as the transition of population among a finite number of languages. Our purpose in this paper is to analyze the underlying features of the search space of the language dynamics. We show experimental results, in which we could observe conditions for fertilization in terms of similarity among languages. Furthermore, the conditions vary depending on the amount of language input. We found a positive correlation between the environment of language acquisition and creolization, and endorse that pidgin communities are suitable for creoles to emerge. This study is considered to contribute to the study of evolutionary models of collaboration.

1 Introduction

In the fields of sociolinguistics, cognitive science, developmental psycholinguistics, and other related fields, many researchers have studied particular languages called pidgins and creoles. Pidgins are simplified tentative languages spoken in multilingual communities. They come into being where people need to communicate but do not have a language in common. Creoles are full-fledged new languages which children of the pidgin speakers acquire as their native languages. Pidgins and creoles are defined as two different stages of language change [Arends *et al.*, 1994]. Interesting is the fact that children growing up hearing syntactically simplified languages such as pidgins develop a mature form as creoles. Observing actual pidgins and creoles, linguists have argued that creoles would appear in specific environments like pidgin communities [DeGraff, 1999]. In general, children inherit language from their parents and neighbors during their acquisition period. However, it has not yet been clarified how children correctly deduce the underlying grammatical rules and consistently acquire the same language [Nowak and Komarova, 2001]. Pidgins and creoles may concern the mechanism for language acquisition of infants. Particularly, some properties of creoles imply the existence of innate universal grammar [Chomsky, 1981]. Thus, the study of pidgins and creoles plays a key role in the clarification of language acquisition [Bickerton, 1990].

The purpose of this paper is to investigate the characteristics of creole using a mathematical formalization of population dynamics [Weibull, 1995]. From efforts in linguistics, it is clear that the emergence of creole is affected by contact with other languages, the distribution of population of each language, and similarities among the languages. Constructing a model including these elements, we derive conditions for the emergence of creole from theoretical and numerical analyses. Thus, our work approaches to the mechanism of language acquisition.

In the stream of simulation studies of language evolution [Cangelosi and Parisi, 2002], the emergence of creole has also been studied. Briscoe [2002] has reported sophisticated models of human language acquisition by means of a multi-agent model. However, because the number of agents was finite, the results were often hard to generalize to explain language phenomena in the real world. Most multi-agent models had suffered from this problem.

To overcome this drawback of multi-agent models from a different viewpoint, Nowak et al. [2001] developed a mathematical theory of the evolutionary dynamics of language called the *language dynamics equation*, in which the change of language is represented as the transition of population among a finite number of languages. However, in the framework of evolutionary dynamics of language, the emergence of creole had not yet been discussed. Nakamura et al. [2003; 2004] modified the language dynamics based on social interaction, and then dealt with the emergence of creole. They showed that dominant creoles emerge under specific conditions of similarity among languages.

In this paper, we develop the analysis of the environment of language acquisition and observe novel conditions of creole. Since language is the basis of collaboration and since collaborating individuals may speak different languages, this formalization of the emergence of a creole as an effect or a prerequisite for collaboration is considered to contribute to the study of evolutionary models of collaboration.

In Section 2, we describe the modified language dynamics model and a learning algorithm, and in Section 3 we define a creole in population dynamics. Section 4 reports our experiments. We discuss the experimental results in Section 5 and conclude in Section 6.

2 Population Dynamics for the Emergence of Creole

In this section, we briefly explain a mathematical model proposed by Nakamura et al. [2004] and consider a learning algorithm for language acquisition.

2.1 Language Dynamics Equation without Fitness

In response to the language dynamics equation by Nowak et al. [2001], Nakamura et al. [2003] assumed that any language could be classified into one of a certain number of grammars. Thus, the population of language speakers is distributed to a finite number (n) of grammars $\{G_1 \ldots G_n\}$. Let x_i be the proportion of speakers of G_i within the total population. Then, the language dynamics is modeled by an equation governing the transition of language speakers among languages.

