# Modeling and Specification of Authentication Protocol (NSLPK) in OTS/CafeOBJ

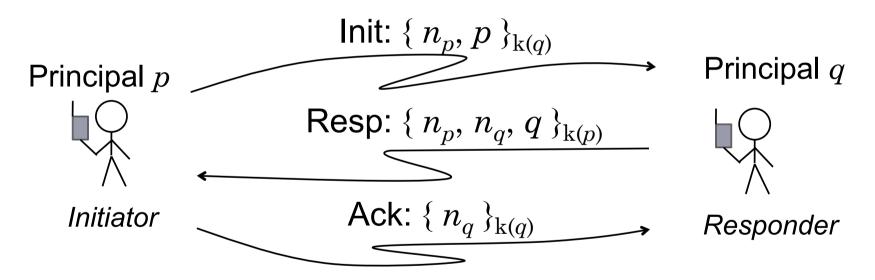
## Lecture Note 06 CafeOBJ Team for JAIST-FSSV2010

## **Topics**

- ♦ NSLPK, an authentication protocol, and Agreement Property that NSLPK should enjoy.
- ◆ A specification of NSLPK.
- Formalization of Agreement Property.

#### **NSLPK**

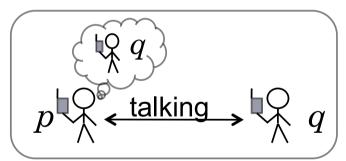
♦ An authentication protocol that is a revised version (by Gavin Lowe in 1995) of NSPK published by Roger Needham and Michael Schroeder in 1978.

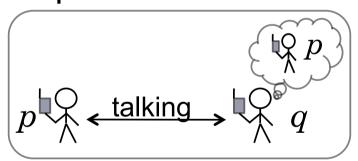


- $\checkmark n_x$ : a nonce (an unguessable random number) made by a pricipal x.
- $\checkmark$   $\{m\}_{k(x)}$ : a ciphertext obtained by encrypting a message (tuple) m with the principal x's public key.

## **Agreement Property**

- Whenever a protocol run is successfully completed by p and q,
  - the principal with which p is communicating is really q, and
  - the principal with which q is communicating is really p even if there are malicious principals.

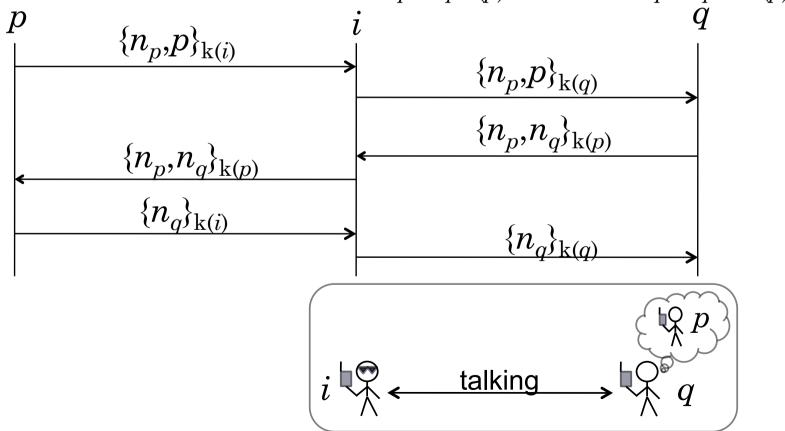




- For verification of the property,
  - the assumptions used are made clear,
  - a transition system (an OTS) of NSLPK is made, and
  - the property is formalized.

#### Lowe's Attack on NSPK

• 17 years passed since it was created till an attack was found by Lowe on NSPK whose difference from NSLPK is only that a Resp message is  $\{n_p, n_q\}_{k(p)}$  but not  $\{n_p, n_q, q\}_{k(p)}$ .



## **Assumptions (1)**

There are an arbitrary number of principals, all of which except for one are trustable.

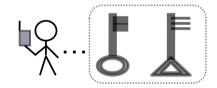
trustable



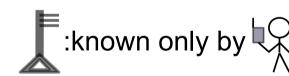


untrustable

 Each principal is given a pair of public & private keys; the public key is known by all principals, while the private key only by the principal.



