Symmetry of Isohedral Tilings of Polyominoes and Polyiamonds as Fundamental Domains

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Polyominoes and polyiamonds are among the simplest shapes for tiles and are easily produced by computer or by hand. A polyomino is a tile made up of n congruent squares joined at their edges. A polyiamond is a tile made up of n congruent equilateral triangles joined at an edge.

There is a rich store of problems concerning these tiles. Most of the problems about these tiles are about their tiling properties. In this work, we focus on isohedral tilings of the plane by these tiles in which the tilings have 3-, 4-, or 6-fold rotation symmetry or 2-fold symmetry and two perpendicular translations. An isohedral tiling of the plane is one in which congruent copies of a single tile fill the plane without gaps or overlaps, and the symmetry group of the tiling acts transitively on the tiles. In our discussion, we assume knowledge of the structure of the symmetry groups **p3**, **p4**, **p6**, **p3m1**, **p31m**, **p4m**, **p4g**, **p6m**, **pmm**, **pmg**, **pgg**, and **cmm** [1].

In our previous paper[2, 3], we have developed an algorithm to enumerate all polyominoes and polyiamonds that are fundamental domains for **p4**, **p3** or **p6** isohedral tilings where we assumed that each tile has no symmetry; however, in some cases, our algorithm produced a tile with symmetry. In this work, we apply, basically, our previous algorithm and enumerate the polyomino and polyiamond tiles that can produce isohedral tilings of the plane having one of the above twelve symmetry groups. This time we enumerate them first by neglecting the symmetry of the tiles as [2, 3] and then incorporate the symmetry of the tiles. It is interesting that there is no polyomino or polyiamond that is a fundamental domain for an isohedral tiling with symmetry **p4m**, **p6m p3m1** or **pmm** because of the restriction of fundamental domain.

At the conference, we shall explain our algorithm and show a complete list of generated tiles.

References

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