Clarity Watchman Route

Azadeh Tabatabaei*　　Ali mohades†

Abstract

In the Classic Watchman Route Problem the goal is to plan the shortest continuous closed route in a simple polygon P such that each point on the boundary of P can be seen. The Classic Watchman Route Problem combines elements of two NP-hard problems, namely, the Art Gallery Problem with Point Guards [1], and the Euclidean Traveling Salesman Problem [2]. Therefore, it was quite surprising When Chin and Ntafos claimed that it was possible to find the shortest Watchman Route that is forced to pass a given point on the boundary of the polygon in polynomial time [3].

Clarity Watchman Route asks for a closed route inside simple polygon P such that each boundary point of P can be seen under incidence constraint from at least one point along the route. When a boundary point w of a planar scene is visible under incidence constraint from another point q of the scene, q sees w with high quality. Clarity Watchman Route is an extension of Classic Watchman Route Problem, the visibility model of which is as follows:

Definition 1. Visibility under Incidence Constraint. Let the open subset $W \subset \Re^2$ describe the workspace layout. Let ∂W be the boundary of W. A point $w \in \partial W$ is visible from point $q \in W$ if the following conditions are true:

1. Line of sight constraint: the open line segment wq joining q and w does not intersect ∂W.

2. Incidence constraint: $\angle(n, v) \leq \tau$, where n is a vector perpendicular to ∂W at w and v a vector oriented from w to q, and $\tau \in [0, \pi/2]$ is an input constant, see figure 1 as an example.

H.H.Gonzalez-Banos and J.C. Latombe used this definition in Art Gallery Problem [4]. A practical instance of this definition, is that, when one watches a monitor directly, the scene is seen more Clearly rather than she/he watches the monitor under unsuitable incidence angle.

Notice in this visibility model, when τ is a very small constant (goes to zero), the sensor sees only little range of the walls which are in the sensor line of sight. So, if $\tau > 30$ degrees, this visibility model is more practical. Throughout this paper we consider that $\tau > 30$ deg.

*Department of Computer Science, Amirkabir University of Technology, aztabatabaei@aut.ac.ir

†Department of Computer Science, Amirkabir University of Technology, mohades@aut.ac.ir

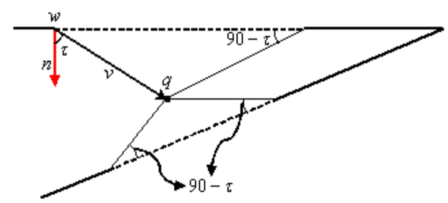

Figure 1: Under visibility constraint only the dotted segments of the scene boundary are visible from point q.

We are interested in finding a closed curve inside polygon P such that when a sensor/robot passes through the route, covers the region boundary with high quality. This extended Watchman Route Problem is defined as follows:

Watchman Route under Visibility Constraint (Clarity Watchman Route). For a given simple polygon P of n vertices, the Clarity Watchman Route Problem asks for a closed route inside P such that each point on the boundary of P can be seen from at least one point along the route under definition 1.

In this paper, each internal angle which is less than $90 - \tau$ degrees in the scene, is called *very acute angle*. So, by assuming $\tau > 30$, the very acute angles are less than 60 degrees.

An *effect* of incidence constraint on Watchman Route is that walls meeting at a very acute angle cannot be covered, by each route does not meet the very acute angle, regardless of nearing the route to the very acute angles. So, *the Clarity Watchman Route passes through very acute angles*, see figure 2.

The effect of incidence constraint on Watchman Route causes *increase* in the *length* of the Clarity Watchman Route, because it has at least one point to the left of (or on) each so-called essential cut [5]. Furthermore, this route must pass through the *very acute angles*.

It is obvious that the Classic Watchman Route, for convex polygons is only one point. While the Clarity Watchman Route for convex polygons is a curve except for some particular polygons (for example some regular convex polygons).

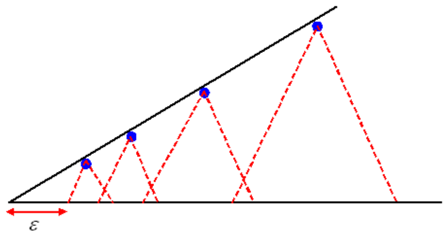

Figure 2: Under incidence limitation, from each route does not meet the very acute angle, there always remains an unseen section ϵ for walls meeting at a very acute angle.

In this paper we compute the Clarity Watchman Route in convex polygons based on their acute angles. Whereas the very acute angles are less than 60 degrees, it is easy to show that:

Any convex polygon has at most two very acute angles.

Thus there are three cases of convex polygons:

Case 1: The convex polygons which have two very acute angles. We demonstrate that, if δ is a polygonal chain that is constructed on an arbitrary segment such that the incidence angles between the chain and the segment are less than $90-\tau$ degrees, then any point on the chain can be covered with the segment points, see figure 3.

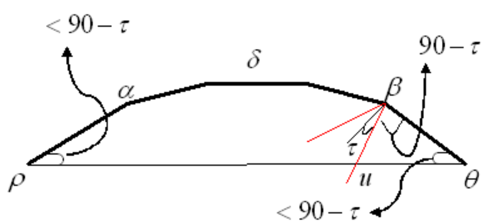

Figure 3: The polygonal chain δ is showed with bold line. Any point on this polygonal chain is covered with at least a point of segment $\rho\theta$ under incidence constraint.

So, in this case, the Clarity Watchman Route is the segment connects the two very acute angles. Thus it can be computed in time *O(n)*.

Case 2: The convex polygons which have one very acute angle. Without loss of generality, assume that the very acute angle is right most vertex. We decompose the polygon into top and bottom polygons, with connecting the very acute angle and left most vertex, as is shown in figure 4. Then we compute two segments d and d' in the polygon such that any path which passes through the very acute vertex and d (d') covers the bottom polygon (top polygon) under incidence constraint, respectively. Then by applying the hourglass method [5] we compute the shortest path which passes through very acute angle s and two segments d and d'. This path is a Clarity Watchman Route, as is shown in figure 4.

In the special case that d dominates d' (d' dominate d) [6], the shortest path which passes through s and d (s and d') is a Clarity Watchman Route.

Case 3: The convex polygons which have no very acute angle. In this case we compute fix Clarity Watchman Route (the route that is forced to pass a given point on the boundary of the polygon). By assuming that the fix point s is right most vertex, similar to case 2 we compute a fix Clarity Watchman Route in the polygon.

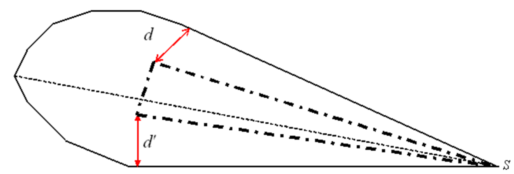

Figure 4: Polygon decomposed into bottom and top polygons, the dotted route covers the polygon under visibility (incidence) constraint.

References

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