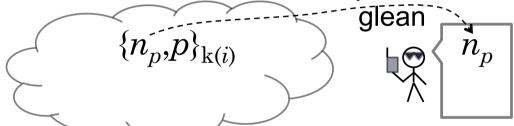




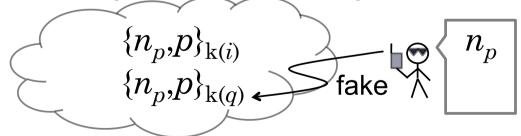
- The cryptosystem used is perfect:
  - Ciphertexts can only be decrypted with the corresponding private keys.
  - Nonces (and private keys and the plaintexts of ciphertexts) cannot be guessed.

## **Assumptions (2)**

- ◆ The behaviors of malicious principals are formalized as the *intruder* proposed by Dolev&Yao; the intruder does the following:
  - to glean as much information as possible from the network,



• to fake messages based on the gleaned information, and



to imitate a trustable principal.

## Making an OTS of NSLPK

- ◆ Formalization of data such as nonces, ciphertexts, etc.
- Formalization of the behaviors of NSLPK.
  - To determine what values are observed.
  - Formalization of sending messages exactly obeying the protocol.
  - Formalization of faking messages based on the gleaned information from the network.

## **Principals & Random Numbers**

♦ Module PRINCIPAL:

```
mod* PRINCIPAL principal-sort Principal {
    [Principal]
    op intruder : -> Principal
    op _=_ : Principal Principal -> Bool {comm}
    eq (P:Principal = P) = true .
    ceq P1:Principal = P2:Principal if P1 = P2 .
}
```

◆ Module RANDOM:

```
mod* RANDOM principal-sort Random {
    [Random]
    op _=_ : Random Random -> Bool {comm}
    eq (R:Random = R) = true .
    ceq R1:Random = R2:Random if R1 = R2 .
}
```

#### **Nonces**

◆ Module NONCE: One constructor is declared.

```
op n: Principal Principal Random -> Nonce {constr} \checkmark n (p,q,r) denotes a nonce made by p for q, where r makes the nonce unique and unguessable.

• n_p in \{n_p,p\}_{\mathbf{k}(q)} is denoted by n (p,q,r_1).

• n_q in \{n_p,n_q,q\}_{\mathbf{k}(p)} is denoted by n (q,p,r_2).
```

 $\checkmark p, q, r \text{ in } n(p,q,r)$  are meta-information.

✓ The following operators are prepared: Init:  $p \rightarrow q \{n_p, p\}_{k(q)}$ 

```
eq creator(n(C,W,R)) = C . 
eq forwhom(n(C,W,R)) = W . 
eq random(n(C,W,R)) = R . 
eq (N1 = N2) = (creator(N1) = creator(N2) and forwhom(N1) = forwhom(N2) and random(N1) = random(N2)) .
```

## Ciphertexts in Init Messages

◆ Module CIPHER1: One constructor is declared.

```
op enc1 : Principal Nonce Principal
                                  -> Cipher1 {constr}
```

- $\checkmark$  enc1 (p, n, q) denotes a ciphertext  $\{n, q\}_{k(n)}$ .
- ✓ The following operators are provided:

```
p \rightarrow q \quad \{n_p, p\}_{k(q)}
                                                    \mathsf{Resp:} \quad q \to p \quad \{n_p, n_q, q\}_{\mathbf{k}(p)}
eq key(enc1(K,N,P)) = K.
                                                    Ack: p \rightarrow q \quad \{n_a\}_{k(a)}
eq nonce (enc1(K, N, P)) = N.
eq principal(enc1(K, N, P)) = P .
eq (E11 = E12) = (key(E11) = key(E12)) and
                          nonce(E11) = nonce(E12) and
                          principal(E11) = principal(E12)) .
```

Init:

## Ciphertexts in Resp Messages

♦ Module CIPHER2: One constructor is declared.

