An Overview of Models and Proof Rules for CafeOBJ Proof Scores

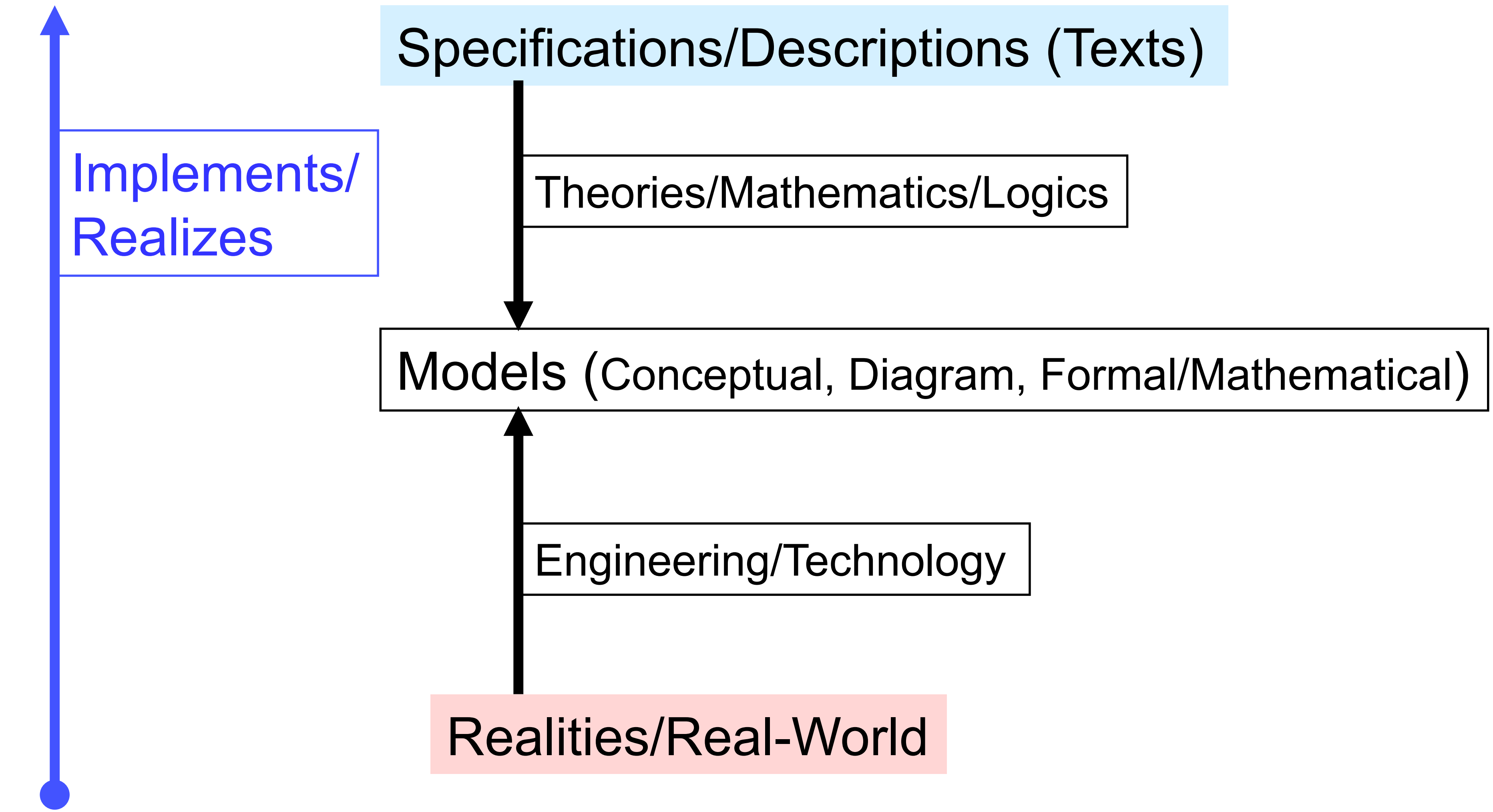
Lecture Note 03a

CafeOBJ Team for JAIST-FSSV2010

Topics

- Specification/Descriptions, Models, and Realities
- Constructor-based Order Sorted Algebra
- Satisfaction of a Property by a Specification
  - SPEC |= prop
- Proof rules for SPEC |= prop and SPEC |- prop