Because Nowak et al. [2001] assumed that language speakers bore offspring in proportion to their successful communication, they embedded a fitness term in their model which determined the birth rate of each language group. The model for creolization excluded fitness, on the assumption that in the real world creoles did not emerge because creole speakers had more offspring than speakers of other pre-existing languages, that is:

$$\frac{dx_j(t)}{dt} = \sum_{i=1}^{n} \overline{q}_{ij}(t)x_i(t) - x_j(t) \ . \qquad (1)$$

In the language dynamics equations, the similarity matrix S and the transition matrix $\overline{Q}(t)$ play important roles: the similarity matrix $S = \{s_{ij}\}$ denotes the probability that a sentence of G_i is accepted by G_j. The transition matrix $\overline{Q}(t) = \{\overline{q}_{ij}(t)\}$ is defined as the probability that a child of G_i speaker obtains G_j by the exposure to his/her parental language and to other languages. Being different from the definition by Nowak et al. [2001], the definition of $\overline{Q}(t)$ depends on the generation parameter t, as well as the S matrix and a learning algorithm.

2.2 Learning Algorithm

In some communities, a child learns language not only from his/her parents but also from other adults, whose language may be different from the parental one. In such a situation, the child is assumed to be exposed to other languages, and thus may acquire the grammar most efficient in accepting multiple language input. In order to assess how often the child is exposed to other languages, we divide the language input into two categories: one is from his/her parents, and the other is from other language speakers. We name the ratio of the latter to the total amount of language input an *exposure ratio* α. This α is subdivided into smaller ratios corresponding to those other languages, where each ratio is in proportion to the population of the language speakers. An example distribution of languages is shown in Figure 1. Suppose a child has parents who speak G_p, s/he receives input sentences from G_p on

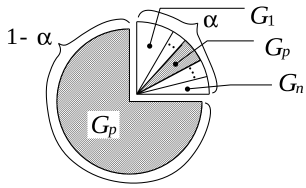


Figure 1: The exposure ratio α

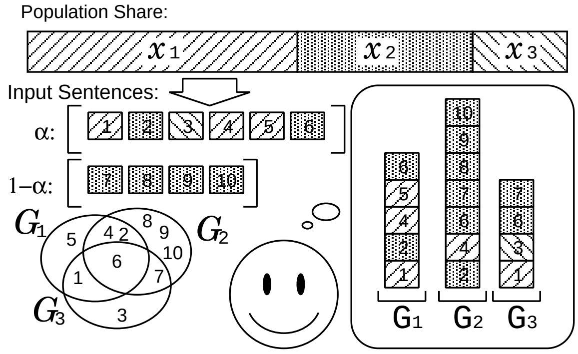


Figure 2: The learning algorithm

the percentage of the shaded part, $\alpha x_p + (1 - \alpha)$, and from non-parental languages $G_i (i \neq p)$ on the percentage, αx_i.

We adopted a batch learning algorithm, which resolves Niyogi [1996]'s problem regarding an unrealistic Markov structure which implies that some children cannot learn certain kinds of language. From the viewpoint of universal grammar, that all conceivable grammars of human beings are restricted to a finite set [Chomsky, 1981], language learning is considered as a choice of a plausible grammar from them. The following algorithm realizes such learning as: 1) In a child's memory, there is supposed to be a score table of grammars. 2) The child receives a sentence uttered by an adult. 3) The acceptability of the sentence is tested using each grammar. The grammar which accepts the sentence scores 1 point. 4) Steps 2) and 3) are repeated until the child receives a fixed number (w) of sentences, which is regarded as sufficient for the estimation of the grammar. 5) The child adopts the grammar with the highest score.