- $\checkmark$  enc2  $(p, n_1, n_2, q)$  denotes a ciphertext  $\{n_1, n_2, q\}_{k(p)}$ .
- ✓ The following operators are provided:

```
eq key(enc2(K,N1,N2,P)) = K . 

eq nonce1(enc2(K,N1,N2,P)) = N1 . 

eq nonce2(enc2(K,N1,N2,P)) = N1 . 

eq principal(enc2(K,N1,N2,P)) = P . 

eq (E21 = E22) = (key(E21) = key(E22) and nonce1(E21) = nonce1(E22) and nonce2(E21) = nonce2(E22) and principal(E21) = principal(E22)) .
```

## Ciphertexts in Ack Messages

◆ Module CIPHER3: One constructor is declared.

```
op enc3 : Principal Nonce -> Cipher3 {constr}
```

- ✓ enc3 (p, n) denotes a ciphertext  $\{n\}_{k(n)}$ .
- ✓ The following operators are provided:

```
\text{Init:} \qquad p \to q \quad \{n_p, p\}_{\mathbf{k}(q)}
                                                                            Resp: q \rightarrow p \{n_p, n_q, q\}_{\mathbf{k}(p)}
                                                                            Ack: p \rightarrow q \quad \{n_a\}_{k(a)}
eq key(enc3(K,N)) = K .
eq nonce (enc3(K, N)) = N.
```

```
eq (E31 = E32) = (key(E31) = key(E32)) and
                 nonce(E31) = nonce(E32).
```

## Messages (1)

- A term denoting a message contains
  - the (seeming) source (sender),
  - the destination (receiver), and
  - the body (ciphertext).

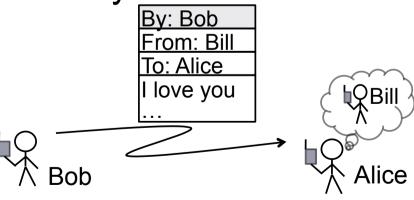


the actual source (creator).

because messages may be faked by the intruder.

This is meta-information.

- Such a term is in the form
  - m(creator,sender,receiver,ciphertext)



From: Bill To: Alice

love you

Alice

## Messages (2)

**♦** Module MESSAGE: Three constructors are declared.

```
[Message1 Message2 Message3 < Message]
op m1 : Principal Principal Principal Cipher1
                                         -> Message1 {constr}
op m2 : Principal Principal Principal Cipher2
                                         -> Message2 {constr}
op m3 : Principal Principal Principal Cipher3
                                         -> Message3 {constr}
\checkmark m1 (p?, p, q, enc1 (...)) denotes an Init message.
✓ m2 (p?, p, q, enc2 (...) ) denotes a Resp message.
✓ m3 (p?, p, q, enc3 (...)) denotes an Ack message.
                   Init: p \rightarrow q \quad \{n_p, p\}_{k(q)}
                   Resp: q \rightarrow p \quad \{n_p, n_q, q\}_{k(p)}
                   Ack: p \rightarrow q \quad \{n_a\}_{k(a)}
```

## Messages (3)

#### ✓ The following operators are provided:

```
eq creator(m1(C,S,R,E1)) = C . eq creator(m2(C,S,R,E2)) = C . eq creator(m3(C,S,R,E3)) = C . eq sender(m3(C,S,R,E1)) = S . eq sender(m2(C,S,R,E1)) = S . eq sender(m3(C,S,R,E2)) = S . eq receiver(m1(C,S,R,E3)) = R . eq receiver(m2(C,S,R,E1)) = R . eq receiver(m3(C,S,R,E2)) = R . eq cipher1(m1(C,S,R,E1)) = E1 . eq cipher2(m2(C,S,R,E2)) = E2 . eq cipher3(m3(C,S,R,E3)) = E3 .
```

