On Ramsey $(K_{1,2}, P_4)-$minimal Graphs

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Abstract

For any given graphs G and H, we write $F \rightarrow (G, H)$ to mean that any red-blue coloring of the edges of F implies that either F contains a red subgraph G or a blue subgraph H. Graph F is *Ramsey* (G, H)*-minimal* if $F \rightarrow (G, H)$ but $F^* \nrightarrow (G, H)$ for any proper subgraph $F^* \subset F$. The class of all (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$. In this paper we determine some graphs in the infinite class $\mathcal{R}(K_{1,2}, P_4)$.

We consider simple graphs, namely finite undirected graphs without loops and multiple edges. Let G and H be graphs. We say that $F \rightarrow (G, H)$ if any red-blue coloring of the edges of F implies that either F contains a red subgraph G or a blue subgraph H. Graph F is *Ramsey* (G, H)*-minimal* if $F \rightarrow (G, H)$ but F lose the property upon removal of any edge. The class of all (G, H)-minimal graphs is denoted by $\mathcal{R}(G, H)$. The pair (G, H) is called *Ramsey-finite* or *Ramsey-infinite* depending upon whether $\mathcal{R}(G, H)$ is finite or infinite. A(G, H)*-coloring* is defined as a 2-coloring where neither a red G nor a blue H occurs. Notations not specifically mentioned will follow the terminology in [7].

The main problems in Ramsey (G, H)-minimal graphs are the determination and the characterization of all graphs F belonging to $\mathcal{R}(G, H)$. It is also interesting to determine whether the class is finite or infinite. Because of their difficulties, there are just a few results on the problems stated above, even for the combination of two small-order graphs or two simply-structured graphs (G, H).

Some recent results related to the determination and characterization of all graphs belonging to $\mathcal{R}(G, H)$ where $G \cong K_{1,2}$ are presented as follows.

Burr et.al [6] proved that $\mathcal{R}(K_{1,2}, H)$ is infinite if H is a bridgeless connected graph. Borowiecki et.al [4] gave some characterizations of all graphs in $\mathcal{R}(K_{1,2}, K_{1,m})$ for $m \geq 3$. Next, Borowiecki et.al [5] characterized all graphs belonging to $\mathcal{R}(K_{1,2}, K_3)$. Then Baskoro et.al [1] showed that $W_{3t+1} \in \mathcal{R}(K_{1,2}, C_3^t)$ for $t \geq 1$. In [3] Baskoro et.al determined some classes of graphs belonging to $\mathcal{R}(K_{1,2}, C_4)$ and stated the problem to determine all graphs in $\mathcal{R}(K_{1,2}, C_4)$ with diameter 2 and 3. Then Baskoro et.al [2] presented some new graphs of diameter 2 in $\mathcal{R}(K_{1,2}, C_4)$. Recently, Vetrík et.al [10] gave an infinite family of graphs in $\mathcal{R}(K_{1,2}, C_4)$ of any diameter ≥ 4.

Faudree and Sheehan [9] defined a restricted infinite class $\mathcal{R}^T(G, H)$, consists of all trees T such that $T \rightarrow (G, H)$ for two arbitrary trees G and H; but $T^* \nrightarrow (G, H)$ for any proper subgraph $T^* \subset T$. They gave some characterizations of T belonging to $\mathcal{R}^T(K_{1,k}, P_n)$ for $k \geq 2$ and $n \geq 4$ and determined some trees in $\mathcal{R}^T(K_{1,2}, P_4)$ and $\mathcal{R}^T(K_{1,2}, P_5)$. Motivated

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by this, we study the Ramsey-minimal graphs (other than trees) in the infinite class $\mathcal{R}(K_{1,2}, P_4)$.

In this paper we define $\mathcal{R}^*(K_{1,2}, P_4) := \mathcal{R}(K_{1,2}, P_4) - \mathcal{R}^T(K_{1,2}, P_4)$. It is clear that all graphs $F \in \mathcal{R}^*(K_{1,2}, P_4)$ must contain some cycles. We divide the class into two subclasses, $\mathcal{R}_1^*(K_{1,2}, P_4)$, where all graphs F in this class contain no leaves; and $\mathcal{R}_2^*(K_{1,2}, P_4)$, where all F contain some leaves.

Here we list some of graphs in the class $\mathcal{R}^*(K_{1,2}, P_4)$.

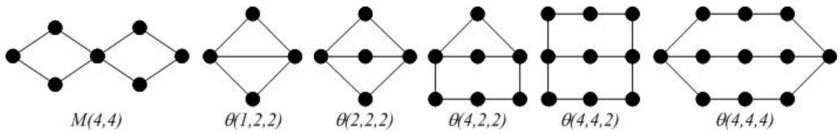

Figure 1: Some graphs in $\mathcal{R}_1^*(P_3, P_4)$

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