The child is exposed to utterances of adult speakers of each language, the percentage of which is determined by the distribution of population and the exposure ratio α, while the S matrix determines the acceptability of a sentence. In Figure 2, we show an example where a child of G_2 speaker obtains G_2 after exposure to a variety of languages. The child receives sentences, which are boxes numbered from 1 to 10. The input sentences are divided into two sets according to the exposure ratio α. One of the sets consists of sentences of all grammars. The number of the sentences of each language is proportional to the population share of the language speakers. For example, the child hears sentences 1, 4 and 5 uttered by G_1 speakers. The other consists of sentences of his/her parents. Therefore, these sentences are acceptable by a particular grammar. Because his/her parental grammar is G_2, for example, the sentences 7 to 10 are randomly chosen from the language of G_2. The child counts acceptable sentences for each grammar. The sentence 1 can be accepted by G_3 as well as G_1, while it is uttered by a G_1 speaker. The Venn diagram in Figure 2 represents that each language shares sentences with others. In this case, because the sentence 1 is acceptable both by G_1 and by G_3, the child adds 1 to both of the counters in his/her mind.

2.3 Revised Transition Probability

Suppose that children hear sentences from adult speakers depending on the exposure ratio and on the distribution of population. A probability that a child whose parents speak G_i accepts a sentence by G_j is expressed by:

$$U_{ij} = \alpha \sum_{k=1}^{n} s_{kj} x_k + (1-\alpha) s_{ij} \quad . \tag{2}$$

After receiving a sufficient number of sentences for language acquisition, the child will adopt the most plausible grammar, as estimated by counting the number of sentences accepted by each grammar. This learning algorithm is simply represented in the following equation. Exposed to a variety of languages in proportion to the population share of adult speakers, children whose parents speak G_i will adopt G_{j^*} in the following manner:

$$j^* = \underset{j}{\operatorname{argmax}} \{U_{ij}\} \quad . \tag{3}$$

When the children hear w sentences, a probability that a child of G_i speaker accepts r sentences with G_j is given by a binomial distribution,

$$g_{ij}(r) = \binom{w}{r} (U_{ij})^r (1-U_{ij})^{w-r} \quad . \tag{4}$$

On the other hand, a probability that the child accepts less than r sentences with G_j is

$$h_{ij}(r) = \sum_{k=0}^{r-1} \binom{w}{k} (U_{ij})^k (1-U_{ij})^{w-k} \quad . \tag{5}$$

From these two probability distributions, the probability that a child of G_i speaker accepts k sentences with G_j, while less than $k-1$ sentences with the other grammars, comes to $g_{ij}(k) \prod_{l=1, l \neq j}^{n} h_{il}(k)$. For a child of G_i speaker to acquire G_j after hearing w sentences, G_j must be the most efficient grammar among n grammars; viz., G_j must accept at least $\lceil \frac{w}{n} \rceil$ sentences. Thus, the probability $\overline{q}_{ij}$ becomes the sum of the probabilities that G_j accepts $w, w-1, \cdots, \lceil \frac{w}{n} \rceil$ sentences. Because each of the sentences is uttered by a speaker and is accepted by at least one grammar, there must be a grammar which accepts $\lceil \frac{w}{n} \rceil$ or more out of w sentences. Thus, if G_j accepts less than $\lceil \frac{w}{n} \rceil$ sentences, the child does not acquire G_j. Therefore, $\overline{q}_{ij}$ becomes:

$$\overline{q}_{ij}(t) = \frac{\displaystyle\sum_{k=\lceil \frac{w}{n} \rceil}^{w} \left\{ g_{ij}(k) \prod_{\substack{l=1 \\ l \neq j}}^{n} h_{il}(k) + R(k,n) \right\}}{\displaystyle\sum_{m=1}^{n} \left[\sum_{k=\lceil \frac{w}{n} \rceil}^{w} \left\{ g_{im}(k) \prod_{\substack{l=1 \\ l \neq m}}^{n} h_{il}(k) + R(k,n) \right\} \right]} \quad , \tag{6}$$

where $R(k,n)$ is the sum total of the probabilities that the child would choose G_j when one or more other grammars accept the same number of sentences as G_j. When there are m candidate grammars including G_j, the probability becomes one divided by m. The following expression is an example when $n = 3$.

