i117: Basic of Programming

4. User-defined data structures (1)

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Roadmap

- How to define data structures
- User-defined lists
- Stacks
- Queues
- Peano natural numbers

How to define data structures

- What are called "structure," "record," etc. are used to define data structures in some other programming languages, such as C and Pascal.
- "*class*" is used to define data structures in Python, which has an aspect of object-oriented programming like Java in which "class" is used to define data structures (or the structure of objects)

User-defined lists

Inductively defined as follows:

(1) nil is a list.

(2) If *e* is an element and *l* is a list, then *e* | *l* is a list.

nil	the empty list
0 | nil	the list that only consists of 0
1 | 0 | nil	the list that consists of 1 and 0 in this order
2 | 1 | 0 | nil	the list that consists of 2, 1 and 0 in this order

User-defined lists

```
class List(object):
  def cons(self,e):
    pass
  def append(self,l):
    pass
  def len(self):
    pass
  def __str__(self):
    pass
```

Honestly, we do not need this class.

This class is used as the role of "*interface*" of Java.

A procedure defined inside a class is generally called a "*method*."

In the Python terminology, the name of such a procedure is called an "*attribute*."

Every method must take **self** (a receiver object of the class) as the first parameter.

__str__ will be explained on the next page.

User-defined lists

```
class Nil(List):
  def cons(self,e):
    return NnList(e,self)
  def append(self,l):
    return l
  def len(self):
    return 0
  def __str__(self):
    return 'nil'
```

Nil() makes an object of class Nil, representing the empty list nil.

Let *nil* refer to the object.

nil.len() returns 0.

Note that len() does not take any parameters, even if its definition has one (**self**).

nil (the receiver of the message) is used as the first parameter.

nil.__str__() returns 'nill'.

str(*nil*) can also be used and returns 'nill'.

User-defined lists

There are two *instance variables* (or attributes) in class NnLIST.

```
class NnList(List):
  def __init__(self,h,t):
    self.head = h
    self.tail = t
  def cons(self,e):
    return NnList(e,self)
  def append(self,l):
    return NnList(self.head,self.tail.append(l))
  def len(self):
    return 1 + self.tail.len()
  def __str__(self):
    return str(self.head) + ' | ' + str(self.tail)
```

Let *lst* be an object (a list) of class NnList. *lst.head* refers to the head (or top) element of the list, and *lst.tail* referes to the remaining part (tail or bottom) of the list.

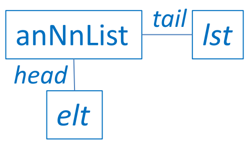
User-defined lists

```
....
def __init__(self,h,t):
  self.head = h
  self.tail = t
....
```

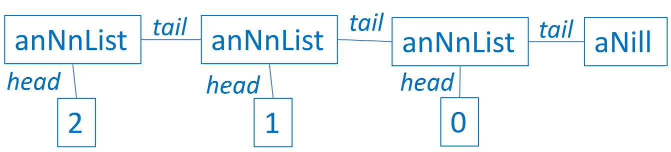
This is called a constructor with which an object (or an instance) of class NnList is made.

NnList(*elt*, *lst*)

It makes the list whose head is *elt* and whose tail is *lst*.



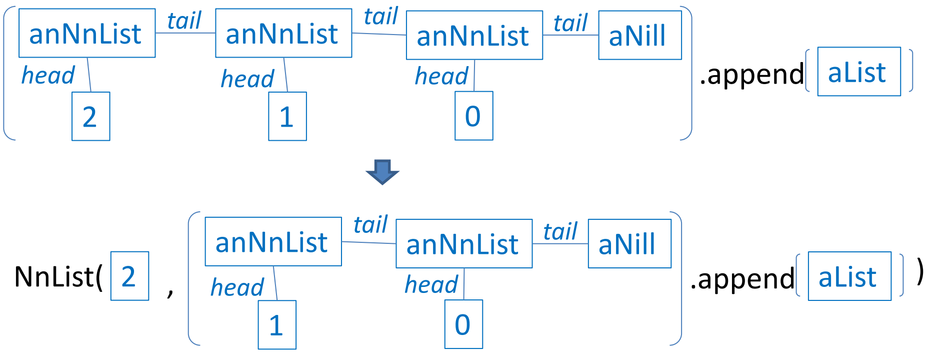
NnList(2, NnList(1, NnList(0, Nil()))) makes the following:



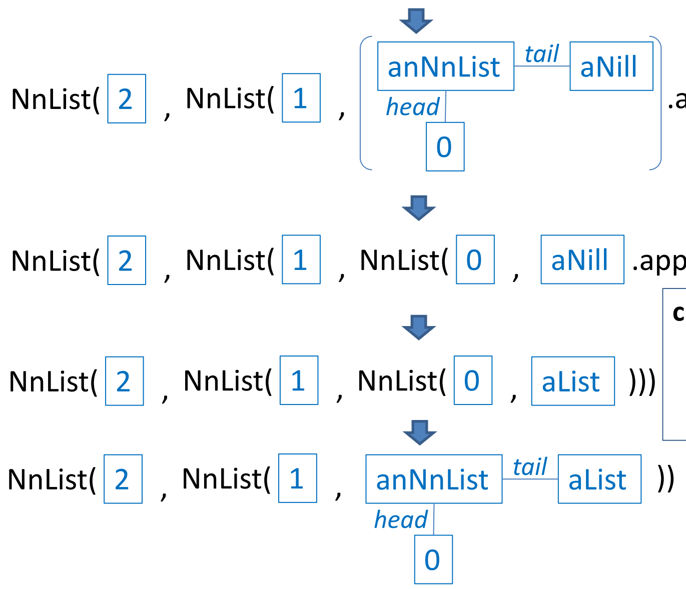
User-defined lists

```
....
def append(self,l):
    return NnList(self.head,self.tail.append(l))
....
```

It concatenates two lists.

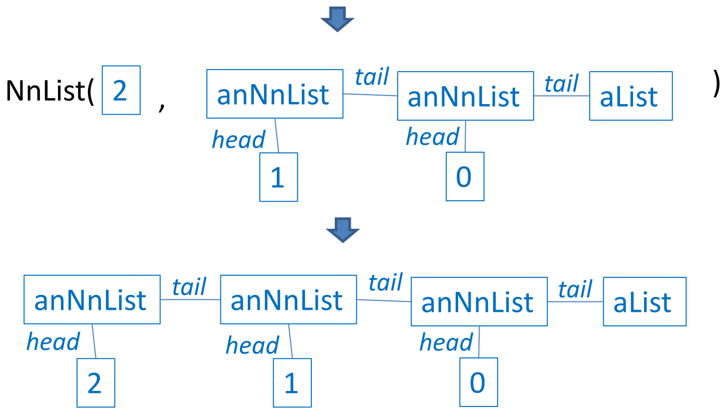


User-defined lists



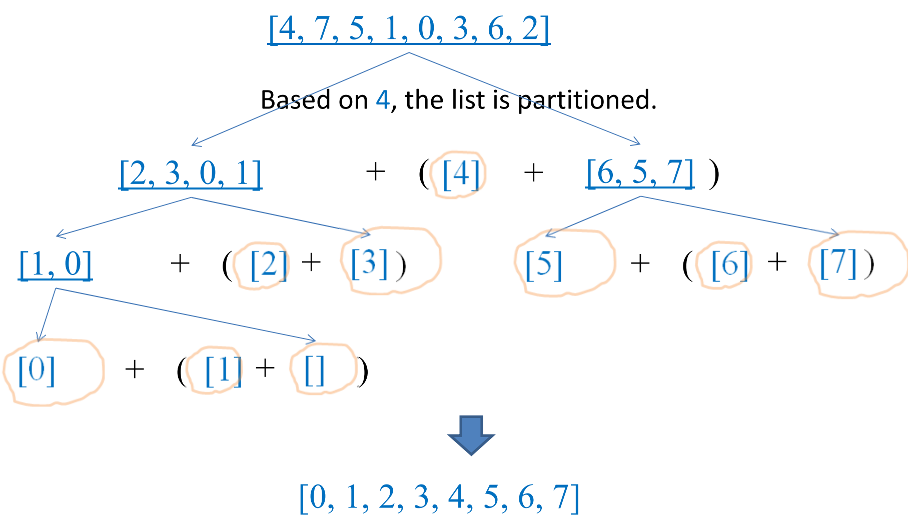

```
class Nil(List): ...
    def append(self,l):
        return l
    ...
```