Specifications, Models, Realities



Specification

An **constructor-based equational specification SPEC** in CafeOBJ (a text in the CafeOBJ notation with only equational axioms) is defined as a pair **(Sig,E)** of order-sorted constructor-based signature **Sig** and a set **E** of conditional equations over **Sig**. A signature **Sig** is defined as a triple **(S,F,F^c)** of an partially ordered set **S** of sorts, an indexed family **F** of sets of **S**-sorted functions/ operations, and a set **F^c** of constructors. **F^c** is a family of subsets of **F**, i.e. $F^c \subseteq F$.

SPEC = ((S,F,F^c),E)

Model: (S,F)-Algebra

A formal/mathematical **model** of a specification **SPEC = ((S,F,F^c),E)** is an reachable order-sorted **algebra A** which has the signature **(S,F)** and satisfies all equations in **E**.

An order-sorted algebra which has a signature (S,F) is called an **(S,F)-algebra.** An **(S,F)-algebra A** interprets a sort symbol s in **S** as a (non empty) set $\mathbf{A_s}$and an operation (function) symbol **f :s1 s2 …sn->s(n+1)** in **F** as a function $\mathbf{A_f : A_{s1},A_{s2},..,A_{sn} \text{->} A_{s(n+1)}}$. The interpretation respects the order-sort constrains.

Model: (S,F,F^C)-Algebra

If a sort $s \in S$ is the co-arity of some operator $f \in F^C$, the sort s is called a **constrained sort**. A sort which is not constrained is called a **loose sort**.

An (S,F)-algebra A is called **(S,F,F^C)-algebra** if any value $v \in A_s$ for any constrained sort $s \in S$ is expressible only using

(1) function A_f for $f \in F^C$

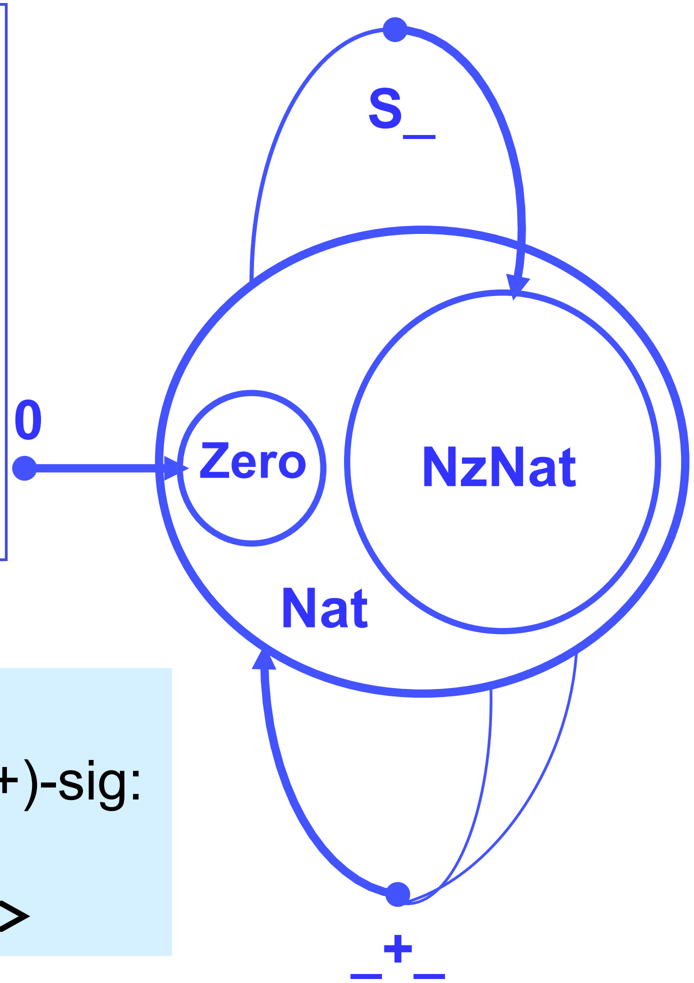
and

(2) function A_g for $g \in F$ whose co-arity is loose sort .

