An Algorithm for Shortest Path in Polygons with Disc Obstacles

Rajendra P Lal                           P. Sai Prasad                                  S V Suryanarayana

University of Hyderabad, India

EXTENDED ABSTRACT

1. Introduction

The problem of finding shortest path among a pair of points inside a polygon is well studied in Computational Geometry. An immediate extension to this is to find a shortest path between a pair of points when the polygon contains obstacles. This has several applications in the field of Robotics, Motion Planning, wire routing etc. This problem (on a plane) is studied by Kim et al [1] assuming the obstacles to be non-intersecting discs with different radii. In this article we present an algorithm to solve the shortest path problem in a polygon having disc obstacles of different radii, using a different approach. Our approach uses convex-hull filter technique [1] and a modified version of rubber-band algorithm [3]. The proposed algorithm (over a restricted set of inputs) significantly improves time complexity $O(n^3)$ as obtained in[1], n being the number of obstacles. An improvement by a linear factor is achieved by parallel processing.

2. Outline of the Algorithm

The first step in our algorithm is to find the shortest path in the polygon in the absence of obstacles, applying rubber-band algorithm [2]. As a preprocessing for the rubber-band algorithm, the given polygon is decomposed into triangles or trapezoids. Now disc obstacles are introduced in the polygon and discs belonging to different triangles are identified. Our algorithm works for a restricted set of inputs: that is no disc intersects with the diagonals produced by the triangulation. But this restriction can be removed up to a large extent by decomposing the polygon into convex polygons. The rubber-band algorithm basically outputs a set of points $\{P_1, P_2, \ldots\}$ on the diagonals of the polygon (fig-1), out of which, shortest path is traced as several connected line segments $\{SP_1, P_1P_2, \ldots PnT\}$. In each triangle, discs that intersect the line segments $PiPj$ are identified using simple algebraic techniques. Then a convex hull of the identified discs and the points $Pi$ and $Pj$ is constructed as illustrated in [1](fig-2). In the process all the discs that are not obstacles in actual sense are eliminated and a convex hull containing the discs that are actually obstacles is created. Now in each triangle, line segments are drawn intersecting the discs extending from one edge of the convex hull to the other. The segments that lie inside the discs are identified for each line that is drawn. These segments are called as restricted segments. These segments are further refined according to their visibility from other segments, using tangents to the discs, called as forbidden segments. Theses forbidden segments are computed for each triangle and stored for use in rubber-band algorithm. Next step of the algorithm is to apply a variant of rubber-band
algorithm to the sets of pairs \{Pi,Pj\}. As a result local shortest paths in each triangle are constructed. Finally all these local paths are merged together to give the sought path.

The important point in this algorithm is that the local paths can be computed in parallel, which reduces the time complexity by a linear factor.

3. References