Crossing Numbers and Skewness of Some 5-Regular Graphs

Gek L. Chia and Chan L. Lee
Institute of Mathematical Sciences
University of Malaya
50603 Kuala Lumpur
Malaysia

Extended Abstract

Let G be a graph. The *crossing number* of G, denoted $cr(G)$, is the minimum number of pairwise intersections of its edges when G is drawn in the plane. The *skewness* of G, denoted $sk(G)$, is defined to be the minimum number of edges in G whose removal results in a planar graph. Clearly $cr(G) \geq sk(G)$.

In [3], Owen determined the values of r and n for which there exists an r-regular planar graph with n vertices. In [1], an attempt was made in classifying 5-regular graphs according to their crossing numbers and with given number of vertices. In this paper, the same problem is considered but with crossing numbers replaced by skewness of graphs. In particular,

(i) we determine the unique 5-regular graph G on 10 vertices with $sk(G) = 1$ and show that the crossing number of this graph is equal to 3.

An immediate corollary of (i) is that any 5-regular graph on 10 vertices has crossing number at least 2, a result that has been observed earlier in [2].

(ii) Also, we prove that if G is a 5-regular non-planar graph on 12 vertices, then $sk(G) \geq 2$.

Clearly, (ii) implies that if G is a 5-regular non-planar graph on 12 vertices, then $cr(G) \geq 2$, a result that has been established in [1].

(iii) We then use the results in (i) and (ii) to determine the unique 5-regular graph on 10 vertices having crossing number 2. This answers a question in [1] in the negative.

An easy consequence of (i) is the following observation.

(iv) For every integer $n \equiv 2 \pmod 4 \geq 10$, there exists a connected 5-regular graph G with n vertices such that $sk(G) = 1$, $cr(G) = 3$.

Further,

(v) for every integer $n \equiv 0 \pmod 4 \geq 8$, we construct a 5-regular graph H with n vertices such that $sk(H) = cr(H) = 2$.

References

[1] G.L. Chia and C.S. Gan, On crossing numbers of 5-regular graphs, in Computing and Combinatorics, *Lecture Notes in Computer Science,* Vol. 2387 Springer-Verlag (2002) 230–237.

[2] G.L. Chia and C.S. Gan, Minimal regular graphs with given girths and crossing numbers, *Discussiones Mathematicae, Graph Theory*, Vol. 24 (2004) 223–237.

[3] A.B. Owen, On the planarity of regular incidence sequences, *J. Combinat. Theory*, Vol. 11 (1971) 201–212.