(S,F,F^C)-algebra can also be called **F^C-reachable algebra**

An example of Signature and its Algebra

```
-- Let (PNAT+)-sig be
-- the  signature of PNAT+
-- sort
[ Zero NzNat < Nat ]
-- operators
op 0 :   -> Nat {constr}
op s_ :  Nat -> NzNat {constr}
op _+_ : Nat Nat -> Nat
```



A (PNAT+)-sig-algebra
Order-Sorted Algebra with Signature (PNAT+)-sig:

`<Nat, NzNat, Zero; 0, s_, _+_>`

Valuation, evaluation

A **valuation** (or an assignment) is a sort preserving map from the (order-sorted) set of variables of a specification to an order-sorted algebra (a model), and assigns values to all variables.

Given a model **A** and a valuation **v**, a **term t** of sort **s,** which may contain variables, is evaluated to a **value** $\mathbf{A_v(t)}$ in $\mathbf{A_s}$

Equation

Given terms t, t',t1,t1',t2,t2'…tn,tn' , a **conditional equation** is a sentence of the form:

t = t' if (t1 = t1') /\ (t2 = t2') /\ …/\ (tn = tn')

An ordinary equation is a sentence of the form:

t = t'

that is n=0.

A conditional equation in CafeOBJ notation:

(t=t' if c)

where t,t' are any terms and c is a Boolean term is an abbreviation of

(t=t' if c = true)

Satisfiability of equation

An ordered-sorted algebra **A** **satisfies** a conditional equation:

t = t' if (t1 = t1') /\ (t2 = t2') /\…/\ (tn = tn')

iff

A_v(t1)=A_v(t1') and A_v(t2)=A_v(t2') and…and A_v(tn)=A_v(tn')
implies A_v(t)=A_v(t')

for any valuation **v** .

The satisfaction of an equation by a model **A** is denoted by
A |= (t = t' if (t1 = t1') /\ (t2 = t2') /\…/\ (tn = tn'))

CafeOBJ _=_ (meta-level equality) and Boolean _=_ (object level equality)

```
If a specification SP includes,
      op _=_ : S S -> Bool .
      eq (X = X) = true .
      ceq (X = Y) if (X = Y) .
then
   SP |= t=t' if (t1=t1')/\ (t2=t2')/\.../\(tn=tn')
iff
   SP |= (t1=t1' and t2=t2' and ...and tn=tn'
              implies t=t') = true  .
```

1. **Object-level equality can be substitutes for meta-level equality**
2. **Every sentence can be written as Boolean term.**

SPEC-algebra

For a specification **SPEC** = (**(S,F,F^{c}), E)**, a **SPEC-algebra** is a **(S,F,F^{C})-algebra** which satisfies all equations in **E** .

Satisfiability of property by specification: SPEC |= prop

A specification **SPEC = ((S,F,F^c),E)** is defined to satisfy a property **p** (a term of sort **Bool)** iff **A |= (p** = true) holes for any **SPEC-algebra A.**

The satisfaction of a predicate **prop** by a specification **SPEC** = **((S,F,F^c),E)** is denoted by:

SPEC |= p or E |= p

A most important purpose of developing a specification **SPEC = ((S,F,F^c),E)** in CafeOBJ is to check whether

SPEC |= prop

holds for a predicate **prop** which describes some important property of the system which **SPEC** specifies.

Proof rules for SPEC |= prop (semantic entailment)

For doing formal verification, it is common to think of syntactic proof theoretic entailment:

SPEC |- prop

which corresponds to semantic entailment:

SPEC |= prop .

We have a sound and *quasi* complete set of proof rules for |- (see Lecture Note 3b for details) which satisfies:

SPEC |- prop iff SPEC |= prop

for unstructured specifications and constitutes a theoretical foundation for proof score construction.