User-defined lists



User-defined lists

Soring with Quicksort



User-defined lists

```
def qsort(lst):
  if lst.len() <= 1:
    return lst
  else:
    pair = partition(lst.head,lst.tail)
    return qsort(pair[0]).append(qsort(pair[1]).cons(lst.head))
```

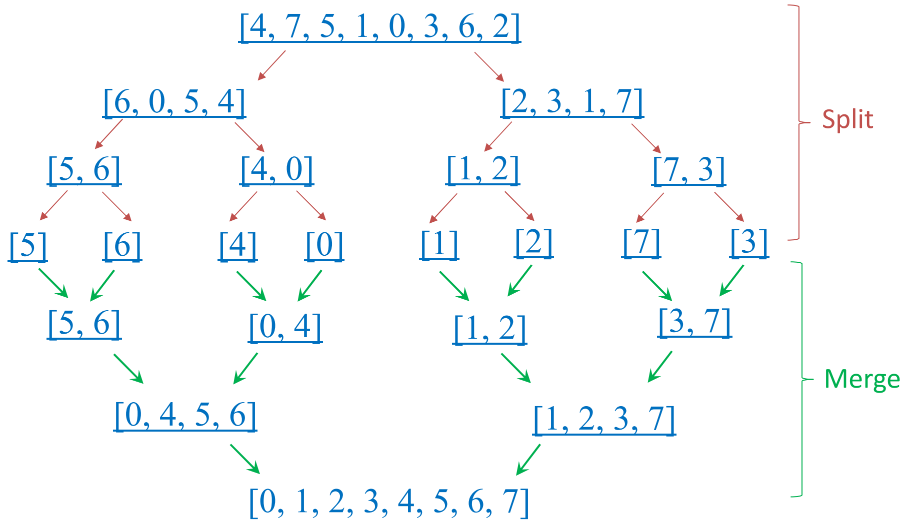
```
def partition(pvt,lst):
  pair = (Nil(), Nil())
  while isinstance(lst,NnList):
    e = lst.head
    lst = lst.tail
    if e < pvt:
      pair = (pair[0].cons(e), pair[1])
    else:
      pair = (pair[0], pair[1].cons(e))
  return pair
```


User-defined lists

```
l0 = Nil()
l1 = NnList(2,l0)
l2 = NnList(6,l1)
l3 = NnList(3,l2)
l4 = NnList(0,l3)
l5 = NnList(1,l4)
l6 = NnList(5,l5)
l7 = NnList(7,l6)
l8 = NnList(4,l7)
print(str(l8))
print(str(qsort(l8)))
```

User-defined lists

Soring with Mergesort



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User-defined lists

```
def msort(lst):
  if lst.len() <= 1:
    return lst
  else:
    l1 = Nil()
    l2 = Nil()
    flag = True
    while isinstance(lst,NnList):
      if flag:
        l1 = l1.cons(lst.head)
      else:
        l2 = l2.cons(lst.head)
      lst = lst.tail
      flag = not flag
    return merge(msort(l1),msort(l2))
```

```
def merge(l1,l2):
  if isinstance(l1,Nil):
    return l2
  elif isinstance(l2,Nil):
    return l1
  else:
    if l1.head < l2.head:
      return NnList(l1.head,merge(l1.tail,l2))
    else:
      return NnList(l2.head,merge(l1,l2.tail))
```

User-defined lists

```
l0 = Nil()
l1 = NnList(2,l0)
l2 = NnList(6,l1)
l3 = NnList(3,l2)
l4 = NnList(0,l3)
l5 = NnList(1,l4)
l6 = NnList(5,l5)
l7 = NnList(7,l6)
l8 = NnList(4,l7)
print(str(l8))
print(str(msort(l8)))
```


Stacks

- A collection of data.
- Used in the *Last-In-First-Out* (*LIFO*) way.
- Three basic operations:
  – *push* puts an element in a stack at the top
  – *pop* deletes the top from a stack (if any)
  – *top* returns the top of a stack (if any)