$$\begin{aligned} R_{ij}(k,3) &= \tfrac{1}{3}\{g_{ij}(k) g_{ij_2}(k) g_{ij_3}(k)\} \\ &+ \tfrac{1}{2}\{g_{ij}(k) g_{ij_2}(k) h_{ij_3}(k) + g_{ij}(k) h_{ij_2}(k) g_{ij_3}(k)\} \\ &(j_2, j_3 \in \{1,2,3\}, j \neq j_2, j_3, j_2 \neq j_3). \end{aligned} \tag{7}$$

3 Creole in Population Dynamics

Creoles are considered as new languages. From the viewpoint of population dynamics, we define a creole as a transition of population of language speakers. A creole is a language which no one spoke in the initial state, but most people have come to speak at a stable generation. Therefore, creole is represented by G_c such that: $x_c(0) = 0, x_c(t) > \theta_c$, where $x_c(t)$ denotes the population share of G_c at a convergent time t, and θ_c is a certain threshold to be regarded as a dominant language. We set $\theta_c = 0.9$ through the experiments.

For convenience, we have mainly observed the behavior of the model using three grammars. The similarity matrix can be expressed as a symmetric matrix such that:

$$S = \begin{pmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{pmatrix} \quad . \tag{8}$$

Here, we regard G_3 as a creole grammar, giving the initial condition as $(x_1(0), x_2(0), x_3(0)) = (0.5, 0.5, 0)$. Therefore, the element a denotes the similarity between two pre-existing languages, and b and c are the similarities between G_1 and the creole, and between G_2 and the creole, respectively. If we assume that a language consists of a finite set of sentences and each sentence is uttered with uniform probability, the similarities a, b and c are represented in a Venn diagram shown in Figure 3.

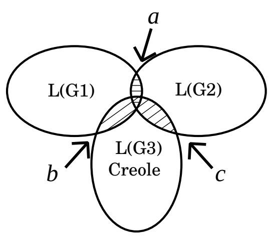
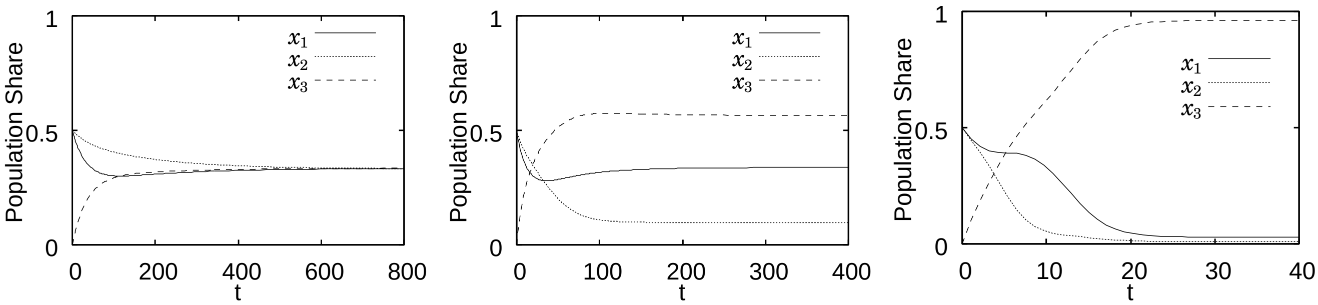


Figure 3: The similarities between languages



(a) $\alpha = 0$ (b) $\alpha = 0.2$ (c) $\alpha = 0.7$

Figure 4: The change of dynamics with regard to the exposure ratio α $((a, b, c) = (0, 0.3, 0.2), w = 3)$

4 Experiments

In this section, we show experimental results of our model. We examine conditions in which a creole appears and comes to be dominant. Nakamura et al. [2003] showed conditions of the similarities for creolization, and argued the relationship between innate human grammars and creoles. Because we look at dependency upon the environment of language acquisition for creolization, we conduct the following experiments: (1) a relationship between the exposure ratio α and population transition, (2) conditions on the similarity among languages for creolization when children learn a language in a particular environment, and (3) a relationship between the exposure ratio α and the amount of language input. We show the experimental results in the following subsections.

