i117: Basic of Programming

7. Arithmetic calculator: virtual machine & compiler

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Roadmap

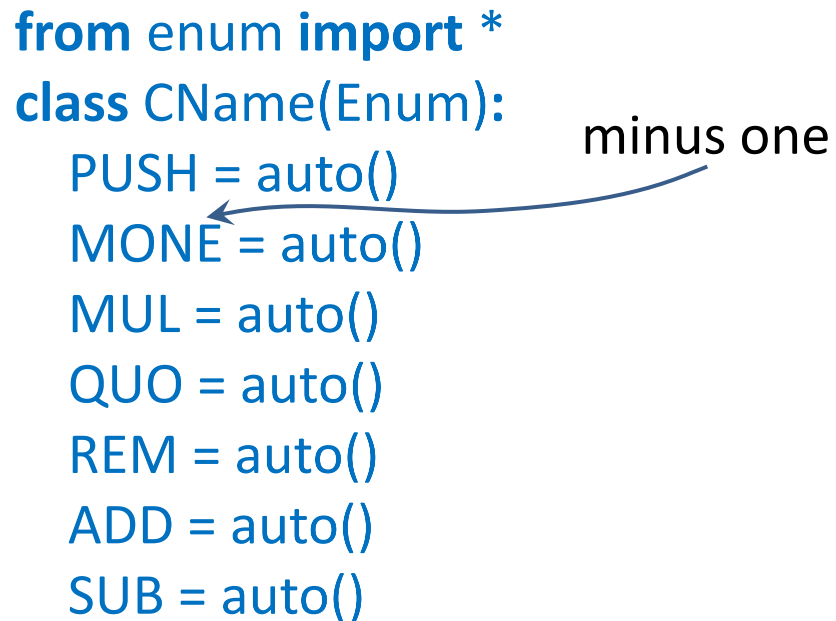
- Virtual machine for an arithmetic calculator
- Compiler for an arithmetic calculator

Virtual machine for an arithmetic calculator

- A virtual machine (VM) is a software system that emulates a physical machine like a microprocessor, such as Java virtual machine (JVM).
- The VM developed here uses a stack (of integers) and then is called a stack machine.
- It has a collection of commands (of instructions) like Java bytecode.
- It handles a list of commands with a stack.

Virtual machine for an arithmetic calculator

We first prepare command names.



```python
from enum import *
class CName(Enum):
    PUSH = auto()
    MONE = auto()
    MUL = auto()
    QUO = auto()
    REM = auto()
    ADD = auto()
    SUB = auto()
    EQ = auto()
    NEQ = auto()
    LT = auto()
    GT = auto()
    AND = auto()
    OR = auto()
    NSC = auto()
```

minus one

no such command

Virtual machine for an arithmetic calculator

```
def __str__(self):
    if self == CName.PUSH:
        return 'push'
    elif self == CName.MONE:
        return 'mone'
    elif self == CName.MUL:
        return 'mul'
    elif self == CName.QUO:
        return 'quo'
    elif self == CName.REM:
        return 'rem'
    elif self == CName.ADD:
        return 'add'
    elif self == CName.SUB:
        return 'sub'
```

```
    elif self == CName.EQ:
        return 'eq'
    elif self == CName.NEQ:
        return 'neq'
    elif self == CName.LT:
        return 'lt'
    elif self == CName.GT:
        return 'gt'
    elif self == CName.AND:
        return 'and'
    elif self == CName.OR:
        return 'or'
    else:
        return 'noSuchCommand'
```

Virtual machine for an arithmetic calculator

We then prepare the commands used for our VM.

```
class Command(object):
    cname = CName.NSC
    num = 0
    def __init__(self, cn, x):
        self.cname = cn
        if cn == CName.PUSH:
            self.num = x
    def __str__(self):
        if self.cname == CName.PUSH:
            return str(self.cname) + '(' + str(self.num) + ')'
        else:
            return str(self.cname)
```