Stacks

```
class Stack(object):
    def push(self,e):
        pass
    def pop(self):
        pass
    def top(self):
        pass
    def __str__(self):
        pass
```

```
class EmptyStack(Stack):
    def push(self,e):
        return NeStack(e,self)
    def pop(self):
        return self
    def top(self):
        return None
    def __str__(self):
        return 'es'
```


Stacks

```
class NeStack(Stack):
    def __init__(self,t,b):
        self.topElt = t
        self.botom = b
    def push(self,e):
        return NeStack(e,self)
    def pop(self):
        return self.botom
    def top(self):
        return self.topElt
    def __str__(self):
        return str(self.topElt) + ' ; ' + str(self.botom)
```

Python does not allow us to use the same name for instance variables ar methods.

Stacks

```
s1 = EmptyStack()
s2 = s1.push(0)
s3 = s2.push(1)
s4 = s3.push(2)
print(str(s4))
print(s4.top())
s5 = s4.pop()
print(str(s5))
print(s5.top())
```


Queues

- A collection of data.
- Used in the *First-In-First-Out* (*FIFO*) way.
- Three basic operations:
  - *enqueue* puts an element in a queue at the end
  - *dequeuer* deletes the top from a queue (if any)
  - *top* returns the top of a queue (if any)
- Implemented with the built-in list in this course.

(Note that there are multiple ways to implement each data structure, such as queues.)

Queues

```
class Queue(object):
    def __init__(self):
        self.elements = []
    def isEmpty(self):
        if self.elements == []:
            return True
        else:
            return False
    def top(self):
        if self.isEmpty():
            return None
        else:
            return self.elements[0]
```

```
    def dequeue(self):
        if not self.isEmpty():
            self.elements = self.elements[1:]
    def enqueue(self,e):
        self.elements = self.elements + [e]
    def __str__(self):
        return str(self.elements)
```


Queues

```
q = Queue()
print(str(q))
print(q.isEmpty())
q.enqueue(1)
q.enqueue(2)
q.enqueue(3)
print(str(q))
print(q.isEmpty())
print(q.top())
q.dequeue()
print(str(q))
q.dequeue()
print(str(q))
```

Peano natural numbers

Natural numbers have been formalized by Giuseppe Peano (1858 – 1932), an Italian mathematician.

Inductively defined as follows:

(1) 0 is a natural number.

(2) If n is a natural number, then the successor of n, denoted $\mathrm{s}(n)$, is also a natural number.

0	$\mathrm{s}(0)$	$\mathrm{s}(\mathrm{s}(0))$	$\mathrm{s}(\mathrm{s}(\mathrm{s}(0)))$	$\mathrm{s}(\mathrm{s}(\mathrm{s}(\mathrm{s}(0))))$
0	1	2	3	4

Peano natural numbers

Addition is defined as follows:

$0 + y = y$
$\mathrm{s}(x) + y = \mathrm{s}(x + y)$

Multiplication is defined as follows:

$0 * y = 0$
$\mathrm{s}(x) * y = y + (x * y)$

Peano natural numbers

```
class Nat(object):
    def plus(self,y):
        pass
    def multiply(self,y):
        pass
    def __str__(self):
        pass
    def equal(self,y):
        pass
```

```
class Zero(Nat):
    def plus(self,y):
        return y
    def multiply(self,y):
        return self
    def __str__(self):
        return '0'
    def equal(self,y):
        if isinstance(y, Zero):
            return True
        else:
            return False
```


Peano natural numbers


```
class NzNat(Nat):
    def __init__(self,p):
        self.prev = p
    def plus(self,y):
        return NzNat(self.prev.plus(y))
    def multiply(self,y):
        return y.plus(self.prev.multiply(y))
    def __str__(self):
        return 's(' + str(self.prev) + ')'
    def equal(self,y):
        if isinstance(y, NzNat):
            return self.prev.equal(y.prev)
        else:
            return False
```

Peano natural numbers

```
zero = Zero()
one = NzNat(zero)
two = NzNat(one)
three = NzNat(two)
four = NzNat(three)
print('zero: ', str(zero))
print('one: ', str(one))
print('two: ', str(two))
print('three: ', str(three))
print('four: ', str(four))
print(str(three), '+', str(four), ' = ', str(three.plus(four)))
print(str(three), '*', str(two), ' = ', str(three.multiply(two)))
```