Classification of the entanglement properties of eight-qubit graph states*

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A n-qubit *graph state* $|G\rangle$ is a pure state associated to a graph $G(V, E)$. The graph G provides both a recipe for preparing $|G\rangle$ and a mathematical characterization of $|G\rangle$ [4].

Graph states play several fundamental roles in quantum information theory, e.g, in quantum error-correction, quantum computation, quantum simulation, multipartite purification schemes, entanglement distillation protocols, Greenberger-Horne-Zeilinger (GHZ) or all-versus-nothing proofs of Bell's theorem. Graph states are essential for quantum communication protocols, including entanglement-based quantum key distribution, teleportation, reduction of communication complexity, and secret sharing.

Graph states also play a fundamental role in the theory of entanglement. For $n \geq 4$ qubits, there is an infinite amount of different, inequivalent classes of n-qubit pure entangled states. The graph state formalism is an useful abstraction which permits a detailed classification of n-qubit entanglement.

Two n-qubit states, $|\phi\rangle$ and $|\psi\rangle$ have the same n-partite entanglement if and only if there are n one-qubit unitary transformations U_i, such that $|\phi\rangle = \bigotimes_{i=1}^{n} U_i |\psi\rangle$. If these one-qubit unitary transformations belong to the Clifford group, then both states are said to be local Clifford equivalent.

Van den Nest *et al.* found that the successive application of a transformation is sufficient to generate the complete equivalence class of graph states under local unitary operations within the Clifford group This simple transformation is Local Complementation (LC). The application of LC on the qubit i acts inverting the neighborhood $\mathcal{N}(i)$ of i.; i.e., adjacent vertices in the neighborhood become non adjacents and vice versa.

Up to 7 qubits, there are 45 classes of graph states that are not equivalent under one-qubit unitary transformations. With 8 qubits, there are 101 new classes. All these classes have been obtained by various researchers (see, e.g., [3]). The purpose here is to classify them according to several relevant physical properties for quantum information theory.

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Following Hein *et al.* (HEB) [4], the criteria for ordering the classes are: (a) number of qubits, (b) minimum number of controlled-Z gates needed for the preparation, (c) the Schmidt measure, and (d) the rank indexes.

We have extended to 8 qubits the classification of the entanglement of graph states proposed by HEB for $n < 8$ qubits. For each of 101 classes we obtain a representative which requires the minimum number of controlled-Z gates for its preparation, and calculate the Schmidt measure for the 8-partite split (which measures the genuine 8-party entanglement of the class) and the Schmidt ranks for all bipartite splits.

This classification will help us to obtain new all-versus-nothing proofs of Bell's theorem and new Bell inequalities. More generally, it will help us to investigate the nonlocality of graph states [2].

Extending the classification of HEB a further step sheds some light on the limitations of the method of classification. The criteria used by HEB to order the classes already failed to distinguish all classes in $n = 7$. On the other hand, the problem of obtaining a minimum set of invariants capable of distinguishing all classes with $n \leq 8$ qubits has been addressed recently [1].

Our results show that there are no 8-qubit graph states with rank indexes $RI_p = [\nu_j^p]_{j=p}^1$ with $\nu_j^p \neq 0$ if $j = p$, and $\nu_j^p = 0$ if $j < p$. These states are robust against disentanglement by a few measurements. Neither there are 7-qubit graph states with this property [4]. This makes more interesting the fact that there is a single 5- and 6-qubit graph state with this property [4].

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References

[1] A. Cabello, A. J. López-Tarrida, P. Moreno, and J. R. Portillo, "Minimum set of invariants characterizing graph states of up to 8 qubits," Phys. Rev. A **80**, 012102 (2009).

[2] A. Cabello, J. R. Portillo, and D. Rodríguez, "Minimum detection efficiency for the perfect correlations of any graph state" (unpublished).

[3] L. E. Danielsen, *Database of Self-Dual Quantum Codes*, http://www.ii.uib.no/ larsed/vncorbits/.

[4] M. Hein, J. Eisert, and H.J. Briegel, "Multi-party entanglement in graph states," Phys. Rev. A **69**, 062311 (2004).

[5] B.D. McKay, *nauty User's Guide (version 1.5)*, Technical report TR-CS-90-02, Computer Science Department, Australian National University, 1990.

[6] M. Van den Nest, J. Dehaene, and B. De Moor, "Graphical description of the action of local Clifford transformations on graph states," Phys. Rev. A **69**, 022316 (2004).