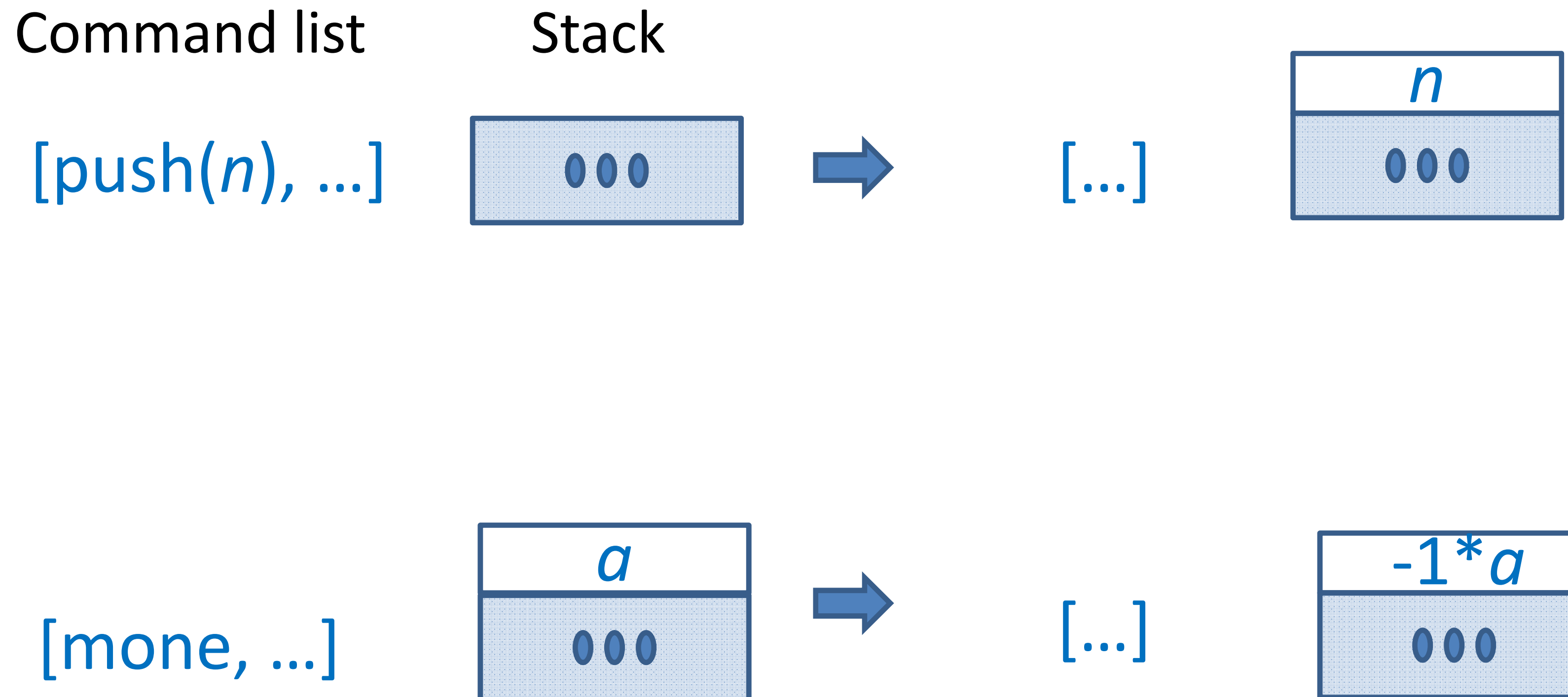
push(3)