#### ✓ Note that cipheri (i = 1,2,3) is declared as follows:

```
op cipheri : Messagei -> Cipheri
```

## Messages (4)

✓ The following operators are provided (cont.):

```
eq (M = M) = true.
eq (M11 = M12) = (creator(M11) = creator(M12) and
                   sender(M11) = sender(M12) and
                   receiver(M11) = receiver(M12) and
                   cipher1(M11) = cipher1(M12)).
eq (M21 = M22) = (creator(M21) = creator(M22)) and
                   sender(M21) = sender(M22) and
                   receiver(M21) = receiver(M22) and
                   cipher2(M21) = cipher2(M22)).
eq (M31 = M32) = (creator(M31) = creator(M32)) and
                   sender(M31) = sender(M32) and
                   receiver(M31) = receiver(M32) and
                   cipher3 (M31) = cipher3 (M32).
eq (M11 = M21) = false.
                                var M : Message
eq (M11 = M31) = false.
                                vars M11 M12 : Message1
eq (M21 = M31) = false.
                                vars M21 M22 : Message2
                                vars M31 M32 : Message3
              JAIST-FSSV2010, March 1-5, 2010, Kanazawa
```

#### Soups (Associative & Commutative Collections)

#### ♦ Module SOUP:

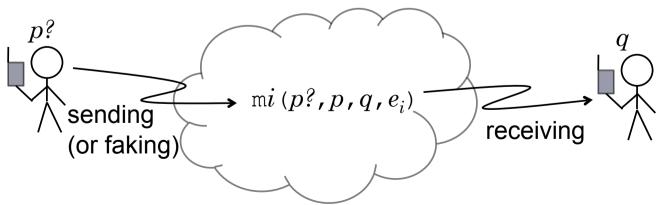
```
mod* SOUP (D :: EQTRIV) principal-sort Soup {
  [Elt.D < Soup]
  op empty : -> Soup {constr}
  op : Soup Soup -> Soup
                           {constr assoc comm id: empty}
  op \in : Elt.D Soup -> Bool
  var S : Soup vars E1 E2 : Elt.D
  eq E1 \in empty = false.
  eq E1 \setminusin (E2 S) = (E1 = E2) or E1 \setminusin S.
where EQTRIV is as follows:
 mod* EQTRIV principal-sort Elt {
   [Elt]
   op = : Elt Elt -> Bool {comm}
   eq (E:Elt = E) = true.
   eq E1:E1t = E2:E1t if E1 = E2.
                JAIST-FSSV2010, March 1-5, 2010, Kanazawa
```

#### **Networks**

Formalized as soups of messages.

```
SOUP(MESSAGE) *{sort Soup -> Network}
```

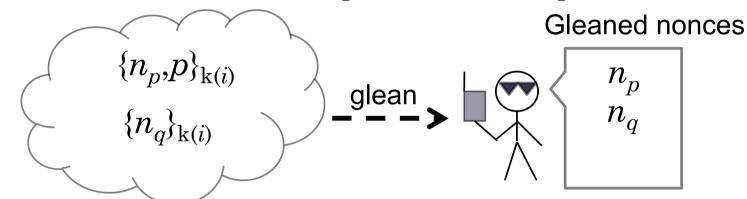
- ✓ Sending a message is formalized as putting it in the soup.
- ✓ If the soup contains  $mi(p?, p, q, e_i)$ , then q can receive it.
- $\checkmark q$  may believe that it originates in p, but it may not be true.
- ✓ Suppose that messages are never deleted from the soup.
- ✓ This assumption may make it possible to do something that
  can never happen in the real world, but covers all possible cases.



#### Soups of Nonces & Soups of Random Numbers

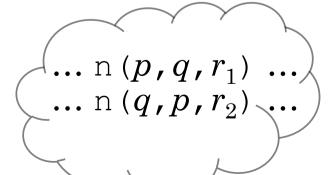
#### Soups of nonces:

SOUP(NONCE) \*{sort Soup -> NonceSoup}



#### Soups of random numbers:

SOUP(RANDOM) \*{sort Soup -> RandSoup}

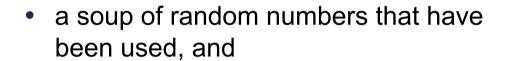


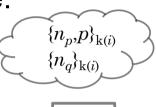
 $egin{array}{c} r_1 \ r_2 \end{array}$ 

Used random numbers

#### **Observable Values**

- The three values are made observable:
  - the network (a soup of messages),





 $egin{array}{c} r_1 \ r_2 \end{array}$ 

 a soup of nonces that have been gleaned by the intruder from the network.