4.1 The Exposure Ratio and Creolization

In order to observe the relationship between contact with languages and creolization, we examine how the exposure ratio α affects the population transition. We arbitrarily set the number of input sentences to $w = 3$, and the similarity among languages to $(a, b, c) = (0, 0.3, 0.2)$ in Eqn (8), respectively. We observe the difference of dynamics, setting the exposure ratio α to 0, 0.2, and 0.7.

Figure 4(a) shows the experimental result at $\alpha = 0$. Growing up hearing language input only from their parents, children infer an appropriate grammar from 3 sentences. Children of G_1 speakers are likely to acquire their parental grammar, while some of them confuse the grammars of G_1 and G_3 with a probability of b^3. Because G_1 is more similar to G_3, which is the new language, than G_2 is, children of G_1 speakers easily acquire G_3. Therefore, the population of G_1 rapidly decreases at the beginning, while that of G_3 increases. The dynamics eventually converge to the same proportion of population for all grammars.

The increase of the exposure ratio α implies opportunity for the language learners to come into contact with languages other than their parental one. Because children as language learners acquire the most acceptable grammar for the input sentences, they are likely to fail to acquire their parental grammar along with α. In other words, the increase of the exposure ratio α tends to accelerate the population transition among languages. Figure 4(b) shows the result at $\alpha = 0.2$. As shown in Figure 4(a), x_1 is less than x_2 at the beginning. Because G_3 is the most similar language to others among all languages, children easily acquire G_3. Therefore, G_3 obtains the highest population. Because G_1 is more similar to G_3 than G_2 is, x_1 exceeds x_2 when G_3 obtains a certain amount of population share.

When $\alpha = 0.7$ in Figure 4(c), G_3 becomes dominant at the stable generations, and obtains the certain amount of population share, that is $\theta_c = 0.9$. We call this phenomenon *creolization* and call G_3 *creole*. Furthermore, the greater α, the more rapidly the dynamics converge.

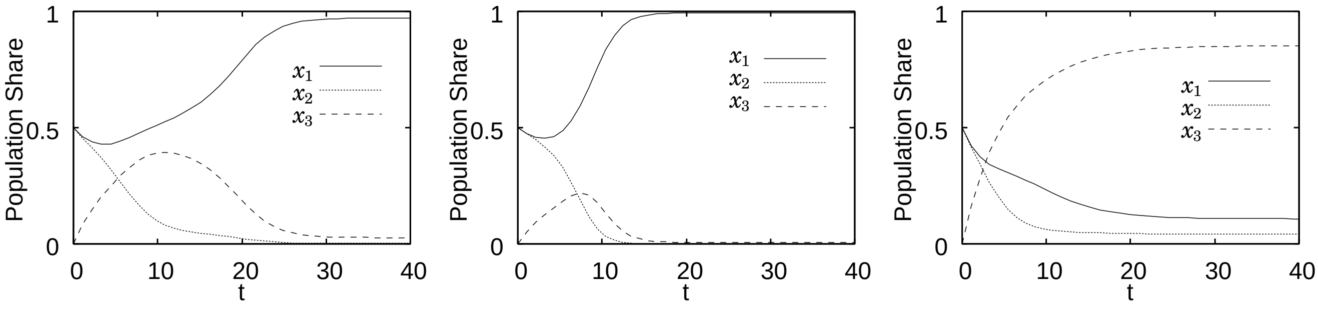
In this section, we observed the relationship between the exposure ratio α and the behavior of dynamics. We can conclude that the increase of the exposure ratio α causes creolization.

4.2 Conditions for Creolization Based on Similarity

The next experiment aims at finding boundaries in the parameter space of the similarity matrix where G_3 becomes dominant. We parameterized elements of the S matrix and plotted the region where creolization occurred. The number of input sentences and the exposure ratio were set to $w = 3$ and $\alpha = 0.7$, respectively. The result is shown in Figure 5, which is a contour diagram of the projection of the parameter space onto the $b - c$ plane. Creoles emerge within the regions surrounded by the lines, each of which is drawn with a different value of a.