add

neq

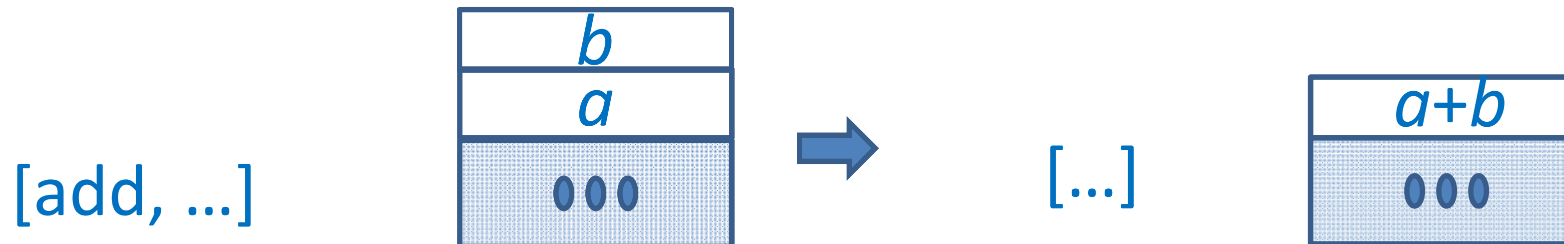
and

Virtual machine for an arithmetic calculator

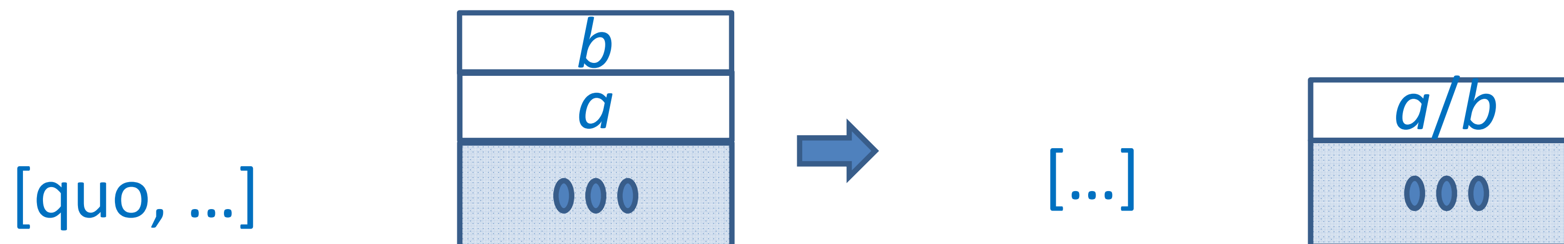


If the stack is empty, then an exception called VMError will be raised.

Virtual machine for an arithmetic calculator



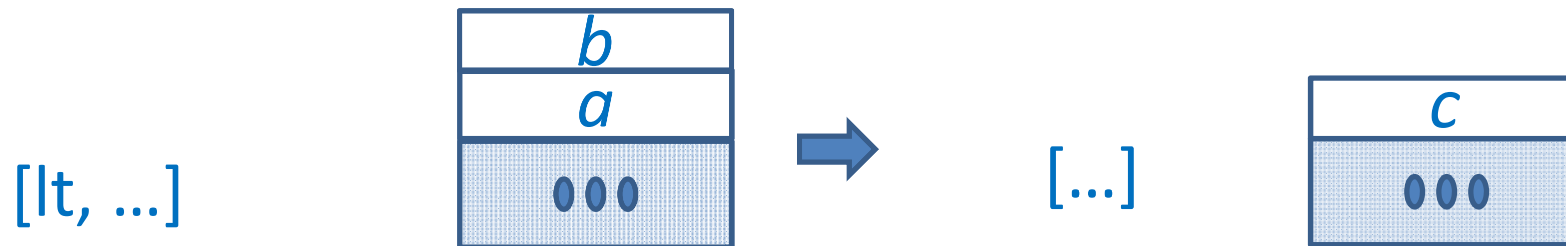
If the stack is empty or only consists of one element, then an exception called VMError will be raised.



If the stack is empty or only consists of one element, then an exception called VMError will be raised.

If b = 0, then an exception called DivisionByZero will be raised.

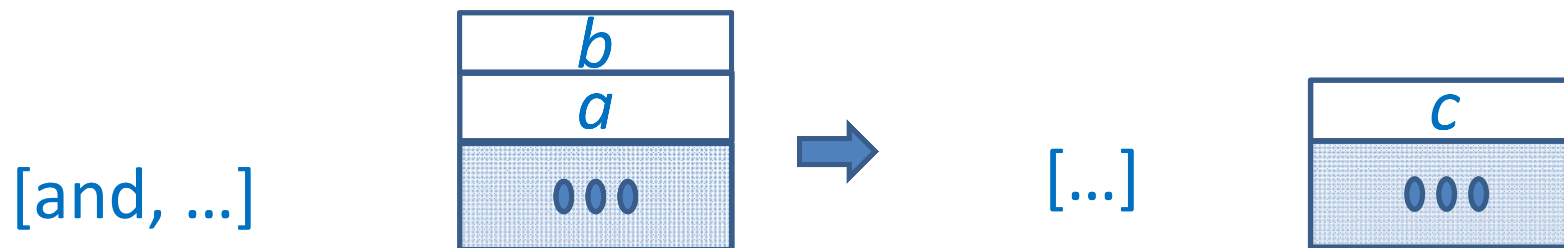
Virtual machine for an arithmetic calculator



$$c = \begin{cases} 1 & \text{if } a < b \\ 0 & \text{otherwise} \end{cases}$$

If the stack is empty or only consists of one element, then an exception called VMError will be raised.

