The Hamiltonian number of some classes of cubic graphs

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Abstract

A Hamiltonian walk in a graph G is a closed spanning walk of minimum length. The length of a Hamiltonian walk in G will be denoted by $h(G)$. Thus if G is a connected graph of order $n \geq 3$, then $h(G) = n$ if and only if G is Hamiltonian. Thus h may be considered as a measure of how far a given graph is from being Hamiltonian. Let G be a connected graph of order n. The Hamiltonian coefficient of G, denoted by $hc(G)$, is defined as $hc(G) = \frac{h(G)}{n}$. It is well known that for every graph G of order n, $hc(G) \leq \frac{2n-2}{n} < 2$, and $hc(G) = \frac{2n-2}{n}$ if and only if G is a tree. Let $\mathcal{CR}(3^n)$ be the class of connected cubic graphs of order n. By putting $h(3^n) = \{h(G) : G \in \mathcal{CR}(3^n)\}$ and call it the range of Hamiltonian numbers in the class of connected graphs of order n. We have found $h(3^n)$ in all situations by proving that if G is a 2-connected cubic graph of order $n \geq 10$ and $h(G) \geq n + 2$, then there exists a connected cubic graph G' of order n containing a cut edge such that $h(G) \leq h(G')$. More precisely we proved that For an even integer $n \geq 4$ and $n \neq 14$. There exists an integer b such that $h(3^n) = \{k \in \mathbb{Z} : n \leq k \leq b\}$. Moreover, an explicit formula for the integer b is given by the following.

1. $b = n$ if and only if $n = 4, 6, 8$.
2. $b = n + 2$ if and only if $n = 10, 12$.
3. If $n = 14 + 2i$ and $i \geq 0$, then $b = 18 + 3i$.

It should be noted that a cubic graph G_i of order $14 + 2i$ with $h(G_i) = 18 + 3i$ is a graph containing as many cut edges as possible. Furthermore, $\frac{h(G_i)}{|v(G_i)|} = \frac{18+3i}{14+2i} < \frac{3}{2}$ and

$$\lim_{i\to\infty} hc(G_i) = \frac{3}{2}.$$

The problem of finding the maximum value of $hc(G)$ in the class of 2-connected cubic graphs of order n is not easy. We introduce three classes of 2-connected cubic graphs with relatively small circumference and obtain several significant results on their Hamiltonian numbers and their Hamiltonian coefficients.

Key Words: Hamiltonian walk, Hamiltonian number, cubic graph.

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