Chordal Bipartite Graphs with High Boxicity
(Extended Abstract)

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Abstract. The boxicity of a graph G is defined as the minimum integer k such that G is an intersection graph of axis-parallel k-dimensional boxes. Chordal bipartite graphs are bipartite graphs that do not contain an induced cycle of length greater than 4. It was conjectured by Otachi, Okamoto and Yamazaki that chordal bipartite graphs have boxicity at most 2. We disprove this conjecture by exhibiting an infinite family of chordal bipartite graphs that have unbounded boxicity.

Key words: Boxicity, chordal bipartite graphs, interval graphs, grid intersection graphs.

1 Introduction

A graph G is an *intersection graph* of sets from a family of sets $\mathcal{F}$, if there exists $f : V(G) \rightarrow \mathcal{F}$ such that $(u, v) \in E(G) \Leftrightarrow f(u) \cap f(v) \neq \emptyset$. An *interval graph* is an intersection graph in which the set assigned to each vertex is a closed interval on the real line. In other words, interval graphs are intersection graphs of closed intervals on the real line. An *axis-parallel k-dimensional box* in $\mathbb{R}^k$ is the Cartesian product $R_1 \times R_2 \times \cdots \times R_k$, where each R_i is an interval of the form $[a_i, b_i]$ on the real line. *Boxicity* of any graph G (denoted by $\mathrm{box}(G)$) is the minimum integer k such that G is an intersection graph of axis-parallel k-dimensional boxes in $\mathbb{R}^k$. Note that interval graphs are exactly those graphs with boxicity at most 1.

Chordal bipartite graphs (CBGs) are bipartite graphs that do not contain an induced cycle of length greater than 4.

2 Our Result

Otachi, Okamoto and Yamazaki [2] proved that P_6-free chordal bipartite graphs have boxicity at most 2. In the same paper, they conjectured that the boxicity of the wider class of chordal bipartite graphs itself is bounded from above by 2. We disprove this conjecture by showing that there exist chordal bipartite graphs with arbitrarily high boxicity. Our result also implies that the class of chordal bipartite graphs is incomparable with the class of "grid intersection graphs" [1].

3 Bipartite Powers

Given a bipartite graph G and an odd positive integer k, we define the graph $G^{[k]}$ to be the bipartite graph with $V\left(G^{[k]}\right) = V(G)$ and $E\left(G^{[k]}\right) = \{(u, v) \mid u, v \in V(G),\ d_G(u, v) \text{ is odd and } d_G(u, v) \leq k\}$. The graph $G^{[k]}$ is called the *k-th bipartite power* of G.

Theorem 1. *For any tree T and an odd positive integer k, $T^{[k]}$ is a CBG.*

Proof. Proof omitted.

4 Boxicity of CBGs

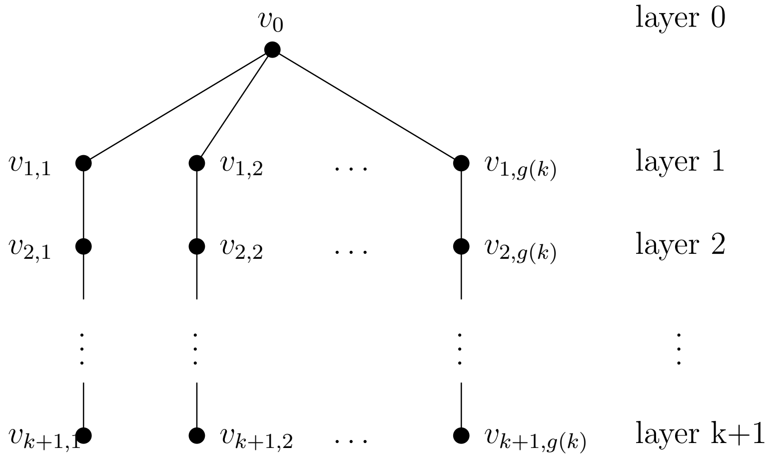

Fig. 1. Tree T_k

Let T_k be the tree shown in figure 1. Here $k \in \mathbb{N}$ is an odd number and $g(k) = \frac{k+1}{2} \cdot (g(k-2) - 1) + 1$ with $g(1) = 2$. Let $G_k = T_k^{[k]}$.

Lemma 1. $\mathrm{box}(G_k) > \frac{k+1}{4}$.

Proof. Proof omitted.

Theorem 2. *For any $b \in \mathbb{N}^+$, there exists a chordal bipartite graph G with $\mathrm{box}(G) > b$.*

Proof. For any odd positive integer k, since G_k is the bipartite power of a tree T_k, G_k is a CBG by Theorem 1. Let $G = G_{(4b-1)}$. Then by Lemma 1, $\mathrm{box}(G) > b$. ■

References

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