Virtual machine for an arithmetic calculator



$$c = \begin{cases} 0 & \text{if } a = 0 \text{ or } b = 0 \\ 1 & \text{otherwise} \end{cases}$$

If the stack is empty or only consists of one element, then an exception called VMError will be raised.

Virtual machine for an arithmetic calculator

If the stack is empty, then an exception called VMError will be raised.

Virtual machine for an arithmetic calculator

```
class DivisionByZero(Exception):
    pass

class VMError(Exception):
    pass
```

```
class Stack(object):
    ...
    def isEmpty(self):
        pass
    def isEmpOrOne(self):
        pass
```

```
class EmptyStack(Stack):
    ...
    def isEmpty(self):
        return True
    def isEmpOrOne(self):
        return True
```

```
class NeStack(Stack):
    ...
    def isEmpty(self):
        return False
    def isEmpOrOne(self):
        return self.botom.isEmpty()
```

Virtual machine for an arithmetic calculator

The procedure l2s converts a list of objects to a string such that the elements are displayed so that humans can read.

```
def l2s(lst):
    s = '['
    l = len(lst)
    for e in lst:
        s = s + str(e)
        l = l - 1
        if l > 0:
            s = s + ', '
    s = s + ']'
    return s
```

Virtual machine for an arithmetic calculator

The VM is defined as follows:

```
class VM(object):
    clist = []
    stk = EmptyStack()

    def __init__(self, cl):
        self.clist = cl
        self.stk = EmptyStack()

    def str(self):
        return 'stack: ' + str(self.stk) + ', command list: ' + l2s(self.clist)
```

Virtual machine for an arithmetic calculator

```
def run(self):
    for com in self.clist:
        if com.cname == CName.PUSH:
            self.stk = self.stk.push(com.num)
        elif com.cname == CName.MONE:
            if self.stk.isEmpty():
                raise VMError('stk is empty for mone')
            x = self.stk.top()
            self.stk = self.stk.pop().push(-1 * x)
        elif com.cname == CName.MUL:
            if self.stk.isEmpOrOne():
                raise VMError('stk consists of 1 or 0 element for mul')
            y = self.stk.top()
            x = self.stk.pop().top()
            self.stk = self.stk.pop().pop().push(x * y)
```

Virtual machine for an arithmetic calculator

```
elif com.cname == CName.QUO:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for quo')
    y = self.stk.top()
    x = self.stk.pop().top()
    if y == 0:
        raise DivisionByZero('division by zero in VM')
    self.stk = self.stk.pop().pop().push(x // y)
elif com.cname == CName.REM:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for rem')
    y = self.stk.top()
    x = self.stk.pop().top()
    if y == 0:
        raise DivisionByZero('division by zero in VM')
    self.stk = self.stk.pop().pop().push(x % y)
```

Virtual machine for an arithmetic calculator

```
elif com.cname == CName.ADD:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for add')
    y = self.stk.top()
    x = self.stk.pop().top()
    self.stk = self.stk.pop().pop().push(x + y)
elif com.cname == CName.SUB:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for sub')
    y = self.stk.top()
    x = self.stk.pop().top()
    self.stk = self.stk.pop().pop().push(x - y)
```

Virtual machine for an arithmetic calculator

```
elif com.cname == CName.EQ:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for eq')
    y = self.stk.top()
    x = self.stk.pop().top()
    if x == y:
        self.stk = self.stk.pop().pop().push(1)
    else:
        self.stk = self.stk.pop().pop().push(0)
elif com.cname == CName.NEQ:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for neq')
    y = self.stk.top()
    x = self.stk.pop().top()
    if x == y:
        self.stk = self.stk.pop().pop().push(0)
    else:
        self.stk = self.stk.pop().pop().push(1)
```