The corresponding operators (called observation or observer operators or functions) are as follows:

```
op network : System -> Network
op rands : System -> RandSoup
op nonces : System -> NonceSoup
```

where System is the sort for the set of states, i.e. the state space.

#### **Initial States**

◆ An arbitrary initial state is denoted by the operator:

```
op init : -> System {constr}

such that

eq network(init) = empty .
 eq rands(init) = empty .
 eq nonces(init) = empty .
```

## Formalization of Sending Messages

Sending Init, Resp and Ack messages according to the protocol is formalized as the operators (called *transition* operators or functions):

- ✓ sdm1 (s, p, q, r) denotes the successor state of s when p sends an Init message to q in s.
- ✓ sdm2 (s, q?, p, q, r, n) denotes the successor state of s when p sends a Resp message to q in s.
- ✓ sdm3  $(s, q?, p, q, n_1, n_2)$  denotes the successor state of s when p sends an Ack message to p in s.

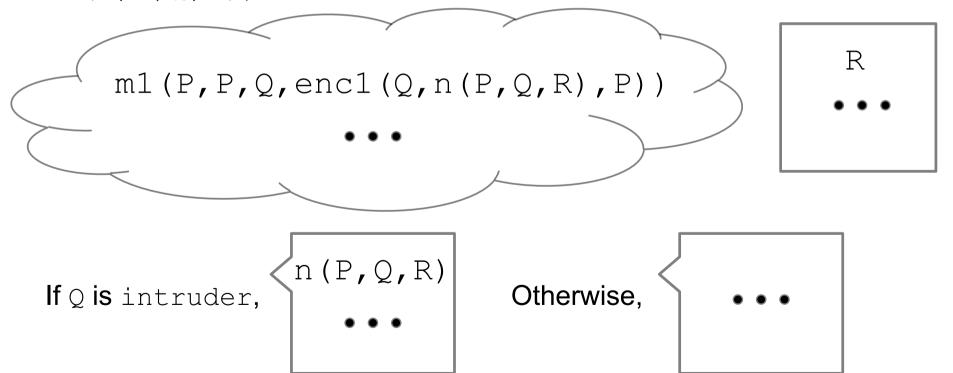
#### | Init: $p \rightarrow q \quad \{n_p, p\}_{k(q)}$

#### Resp: $q \rightarrow p \quad \{n_p, n_q, q\}_{k(p)}$

#### Ack: $p \rightarrow q \quad \{n_q\}_{k(q)}$

## **Sending Init Messages (1)**

- ◆ The effective condition of sdm1 is that a random number is fresh.
  - eq c-sdm1(S,P,Q,R) = not(R \in rands(S)).
- ♦ When c-sdm1 (S,P,Q,R) holds, in the successor state sdm1 (S,P,Q,R),



#### $\text{Init:} \qquad p \to q \quad \{n_p, p\}_{\mathbf{k}(q)}$

#### $\text{Resp:}\quad q\to p \quad \{n_p,n_q,q\}_{\mathbf{k}(p)}$

#### Ack: $p \rightarrow q \{n_q\}_{k(q)}$

## **Sending Init Messages (2)**

#### ◆ The equations for sdm1:

Note that we need to have the last one because we have to explicitly declare that nothing changes if c-sdm1 (S,P,Q,R) does not hold.

