MINIMIZATION OF DISTANCE TRAVELED IN SURVEILLANCE OF A POLYGONAL REGION FROM THE BOUNDARY

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Abstract. Suppose there are "intruders" who can move freely within a dark polygonal region. We study a robot which is equipped with a flashlight and can move along the boundary in order to illuminate any intruder, who tries to evade the flashlight beam. Assuming that the shape of the given polygon makes such detection always possible, we want to minimize the total distance traveled by the robot until all intruders are detected. We present an $O(n \log n)$ time and $O(n)$ space algorithm for optimizing this metric, where n is the number of vertices of the given polygon. □

In the *polygon search* problem, *intruders* move freely in a polygonal area, and *searchers* try to detect them by illuminating them with flashlights [1]. Intruders can move faster than the sweeping speed of a flashlight. This paper considers only one *boundary searcher* with one flashlight whose movement is restricted to the polygon boundary. Our objective is to minimize the distance on the boundary that needs to be traveled by the searcher to illuminate all intruders, assuming that the polygon is searchable. Our motivation is that the energy required by the searching robot is an increasing function of the distance it travels. Recently, Fukami *et al.* designed an algorithm for finding the minimum distance schedule that runs in $O(n^2)$ time and uses $O(n^2)$ space [2]. Our algorithm is based on a totally different approach, and runs in $O(n \log n)$ time and uses $O(n)$ space.

A polygon P is defined by a sequence of vertices and the edges that connect adjacent vertices. The *boundary* of P, denoted by ∂P, consists of all its edges and vertices. The vertices immediately preceding and succeeding vertex v in the clockwise direction are denoted by $\mathit{Pred}(v)$ and $\mathit{Succ}(v)$, respectively. For a reflex vertex r, extend the edge $(\mathit{Succ}(r), r)$ in the direction $\mathit{Succ}(r) \to r$, and let $B(r) \in \partial P$ denote the point where it hits ∂P for the first time. We call $B(r)$ the *backward extension point* associated with r. Similarly, the point $F(r) \in \partial P$ where the extension of $(\mathit{Pred}(r), r)$ in the direction $\mathit{Pred}(r) \to r$ hits ∂P for the first time is called the *forward extension point* associated with r.

Let $x, y \in \mathcal{R}$, where $\mathcal{R}$ is the set of all real numbers. The *visibility space*, denoted by $\mathcal{V}$, consists of the infinite area between and including the lines $Y = X$ (*start line* $\mathcal{S}$) and $Y = X - |\partial P|$ (*goal line* $\mathcal{G}$), as shown in Fig. 0.1, where $|\partial P|$ denotes the length of the boundary of P [3]. We have $(x, y) \in \mathcal{V}$ if and only if $x - |\partial P| \le y \le x$. We assume that an arbitrary point on ∂P has been chosen as the origin. Let $0 \le x', y' < |\partial P|$. Then x' and $x = x' + k|\partial P|$ represent the same point on ∂P from the origin for any integer k. Similarly, y' and $y = y' + k|\partial P|$ represent the same point on ∂P for any integer k. The *visibility diagram* (or *V-diagram*) for a given polygon is drawn in $\mathcal{V}$ by shading some areas in it gray as follows: point $(x, y) \in \mathcal{V}$ is gray if points x and y are not mutually visible [3]. Fig. 0.1(b) is the V-diagram for the polygon in Fig. 0.1(a). In the V-diagram, coordinate points $([v] + k|\partial P|, [v] + k'|\partial P|)$ on line $Y = X$ and $Y = X - |\partial P|$ are labeled by v, where k, k' are integers and $[v]$ is the distance along the polygon boundary from vertex 1, which is chosen as the origin, to vertex v.

Without loss of generality, we can assume that the left side of the beam is kept clear of intruders [4]. We represent the searcher (resp. beam head) position by y (resp. x). A path from start line $\mathcal{S}$ to goal line $\mathcal{G}$ is called a *search path* if, wherever it crosses a gray area, it does so horizontally from right to left. Crossing a gray area from right to left corresponds to a beam head jump from a point on ∂P to another point, which is

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Supported in part by UWM Research Growth Initiative.

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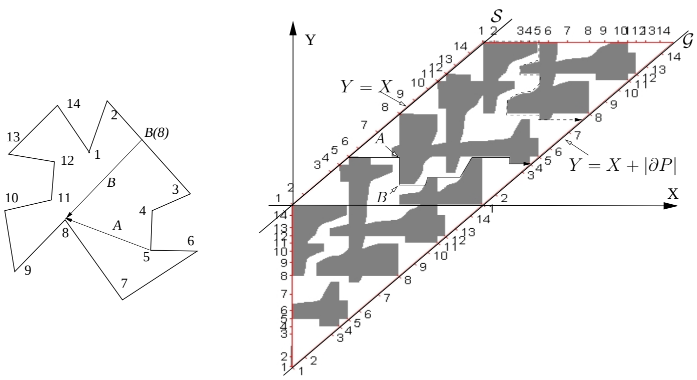

(a) (b)

FIG. 0.1. *(a) A polygon; (b) Canonical search paths in the V-diagram.*

closer to the searcher position counterclockwise, making the cleared area smaller. It is known that P is searchable if and only if there is a search path [4, 5]. Assuming that P is searchable, we want to find a schedule that minimizes the distance traveled by the searcher. It is easy to see that the distance is the sum of the vertical movements of the searcher in the V-diagram.

A *canonical search path* moves horizontally to the right (in the V-diagram) in a white region whenever possible, and when it hits the left face of a gray region it moves along the face either clockwise or counterclockwise, until a rightward move becomes possible. It is easy to show that there is a canonical search path that represents a minimum distance schedule. The paths in Fig. 0.1(b) shown by sequences of solid and dashed line segments each represents a canonical search path. A *configuration* is a (searcher position, beam head position) pair. The solid path, for example, represents a schedule in which the searcher starts at vertex 5, rotates the beam to vertex 8 (configuration A), and then moves to $B(8)$ (configuration B), etc., in Fig. 0.1(a). We want to find a canonical search path with the minimum cumulative vertical distance in the V-diagram. We designate certain configurations, such as B, as *landmarks*, and find the optimal "local" paths among nearby landmarks in $O(n \log n)$ time. We then combine locally optimal paths into a globally optimal path, using a shortest path algorithm. Since our output schedule is given at a higher level than "instructions" used in [6], their bound of $\Omega(n^2)$ on the schedule length does not apply.

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