Virtual machine for an arithmetic calculator

```
elif com.cname == CName.LT:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for lt')
    y = self.stk.top()
    x = self.stk.pop().top()
    if x < y:
        self.stk = self.stk.pop().pop().push(1)
    else:
        self.stk = self.stk.pop().pop().push(0)
elif com.cname == CName.GT:
    if self.stk.isEmpOrOne():
        raise VMError('stk consists of 1 or 0 element for gt')
    y = self.stk.top()
    x = self.stk.pop().top()
    if x > y:
        self.stk = self.stk.pop().pop().push(1)
    else:
        self.stk = self.stk.pop().pop().push(0)
```

Virtual machine for an arithmetic calculator

```
elif com.cname == CName.AND:
   if self.stk.isEmpOrOne():
      raise VMError('stk consists of 1 or 0 element for and')
   y = self.stk.top()
   x = self.stk.pop().top()
   if x == 0 or y == 0:
      self.stk = self.stk.pop().pop().push(0)
   else:
      self.stk = self.stk.pop().pop().push(1)
elif com.cname == CName.OR:
   if self.stk.isEmpOrOne():
      raise VMError('stk consists of 1 or 0 element for or')
   y = self.stk.top()
   x = self.stk.pop().top()
   if x == 0 and y == 0:
      self.stk = self.stk.pop().pop().push(0)
   else:
      self.stk = self.stk.pop().pop().push(1)
```

Virtual machine for an arithmetic calculator

```
        else:
            raise VMError("An invalid command was met!")
    if self.stk.isEmpty():
        raise VMError('stk is empty when vm terminates!')
    return self.stk.top()
```

Virtual machine for an arithmetic calculator

```
push3 = Command(CName.PUSH,3)
push4 = Command(CName.PUSH,4)
push5 = Command(CName.PUSH,5)
add = Command(CName.ADD,None)
mul = Command(CName.MUL,None)
cl = [push3, push4, push5, mul, add]
print(l2s(cl))
vm = VM(cl)
print('VM result: ', vm.run())
```

```
push3 = Command(CName.PUSH,3)
add = Command(CName.ADD,None)
cl = [push3, add]
print(l2s(cl))
vm = VM(cl)
try:
    print('VM result: ', vm.run())
except VMError as em:
    print(em)
```

Virtual machine for an arithmetic calculator

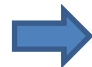
```
push3 = Command(CName.PUSH,3)
push4 = Command(CName.PUSH,4)
push2 = Command(CName.PUSH,2)
add = Command(CName.ADD,None)
quo = Command(CName.QUO,None)
cl = [push3, push4, add, push2, quo]
print(l2s(cl))
vm = VM(cl)
print('VM result: ', vm.run())
```

```
push3 = Command(CName.PUSH,3)
push4 = Command(CName.PUSH,4)
push0 = Command(CName.PUSH,0)
add = Command(CName.ADD,None)
rem = Command(CName.QUO,None)
cl = [push3, push4, add, push0, rem]
print(l2s(cl))
vm = VM(cl)
try:
    print('VM result: ', vm.run())
except DivisionByZero as em:
    print(em)
```

Compiler for an arithmetic calculator

- We will make a compiler that generates a list of commands from an arithmetic expression parse tree.

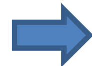
aNum: *n*



[push(*n*)]