## Sending Resp Messages (1)

$$p \rightarrow q \quad \{n_p, p\}_{k(q)}$$

Resp:  $q \rightarrow p$ 

 $\{n_p, n_q, q\}_{\mathbf{k}(p)}$ 

Ack:

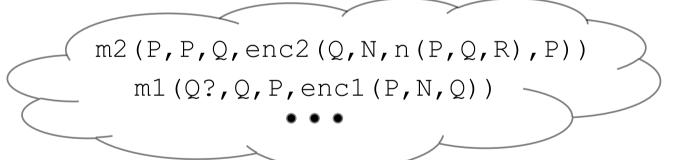
Init:

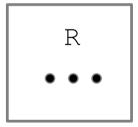
$$p \rightarrow q$$

◆ The effective condition of sdm2 is that there exists an Init message in the network and a random number is fresh.

eq c-sdm2(S,Q?,P,Q,R,N)

- $= (m1(Q?,Q,P,enc1(P,N,Q)) \setminus in network(S) and$  $not(R \in rands(S))$ .
- ♦ When c-sdm2 (S,Q?,P,Q,RN) holds, in the successor state sdm2(S,Q?,P,Q,R,N),

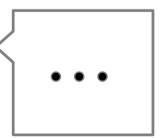




If Q is intruder,

n(P,Q,R)

Otherwise,



#### | Init: $p \rightarrow q \quad \{n_p, p\}_{\mathbf{k}(q)}$

#### Resp: $q \rightarrow p \quad \{n_p, n_q, q\}_{k(p)}$

Ack:  $p \rightarrow q \quad \{n_q\}_{k(q)}$ 

## Sending Resp Messages (2)

#### ◆ The equations for sdm2:

#### $| \text{Init:} p \rightarrow q \{n_p, p\}_{k(q)}$

#### Resp: $q \rightarrow p \quad \{n_p, n_q, q\}_{k(p)}$

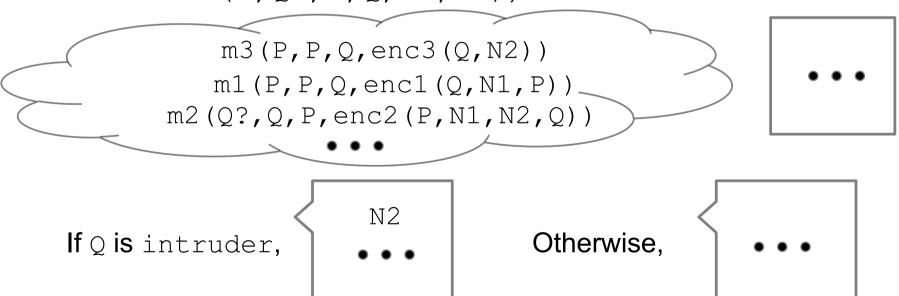
#### Ack: $p \rightarrow q \quad \{n_q\}_{k(q)}$

## Sending Ack Messages (1)

♦ The effective condition of sdm3 is that there exist an Init message and a Resp message in the network.

```
eq c-sdm3(S,Q?,P,Q,N1,N2)
= m1(P,P,Q,enc1(Q,N1,P)) \setminus in network(S) and
m2(Q?,Q,P,enc2(P,N1,N2,Q)) \setminus in network(S).
```

♦ When c-sdm3(S,Q?,P,Q,N1,N2) holds, in the successor state sdm3(S,Q?,P,Q,N1,N2),



#### Init: $p \rightarrow q \quad \{n_p, p\}_{k(q)}$

#### Resp: $q \rightarrow p \quad \{n_p, n_q, q\}_{k(p)}$

#### Ack: $p \rightarrow q \quad \{n_q\}_{k(q)}$

## Sending Ack Messages (2)

#### ◆ The equations for sdm3:

## Formalization of Faking Messages

◆ The intruder may fake messages based on the nonces and ciphertexts gleaned from the network, which is formalized as the transition operators:

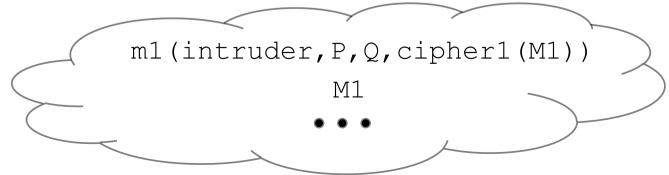
<sup>✓</sup> fkm11  $(s, p, q, m_1)$  denotes the successor state of s when the intruder fakes based on  $m_1$  an Init message, which seems to have been sent by p to q, in s.

