Computer-aided Design of Pop-up Books with Two-dimensional V-fold Structures

Takuya Hara [*] Kokichi Sugihara [†]

We are studying a computer-aided method to automatically design a pop-up book that realizes an arbitrary solid. Many beautiful and complicated pop-up books have been designed by skillful designers and it is difficult for nonprofessional people to design such books. To design them by computers, some mathematical models of pop-up books have been studied [1][2]. Because of NP-hardness of pop-up books shown by Uehara and Teramoto [5], it is also difficult to design complex pop-up books by computers. The methods to design such books based on simple models have been studied. Mitani et al. proposed the methods to automatically design pop-up cards from three-dimensional polygon models based on voxels [3] and lattice-type cross sections [4]. On the other hand, we are studying the automatic methods based on V-folds. A V-fold is one of the simplest models of pop-up books. As Figure 1 shows, a V-shaped pop-up piece is attached to two facing pages. In the V-fold, there are four folding lines. To open and close pages while keeping all faces planar, it is necessary to set these folding lines either all parallel or all concurrent [2]. If all folding lines are parallel, this V-fold can be regarded as a two-dimensional structure. We focus on the two-dimensional V-fold and propose a method to design a two-dimensional pop-up structure that realizes any given polygon by accumulating V-folds.

A two-dimensional V-fold consists of two half lines l and r and two segments, which are connected by pin joints as shown in Figure 2. Let us name the four points as A, B, O, and P, as indicated in the figure. If $PA + AO = PB + BO$ holds, then the example is foldable. Such a convex quadrilateral as $PAOB$ is called a *foldable quadrilateral*. A V-fold is also uniquely erectable, i.e. when the angle AOB is decided, the positions of segments PA, PB are uniquely determined. If the structure is foldable and uniquely erectable, it is called a pop-up structure. It is possible to construct a larger pop-up structure by accumulating such V-folds recursively, as Figure 3 shows.

On the basis of this observation, we next propose a method for realizing an arbitrary polygon such as a two-dimensional pop-up structure with recursive V-folds. An example is shown in Figures 4, 5, and 6. Suppose that the polygon shown in Figure 4 is given. We design the pop-up structure that has

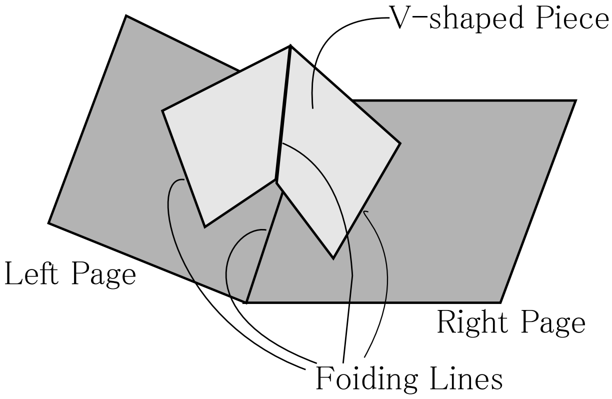

Figure 1: An example of a three-dimensional V-fold.

*Graduate School of Information Science and Technology, the University of Tokyo. takuya_hara@mist.i.u-tokyo.ac.jp
†Meiji Institute for Advanced Study of Mathematical Sciences. kokichis@isc.meiji.ac.jp

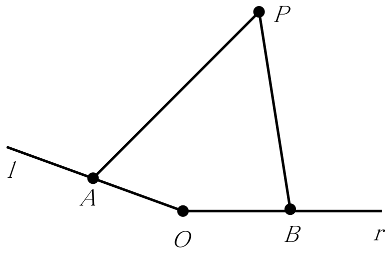

Figure 2: An example of a two-dimensional V-fold.

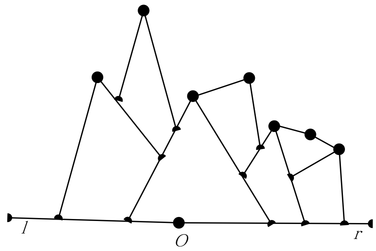

Figure 3: An example of recursive accumulation of V-folds.

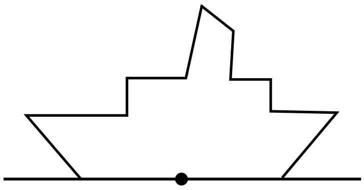

Figure 4: A given polygon.

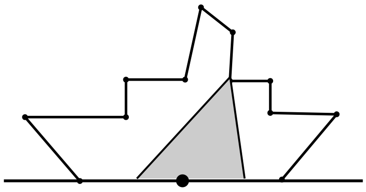

Figure 5: Dividing into three polygons.

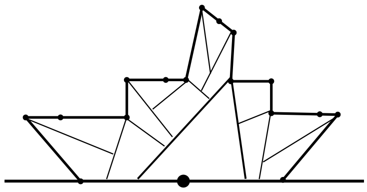

Figure 6: Recursive repetition.

a circumference in the shape of the given polygon. To achieve this, we divide the given polygon into a foldable quadrilateral and one or two small polygons, as Figure 5 illustrates.[1] Next, we apply the same division to two polygons recursively. When the polygon is a triangle, we divide a segment into two segments and make it a foldable quadrilateral.

Extending this method to a three-dimensional structure, it is possible to make a pop-up book in the shape of an arbitrary prism. Moreover, a pyramid-shaped pop-up book can be designed by replacing the length of a segment with the angle around the top of the pyramid. Combining pop-up structures, we will be able to design various pop-up books. In future, we would like to establish the method to design pop-up books from three-dimensional polygon models.

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[1] Here, it is possible to choose many other foldable quadrilaterals. The method to decide the best quadrilateral must be studied in future.