On the Ramsey numbers for the union of graphs

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Extended Abstract

Throughout this paper, we consider finite undirected graphs without loops and multiple edges. For given graphs G and H, a graph F is called a (G,H)-*good graph* if F contains no G and $\overline{F}$ contains no H. A (G,H,n)-*good graph* denotes a (G,H)-good graph with n vertices. The *Ramsey number* $R(G,H)$ is defined as the smallest natural number n such that no (G,H,n)-good graph exists.

Let $k \geq 1$ be an integer. For $i = 1, 2, ..., k$, let G_i be a connected graph with the vertex set V_i and the edge set E_i. The *union* $G = \bigcup_{i=1}^{k} G_i$ has the vertex set $V = \bigcup_{i=1}^{k} V_i$ and the edge set $E = \bigcup_{i=1}^{k} E_i$. In particular, if $G_1 = G_2 = ... = G_k = F$ then $G = kF$. The problem of determining the Ramsey number for the union of graphs has been extensively investigated, see for instances [1, 2, 4].

Let H be a graph with the chromatic number $h \geq 2$ and the chromatic surplus $s \geq 1$. The chromatic surplus of H is the minimum cardinality of a color class taken over all proper h-colorings of H. A connected graph G is said to be H-*good* if $R(G,H) = (o(G)-1)(h-1)+s$, where $o(G)$ is the order of G. By using this terminology, Bielak [2] gave the exact Ramsey numbers for the union of $G = \bigcup_{i=1}^{k} G_i$ versus H when each component G_i is H-good with $s = 1$.

Let S_n be a star on n vertices and W_n be a wheel on $n+1$ vertices. Note that the chromatic number and surplus of W_6 (W_8) are 3 (3) and 1 (1), respectively. Chen et al. [3] have proved that S_n is not W_6-good for $n \geq 3$. Zhang *et al.* [5] also showed that S_n is not W_8-good for even $n \geq 6$. Then, the theorem proposed by Bielak in [2] cannot be used to determine the Ramsey number of a disjoint union of stars versus W_6 or W_8. In this paper, we will determine these Ramsey numbers, namely $R(\bigcup_{i=1}^{k} S_{n_i}, W_6)$ and $R(\bigcup_{i=1}^{k} S_{n_i}, W_8)$. We also obtain the

generalization of the theorem proposed by Hasmawati *et al.* in [4].

Our main results are presented in the following.

Theorem 1 *Let $k \geq 2$ be an integer. Let H be a graph with the chromatic number $h \geq 2$ and the chromatic surplus $s \geq 1$. For $i = 1, 2, \cdots, k$, let G_i be a connected graph satisfying $R(G_1, H) \geq R(G_2, H) \geq ... \geq R(G_k, H)$. Let $G = \bigcup_{i=1}^{k} G_i$. If $o(G_i) \geq R(G_i, H) - R(G_{i+1}, H)$ for every $i = 1, 2, ..., k-1$ then*

$$R(G, H) \leq R(G_k, H) + \sum_{i=1}^{k-1} o(G_i). \tag{1}$$

Furthermore, if $o(G_k) = \min\{o(G_i) | i = 1, 2, \cdots, k\} \geq s$, and G_k is H-good then

$$R(G, H) = R(G_k, H) + \sum_{i=1}^{k-1} o(G_i). \tag{2}$$

Lemma 1 *For $n \geq 3$ and $k \geq 1$, $R(kS_n, W_6) = (k+1)n + 1$.*

Lemma 2 *For even $n \geq 6$ and $k \geq 1$, $R(kS_n, W_8) = (k+1)n + 2$.*

Theorem 2 *Let $k \geq 1$ be an integer. Let $n_k \geq n_{k-1} \geq ... \geq n_1 \geq 3$ be integers and $G = \bigcup_{i=1}^{k} l_i S_{n_i}$ with $l_i \geq 1$ for every $i = 1, 2, ..., k$. Then,*

$$R(G, W_6) = \max_{1 \leq i \leq k} \Big\{(n_i + 1) + \sum_{j=i}^{k} l_j n_j\Big\}. \tag{3}$$

Theorem 3 *Let $k \geq 1$ be an integer. Let $n_k \geq n_{k-1} \geq ... \geq n_1 \geq 6$ be integers and $G = \bigcup_{i=1}^{k} l_i S_{n_i}$ with $l_i \geq 1$ for every $i = 1, 2, ...k$. If n_i is even for every $i = 1, 2, ..., k$ then*

$$R(G, W_8) = \max_{1 \leq i \leq k} \Big\{(n_i + 2) + \sum_{j=i}^{k} l_j n_j\Big\}. \tag{4}$$

Keywords: (G, H)-good graph, H-good, Ramsey number, union graph, wheel.

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