## Faking Init Messages (1)

♦ The effective condition of fkm11 is that there exists an Init message in the network.

```
eq c-fkm11(S,P,Q,M1) = M1 \in network(S) .
```

**♦** The equations for fkm11:



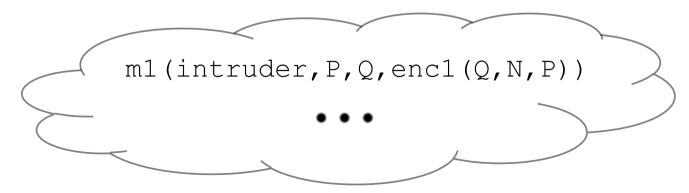
## Faking Init Messages (2)

◆ The effective condition of fkm12 is that a nonce is available to the intruder.

```
eq c-fkm12(S,P,Q,N) = N \in nonces(S) .
```

♦ The equations for fkm12:

```
ceq network(fkm12(S,P,Q,N))
= m1(intruder,P,Q,enc1(Q,N,P)) network(S)
if c-fkm12(S,P,Q,N) .
eq rands(fkm12(S,P,Q,N)) = rands(S) .
eq nonces(fkm12(S,P,Q,N)) = nonces(S) .
ceq fkm12(S,P,Q,N) = S if not c-fkm12(S,P,Q,N)
```

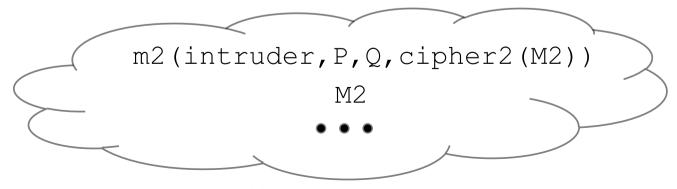


## Faking Resp Messages (1)

♦ The effective condition of fkm21 is that there exists a Resp message in the network.

```
eq c-fkm21(S,P,Q,M2) = M2 \in network(S) .
```

**♦** The equations for fkm21:



## Faking Resp Messages (2)

◆ The effective condition of fkm22 is that two different nonces are available to the intruder.

```
eq c-fkm22(S,P,Q,N1,N2) = N1 \in nonces(S) and
N2 \in nonces(S) and not(N1 = N2)
```

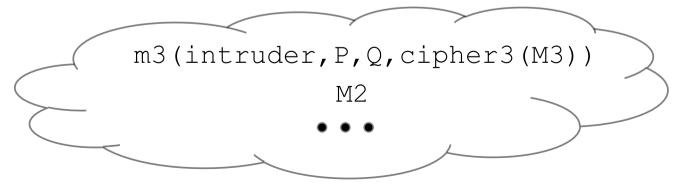
**♦** The equations for fkm22:

## Faking Ack Messages (1)

♦ The effective condition of fkm31 is that there exists an Ack message in the network.

```
eq c-fkm31(S,P,Q,M3) = M3 \in network(S) .
```

♦ The equations for fkm31:



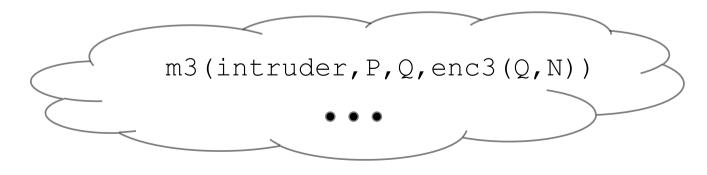
## Faking Ack Messages (2)

◆ The effective condition of fkm32 is that a nonce is available to the intruder.

```
eq c-fkm32(S,P,Q,N) = N \in nonces(S) .
```

♦ The equations for fkm32:

```
ceq network(fkm32(S,P,Q,N))
= m3(intruder,P,Q,enc3(Q,N)) network(S)
if c-fkm32(S,P,Q,N) .
eq rands(fkm32(S,P,Q,N)) = rands(S) .
eq nonces(fkm32(S,P,Q,N)) = nonces(S) .
ceq fkm32(S,P,Q,N) = S if not c-fkm32(S,P,Q,N) .
```



