On Ramsey \((K_{1,2}, P_4)\)-minimal Graphs

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Abstract

For any given graphs \(G\) and \(H\), we write \(F \rightarrow (G, H)\) to mean that any red-blue coloring of the edges of \(F\) implies that either \(F\) contains a red subgraph \(G\) or a blue subgraph \(H\). Graph \(F\) is Ramsey \((G, H)\)-minimal if \(F \rightarrow (G, H)\) but \(F^* \nrightarrow (G, H)\) for any proper subgraph \(F^* \subset F\). The class of all \((G, H)\)-minimal graphs is denoted by \(\mathcal{R}(G, H)\). In this paper we determine some graphs in the infinite class \(\mathcal{R}(K_{1,2}, P_4)\).

We consider simple graphs, namely finite undirected graphs without loops and multiple edges. Let \(G\) and \(H\) be graphs. We say that \(F \rightarrow (G, H)\) if any red-blue coloring of the edges of \(F\) implies that either \(F\) contains a red subgraph \(G\) or a blue subgraph \(H\). Graph \(F\) is Ramsey \((G, H)\)-minimal if \(F \rightarrow (G, H)\) but \(F\) lose the property upon removal of any edge. The class of all \((G, H)\)-minimal graphs is denoted by \(\mathcal{R}(G, H)\). The pair \((G, H)\) is called Ramsey-finite or Ramsey-infinite depending upon whether \(\mathcal{R}(G, H)\) is finite or infinite.

A \((G, H)\)-coloring is defined as a 2-coloring where neither a red -coloring nor a blue -coloring implies that either \(G\) nor a blue \(H\) occurs. Notations not specifically mentioned will follow the terminology in [7].

The main problems in Ramsey \((G, H)\)-minimal graphs are the determination and the characterization of all graphs \(F\) belonging to \(\mathcal{R}(G, H)\). It is also interesting to determine whether the class is finite or infinite. Because of their difficulties, there are just a few results on the problems stated above, even for the combination of two small-order graphs or two simply-structured graphs \((G, H)\).

Some recent results related to the determination and characterization of all graphs belonging to \(\mathcal{R}(G, H)\) where \(G \cong K_{1,2}\) are presented as follows.

Burr et.al [6] proved that \(\mathcal{R}(K_{1,2}, H)\) is infinite if \(H\) is a bridgeless connected graph. Borowiecki et.al [4] gave some characterizations of all graphs in \(\mathcal{R}(K_{1,2}, K_{1,m})\) for \(m \geq 3\). Next, Borowiecki et.al [5] characterized all graphs belonging to \(\mathcal{R}(K_{1,2}, K_{3})\). Then Baskoro et.al [1] showed that \(W_{3t+1} \in \mathcal{R}(K_{1,2}, C_4)\) for \(t \geq 1\). In [3] Baskoro et.al determined some classes of graphs belonging to \(\mathcal{R}(K_{1,2}, C_4)\) and stated the problem to determine all graphs in \(\mathcal{R}(K_{1,2}, C_4)\) with diameter 2 and 3. Then Baskoro et.al [2] presented some new graphs of diameter 2 in \(\mathcal{R}(K_{1,2}, C_4)\). Recently, Vetřík et.al [10] gave an infinite family of graphs in \(\mathcal{R}(K_{1,2}, C_4)\) of any diameter \(\geq 4\).

Faudree and Sheehan [9] defined a restricted infinite class \(\mathcal{T}^*(G, H)\), consists of all trees \(T\) such that \(T \rightarrow (G, H)\) for two arbitrary trees \(G\) and \(H\); but \(T^* \nrightarrow (G, H)\) for any proper subgraph \(T^* \subset T\). They gave some characterizations of \(T\) belonging to \(\mathcal{T}^*(K_{1,k}, P_n)\) for \(k \geq 2\) and \(n \geq 4\) and determined some trees in \(\mathcal{T}^*(K_{1,2}, P_4)\) and \(\mathcal{T}^*(K_{1,2}, P_5)\). Motivated

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by this, we study the Ramsey-minimal graphs (other than trees) in the infinite class \( R(K_{1,2}, P_4) \).

In this paper we define \( R^*(K_{1,2}, P_4) := R(K_{1,2}, P_4) - R^T(K_{1,2}, P_4) \). It is clear that all graphs \( F \in R^*(K_{1,2}, P_4) \) must contain some cycles. We divide the class into two subclasses, \( R^*_1(K_{1,2}, P_4) \), where all graphs \( F \) in this class contain no leaves; and \( R^*_2(K_{1,2}, P_4) \), where all \( F \) contain some leaves.

Here we list some of graphs in the class \( R^*(K_{1,2}, P_4) \).

![Figure 1: Some graphs in \( R^*_1(P_3, P_4) \)](image)

References


