Convex Decompositions of Point Sets in the Plane

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Extended Abstract

Let P be a set of n points in general position in the plane. A set Π of convex polygons with vertices in P and with pairwise disjoint interiors is a *convex decomposition* of P if their union is the convex hull $\mathrm{CH}(P)$ of P and no point of P lies in the interior of any polygon in Π. Let $u(P)$ denote the minimum number m such that there is a convex decomposition Π of P with m elements, and let $U(n)$ denote the minimum number m' such that for any set P of n points in general position in the plane, there is a convex decomposition Π of P with m' elements.

J. Urrutia [5] conjectured that $U(n) \leq n+1$ for $n \geq 3$. Later, V. Neumann-Lara, E. Rivera-Campo and J. Urrutia [4] showed that $U(n) \leq \frac{10n-18}{7}$ for $n \geq 3$, and recently K. Hosono [3] showed that $U(n) \leq \lceil \frac{7}{5}(n-3) \rceil + 1$ for $n \geq 3$. On the other hand, O. Aichholzer and H. Krasser [1] gave a lower bound: $U(n) \geq n+2$ for $n \geq 13$, and J. García-López and C.M. Nicolás [2] showed that $U(n) > \frac{12}{11}n-2$ for $n \geq 4$.

In this talk we give a new upper bound of $U(n)$:

Theorem 1 $U(n) \leq \frac{4}{3}n - 2$ *for* $n \geq 3$.

To show this, we let $b(P)$ denote the number of points of P lying on the boundary of $\mathrm{CH}(P)$, and show the following proposition:

Proposition 1 $u(P) \leq \frac{4}{3}|P| - b(P) + 1$ *for any set* P *of points in general position in the plane.*

References

[1] O. Aichholzer and H. Krasser, The point set order type data base: A collection of applications and results, *Proc. 13th Canadian Conference on Computational Geometry*, University of Waterloo, Waterloo, 2001, 17–20.

[2] J. García-López, C.M. Nicolás, Planar point sets with large minimum convex partitions, Proc. 22nd European Workshop on Computational Geometry, Delphi, 2006, 51–54.

[3] K. Hosono, On convex decompositions of a planar point set, *Discrete Mathematics* **309**(6) (2009), 1714–1717.

[4] V. Neumann-Lara, E. Rivera-Campo, J. Urrutia, A note on convex decompositions of point sets in the plane, *Graphs and Combinatorics* **20**(2) (2004), 223–231.

[5] J. Urrutia, Open problem session, *10th Canadian Conference on Computational Geometry*, McGill University, Montreal, 1998.