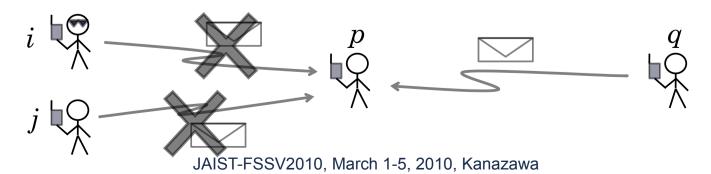
## **Constructors of State Space**

- ◆ The (reachable) state space (System) is constructed from the constructors init, sdm1, sdm2, sdm3, fkm11, fkm12, fkm21, fkm22, fkm31 and fkm32.
- ◆ Theorems on System (or invariant properties of the OTS of NSLPK) can be proved by simultaneous structural induction of the reachable state space (System).

```
 \left[ \begin{array}{c} \text{NSLPKU}\{p_j(\mathbf{s}) = \text{true for } j = 1, \dots, n\} \\ | -_{\{\mathbf{s}, \mathbf{p}, \mathbf{q}, \mathbf{r}\}} p_i(\text{sdm1}(\mathbf{s}, \mathbf{p}, \mathbf{q}, \mathbf{r})) \\ \text{NSLPK}| - (\forall \mathbf{S} : \text{System}) p_l(\mathbf{S}) \text{ for any } l \in \{1, \dots, n\} \end{array} \right]
```

## Formalization of Agreement Property (1)

- Whenever a protocol run is successfully completed by p and q,
  - the principal with which p is communicating is really q, and
  - the principal with which q is communicating is really p.
- ◆ The property can be rephrased based on the specification of NSLPK as follows:
  - Whenever p receives a Resp message that is what p really expects, the Resp message originates in q, and
  - whenever q receives an Ack message that is what q really expects, the Ack message originates in p.



## Formalization of Agreement Property (2)

- Precisely the property is described as follows:
  - If p is not the intruder, then whenever p has sent an Init
    message to q and receives a valid Resp message, which
    seems to have been sent by q, the Resp message
    originates in q, and
  - If q is not the intruder, then whenever q has sent a Resp message to p and receives a valid Ack message, which seems to have been sent by p, the Ack message originates in p.

Note that if p (or q) is the intruder, then the intruder can fake a valid Resp (or Ack) message, which does not originate in q (or p). If two nonces n,n are available to the intruder, then the intruder can fake the messages:

```
m1 (intruder, intruder, q, enc1 (q, n, intruder)) m2 (intruder, q, intruder, enc2 (intruder, n, n, q))
```

## Formalization of Agreement Property (3)

◆ That there exists a message in the network means that it has been sent by (or originates in) the creator, and any messages in the network whose receivers are p can be received by p. So, the property is formalized:

```
eq inv1(S,P,Q,Q?,R,N)
= (not(P = intruder) and
    m1(P,P,Q,enc1(Q,n(P,Q,R),P)) \in network(S) and
    m2(Q?,Q,P,enc2(P,n(P,Q,R),N,Q)) \in network(S)
    implies
    m2(Q,Q,P,enc2(P,n(P,Q,R),N,Q)) \in network(S)).

eq inv2(S,P,Q,P?,R,N)
= (not(Q = intruder) and
    m2(Q,Q,P,enc2(P,N,n(Q,P,R),Q)) \in network(S) and
    m3(P?,P,Q,enc3(Q,n(Q,P,R))) \in network(S)
    implies
    m3(P,P,Q,enc3(Q,n(Q,P,R))) \in network(S)).
```

### **Summary**

- NSLPK has been used as an example to discuss what to prepare for verification that a system enjoys a property:
  - Make the assumptions clear.
  - Create a transition system (an OTS) of the system.
    - √ Formalize data used.
    - ✓ Determine what values are observed.
    - Determine what actions of the system are formalized as transitions.
    - You may want to take into account the property.
  - Formalize the property based on the specification of the OTS.