Regular Graphs with Maximum Forest Number

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Abstract

Let G be a graph and $F \subseteq V(G)$. Then F is called an induced forest of G if $\langle G \rangle$ contains no cycle. The *forest number*, $f(G)$, of G is defined by

$$f(G) := \max\{|F| : F \text{ is an induced forest of } G\}.$$

It was proved by the second author in [6] that if G is an r-regular graph of order n, then $f(G) \leq \lfloor \frac{nr-2}{2(r-1)} \rfloor$. It was also proved that the bound is sharp by constructing an r-regular graph H of order n with $f(H) = \lfloor \frac{nr-2}{2(r-1)} \rfloor$.

In this paper we consider the problem of determining which r-regular graphs G of order n have the forest number $\lfloor \frac{nr-2}{2(r-1)} \rfloor$. The problem was asked by Bau and Beineke [1] for $r = 3$ and, in this particular case, it was answered by the second author in [7]. We are able to answer the problem for all $r \geq 4$. More precisely, we are able to obtain an algorithm of finding all r-regular graphs G of order n with $f(G) = \lfloor \frac{nr-2}{2(r-1)} \rfloor$. Furthermore, we prove that if $\mathcal{R}(r^n; f = \lfloor \frac{nr-2}{2(r-1)} \rfloor)$ is the set of all r-regular graphs G of order n with $f(G) = \lfloor \frac{nr-2}{2(r-1)} \rfloor$ and $G_1, G_2 \in \mathcal{R}(r^n; f = \lfloor \frac{nr-2}{2(r-1)} \rfloor)$, then there exists a sequence of switchings $\sigma_1, \sigma_2, \ldots, \sigma_t$ such that for each $i = 1, 2, \ldots, t$, $G_1^{\sigma_1\sigma_2\ldots\sigma_i} \in \mathcal{R}(r^n; f = \lfloor \frac{nr-2}{2(r-1)} \rfloor)$ and $G_1^{\sigma_1\sigma_2\ldots\sigma_t} = G_2$.

Keywords: Degree sequence, Forest number, Regular graphs

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