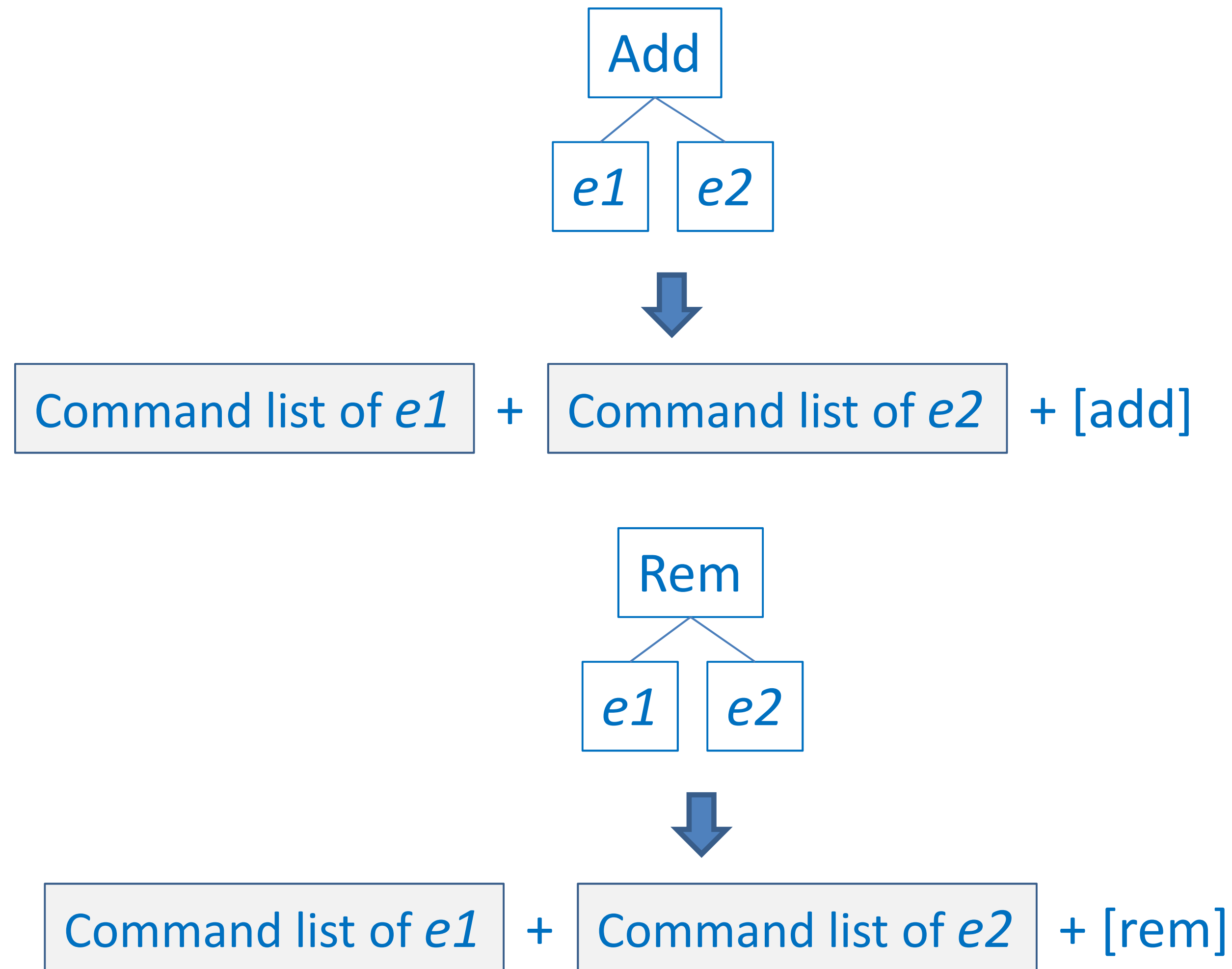
Umi

e

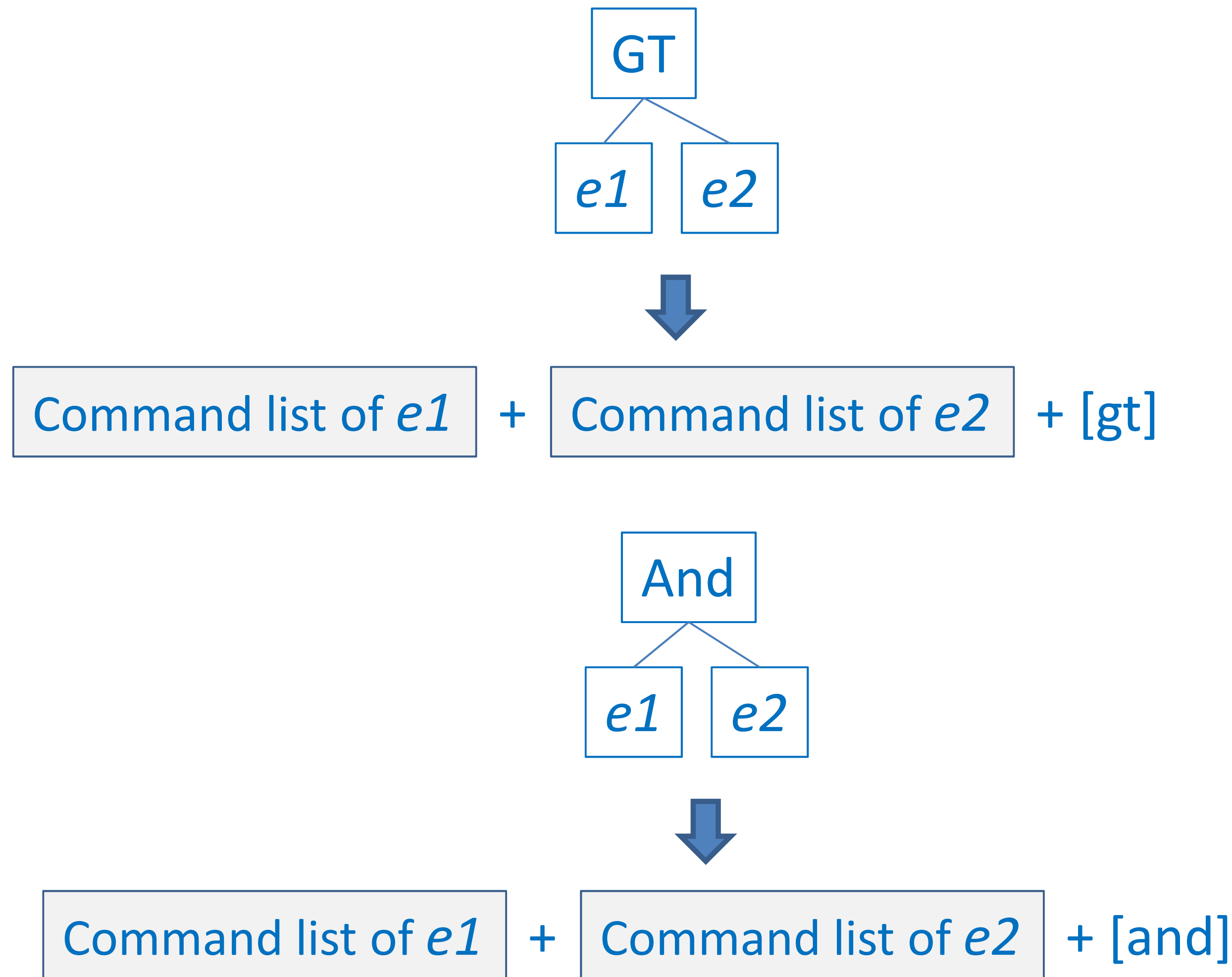


Command list of *e* + [mone]

Compiler for an arithmetic calculator



Compiler for an arithmetic calculator



Compiler for an arithmetic calculator

```
class ExpParseTree(object):
    …
    def compile(self):
        pass
```

```
class NumParseTree(ExpParseTree):
    num = 0
    …
    def compile(self):
        return [Command(CName.PUSH,self.num)]
```

Compiler for an arithmetic calculator

```
class UmiParseTree(ExpParseTree):
   exp = ExpParseTree()
   ...
    def compile(self):
      cl1 = self.exp.compile()
      return cl1 + [Command(CName.MONE,None)]
```

```
class AddParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   ...
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.ADD,None)]
```

Compiler for an arithmetic calculator

```
class SubParseTree(ExpParseTree):
  exp1 = ExpParseTree()
  exp2 = ExpParseTree()
  …
   def compile(self):
     cl1 = self.exp1.compile()
     cl2 = cl1 + self.exp2.compile()
     return cl2 + [Command(CName.SUB,None)]
```

```
class MulParseTree(ExpParseTree):
  exp1 = ExpParseTree()
  exp2 = ExpParseTree()
  …
   def compile(self):
     cl1 = self.exp1.compile()
     cl2 = cl1 + self.exp2.compile()
     return cl2 + [Command(CName.MUL,None)]
```

Compiler for an arithmetic calculator

```
class QuoParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   …
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.QUO,None)]
```

```
class RemParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   …
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.REM,None)]
```

Compiler for an arithmetic calculator

```
class LTParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   …
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.LT,None)]
```

```
class GTParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   …
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.GT,None)]
```