In the previous section, we showed an example of dominant creoles in Figure 4(c). The S matrix was set to $(a, b, c) = (0, 0.3, 0.2)$, which corresponds to the point labeled (i) in Figure 5. The value of $a = 0$ denotes that there is no common sentence in G_1 and G_2. Figure 6(a) shows a result when the value a increases slightly. Because G_1 and G_2 share common sentences, the language learners of G_1 are likely to acquire G_2 and vice versa. In other words, the direct transition of population between G_1 and G_2 frequently occurs. Therefore, the similarity between pre-existing languages prevents the language learners acquiring the new language. Instead of G_3, the similar language G_1 becomes dominant.

The point labeled (ii) in Figure 5 denotes that neither G_1 nor G_2 is similar to G_3. The language learners are affected by the population distribution of languages in terms of language acquisition, rather than by the advantage of language



(a) $(a, b, c) = (0.1, 0.3, 0.2)$ (b) $(a, b, c) = (0, 0.2, 0.1)$ (c) $(a, b, c) = (0, 0.4, 0.3)$

Figure 6: Examples of dynamics when a dominant creole does not emerge ($\alpha = 0.7$, $w = 3$)

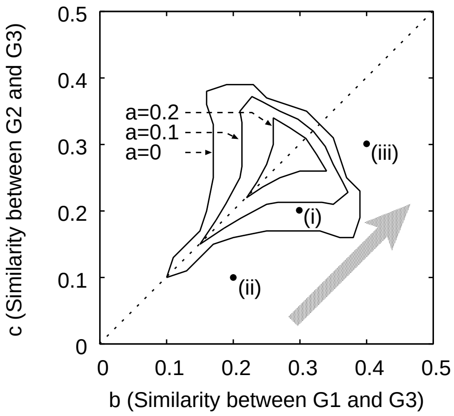


Figure 5: Conditions for creolization on similarity among languages ($\alpha = 0.7$, $w = 3$)

similarity. Because G_3 does not obtain a certain amount of population share, it diminished at the stable generation.

In Figure 6(c) with $(a, b, c) = (0, 0.40, 0.3)$, we observed that G_3 remained the most populous language although x_3 was a little less than $\theta_c = 0.9$. If θ_c was lower, G_3 would be regarded as a creole. And then, the sector form in Figure 5 as the condition for creolization would become wider. Because G_3 is very similar to both G_1 and G_2, the population of G_3 frequently flows into them and vice versa. Therefore, it is difficult for G_3 to keep the population share to be regarded as a dominant creole.

In this section, we observed the conditions for creoles to become dominant, setting the number of input sentences w and the exposure ratio α to certain fixed values. The results in the previous section, shown in Figure 4, correspond to the point labeled (i), that is $(a, b, c) = (0, 0.3, 0.2)$, in Figure 5. Therefore, the condition of creolization based on similarity varies depending on the exposure ratio α. In the next section, we examine how the number of input sentences affects the emergence of creole.

4.3 Language Input and Creolization

The increase of the number of input sentences w signifies that the learners improve the learning accuracy of their parental

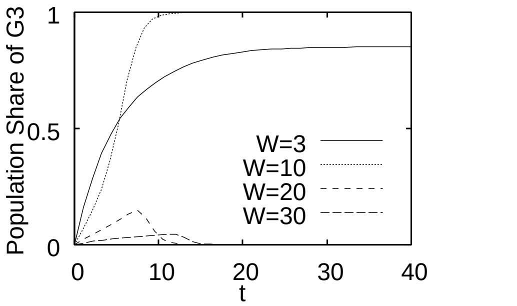


Figure 7: Behaviors of x_3 with regard to the number of input sentences ($(a, b, c) = (0, 0.4, 0.3)$, $\alpha = 0.7$)

grammars. Because this decelerates the population transition among languages, the convergence of dynamics tends to be slow. For example, if we set the number of input sentences to $w = 30$ and the other parameters to the same values as in Figure 4(a), the dynamics does not converge even at a million generations. Let us observe how creole emerges when the language learners hear enough language input to infer a suitable language for the community. We look at the point labeled (iii) in Figure 5, examining the number of input sentences w in the range from 3 to 30.