Compiler for an arithmetic calculator

```
class EQParseTree(ExpParseTree):
  exp1 = ExpParseTree()
  exp2 = ExpParseTree()
  …
   def compile(self):
     cl1 = self.exp1.compile()
     cl2 = cl1 + self.exp2.compile()
     return cl2 + [Command(CName.EQ,None)]
```

```
class NEQParseTree(ExpParseTree):
  exp1 = ExpParseTree()
  exp2 = ExpParseTree()
  …
   def compile(self):
     cl1 = self.exp1.compile()
     cl2 = cl1 + self.exp2.compile()
     return cl2 + [Command(CName.NEQ,None)]
```

Compiler for an arithmetic calculator

```
class AndParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   ...
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.AND,None)]
```

```
class OrParseTree(ExpParseTree):
   exp1 = ExpParseTree()
   exp2 = ExpParseTree()
   ...
    def compile(self):
      cl1 = self.exp1.compile()
      cl2 = cl1 + self.exp2.compile()
      return cl2 + [Command(CName.OR,None)]
```

Compiler for an arithmetic calculator

3 + 4 * 5

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
e1 = MulParseTree(four,five)
e2 = AddParseTree(three,e1)
print(e2)
print(l2s(e2.compile()))
print(VM(e2.compile()).run())
```

(3 + 4) * 5

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
e1 = AddParseTree(three,four)
e2 = MulParseTree(e1,five)
print(e2)
print(l2s(e2.compile()))
print(VM(e2.compile()).run())
```

Compiler for an arithmetic calculator

3 + -(4 * 5)

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
e1 = MulParseTree(four,five)
e2 = UmiParseTree(e1)
e3 = AddParseTree(three,e2)
print(e3)
print(l2s(e3.compile()))
print(VM(e3.compile()).run())
```

3 + (-4 * 5)

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
e1 = UmiParseTree(four)
e2 = MulParseTree(e1,five)
e3 = AddParseTree(three,e2)
print(e3)
print(l2s(e3.compile()))
print(VM(e3.compile()).run())
```

Compiler for an arithmetic calculator

((3 + 4) * 5) / 3

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
e1 = AddParseTree(three,four)
e2 = MulParseTree(e1,five)
e3 = QuoParseTree(e2,three)
print(e3)
print(l2s(e3.compile()))
print(VM(e3.compile()).run())
```

((3 + 4) * 5) % 0

```
three = NumParseTree(3)
four = NumParseTree(4)
five = NumParseTree(5)
zero = NumParseTree(0)
e1 = AddParseTree(three,four)
e2 = MulParseTree(e1,five)
e3 = RemParseTree(e2,zero)
print(l2s(e3.compile()))
try:
    print(VM(e3.compile()).run())
except DivisionByZero as em:
    print(em)
```

Compiler for an arithmetic calculator

```
3 < 5 || 3 = 5
```

```
three = NumParseTree(3)
five = NumParseTree(5)
e1 = LTParseTree(three,five)
e2 = EQParseTree(three,five)
e3 = OrParseTree(e1,e2)
print(e3)
print(l2s(e3.compile()))
print(VM(e3.compile()).run())
```

```
5 < 5 || 5 = 5
```

```
five = NumParseTree(5)
e1 = LTParseTree(five,five)
e2 = EQParseTree(five,five)
e3 = OrParseTree(e1,e2)
print(e3)
print(l2s(e3.compile()))
print(VM(e3.compile()).run())
```

Compiler for an arithmetic calculator

4 < 3 || 4 = 4 && 0 > -1 && (3 = 4 || 3 !=4)

```
zero = NumParseTree(0)
one = NumParseTree(1)
three = NumParseTree(3)
four = NumParseTree(4)
e1 = LTParseTree(four,three)
e2 = EQParseTree(four,four)
e3 = OrParseTree(e1,e2)
e4 = UmiParseTree(one)
e5 = GTParseTree(zero,e4)
e6 = AndParseTree(e3,e5)
e7 = EQParseTree(three,four)
e8 = NEQParseTree(three,four)
e9 = OrParseTree(e7,e8)
e10 = AndParseTree(e6,e9)
print(e10)
print(l2s(e10.compile()))
print(VM(e10.compile()).run())
```