Figure 7 represents the difference among the behaviors of x_3 for each w. As we have mentioned in the previous section, when the learners hearing only 3 sentences choose a grammar, the population share of G_3 does not exceed the threshold $\theta_c = 0.9$ at the stable generations. Because of the high similarity between G_3 and pre-existing grammars, the learners are likely to fail to acquire their parental grammar. As a result, these languages coexist.

When $w = 10$, G_3 becomes a dominant creole. In this situation, the language input $w = 10$ is enough for the learners to get over the confusion of selection. As a result, the behavior is similar to the result of the point labeled (i) in Figure 5 (See Figure 4(c)). Therefore, we consider that there is a correlation between the number of input sentences and the similarity among languages.

If we further increase the number of input sentences to $w = 20$ and $w = 30$, the population of G_3 diminished and eventually disappeared at the stable generations. The dynamics come to behave in the same way as at the point labeled (ii) in Figure 5. In other words, the increase in the number of input sentences makes the fan-shaped region of creolization shift along with the arrow in Figure 5.

In this series of experiments, we looked at the environment of language acquisition for infants. As a result, we found a correlation between the number of input sentences and the similarity among languages.

5 Discussion

Let us discuss similarity among languages. According to Nowak et al. [2001], a similarity matrix is an abstract form of a universal grammar to be mapped into the language dynamics. Therefore, the similarity is predefined and fixed as long as we deal with language phenomena within a short time scale, such as social interaction. In this paper, we assumed that vocabularies were shared in common among all languages, and that the similarity was intended for that of syntax, which is provided by the universal grammar. In general, when language users communicate with people speaking a different language, they accept some sentences which consist of incomprehensible words or use a non-permissible syntax. If the degree of their behavior is taken into account for the similarity among languages, the similarity would vary depending on it. Thus, pidginization is considered as a process for languages to become similar to each other.

Bickerton [1983] has reported that there is a similarity between utterances of creole speakers and of infants in a normal situation. Infants in the stage of language acquisition often utter ungrammatical but syntactically reasonable sentences. He has also argued that if the infants grew up without receiving language input anymore, they would acquire the same language as creoles. On the contrary, children in a pidgin community mostly hear pidgin in their acquisition period, where most of the sentences may be syntactically immature. However, the amount of language input is the same as for other infants who acquire their native languages in a normal environment. We argue that our experimental results support Bickerton's hypothesis. When language learners receive a small amount of language input, creoles emerge under conditions where languages are not similar to each other. In the same way, if a pidgin community is developed and then the languages become similar to each other, creoles are spoken by infants growing up with a large amount of language input.

6 Conclusion

In this paper, we showed the emergence of creole in population dynamics of languages, and argued that the emergence is affected by the environment of language acquisition as well as by the similarity among languages. In previous work, Nakamura et al. [2003; 2004] have argued that there are conditions for creolization based on similarity among languages and on the degree of language contact. In this paper, we focused on the amount of language input, from which infants infer a suitable grammar.

We supposed that a child was exposed to a variety of languages in a pidgin community, and chose a grammar after hearing a small amount of language input. From the experiments, we observed a correlation between the number of input sentences and the similarity among languages. Creoles emerged within a certain range of similarity. On the contrary, when the pre-existing languages are not similar to creole, one of them is likely to become dominant. In addition, when the pre-existing languages are very similar to a creole, all languages tend to coexist and the creole does not dominate the community. These conditions on the similarity among languages vary depending on the amount of language input. We observed that in a situation where children received a large amount of language input, creoles came to be more similar to the pre-existing languages.

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