Group divisible designs with two associate classes

Narong Punnim[1] and Chariya Uiyyasathian[2]

[1]Department of Mathematics, Srinakharinwirot University,
Sukhumvit 23, Bangkok 10110, Thailand.
[2]Department of Mathematics, Faculty of Science,
Chulalongkorn University, Bangkok, 10330, Thailand.
[1,3]Center of Excellent in Mathematics, CHE, Sri Ayutthaya Rd.
Bangkok 10400, Thailand.
Email: [1]narongp@swu.ac.th and [2]chariya.u@chula.ac.th

Abstract

A *group divisible design* GDD$(v = v_1 + v_2 + \ldots + v_g, g, k, \lambda_1, \lambda_2)$ is an ordered pair $(V, \mathcal{B})$ where V is a v-set of symbols and $\mathcal{B}$ is a collection of k-subsets (called *blocks*) of V satisfying the following properties: the v-set is divided into g groups of size $v_1, v_2, \ldots, v_g$; each pair of symbols from the same group occurs in exactly λ_1 blocks in $\mathcal{B}$; and each pair symbols from different groups occurs in exactly λ_2 blocks in $\mathcal{B}$. Pairs of symbols occurring in the same group are known to statisticians as *first associates*, and pairs occurring in different groups are called *second associates*. The existence of such GDDs has been of interest over the years, going back to at least the work of Bose and Shimamoto in 1952 who began classifying such designs [1]. More recently, much work has been done on existences of such designs when $\lambda_1 = 0$ (see [2] for a summary). More work intends to solve the existence problem of a GDD$(v = m + n, 2, 3, \lambda_1, \lambda_2)$ for some m, n, λ_1 and λ_2, for instance see [3], [9], [11], [12]. If $\lambda \geq 2$, the existence problem of GDD$(v = m + n, 2, 3, \lambda, 2)$ is completely solved in [14]. However, when $\lambda = 1$, the problem is much harder.

The necessary conditions can be easily obtained by describing it graphically. Let λK_v denote the graph on v vertices in which each pair of vertices is joined by λ edges. For graphs G_1 and G_2, the graph $G_1 \vee_\lambda G_2$ is formed from the union of G_1 and G_2 by joining each vertex in G_1 to each vertex in G_2 with λ edges. An existence of a GDD$(v = m + n, 2, 3, \lambda_1, \lambda_2)$ is equivalent to an existence of a K_3-decomposition of $\lambda_1 K_m \vee_{\lambda_2} \lambda_1 K_n$.

In this paper we give necessary conditions on m and n for an existence of a GDD$(v = m + n, 2, 3, 1, 2)$, along with sufficient conditions for all $m \leq \frac{n}{2}$. Furthermore, some construction techniques using graph labelings and latin squares to construct a GDD$(v = 9 + 15, 2, 3, 1, 2)$ and a GDD$(v = 25 + 33, 2, 3, 1, 2)$ are presented.

Key Words: Group Divisible Designs, Graph Decompositions, Graph Labelings, Latin Squares

References

[1] D. Raghavarao, Construction and Combinatorial Problems in Design of Experiments, Dover, New York, 1988.

[2] C. J. Colbourn and D. H. Dinitz (eds), Handbook of Combinatorial Designs (2nd Edition), Chapman and Hall, CRC Press, Boca Raton, 2007.

[3] S. I. El-Zanati, N. Punnim and C. A. Rodger, Gregarious GDDs with Two Associate Classes, *Graphs and Combinatorics*, to appear.

[4] H. L. Fu, C. A. Rodger and D.G. Sarvate, The existence of group divisible designs with first and second associates, having block size 3, Ars Combinatoria 54 (2000), 33-50.

[5] H. L. Fu and C. A. Rodger, Group divisible designs with two associate classes: $n = 2$ or $m = 2$, J. Combin. Theory (A), 83 (1998), 94-117.

[6] J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 16 (2009), #DS6.

[7] L. Heffter, Über Nachbarconfigurationen, Tripesysteme and metacyklische Grupen, Deutsche Mathem. Vereining. Jahresber., 5 (1896), 67-69.

[8] A. Kotzig and A. Rosa, Nearly Kirkman triple system, Proceedings of the Fifth SE Conference on Combinatorics, Graph Theory and Computing, Boca Raton, FL, 1974, pp. 607-614.

[9] W. Lapchinda and N. Pabhapote, Group Divisible Designs with two Associate Classes and $\lambda_1 - \lambda_2 = 1$, *IJPAM*, to appear.

[10] C. C. Lindner and C. A. Rodger, Design Theory, CRC Press, Boca Raton, 1997.

[11] N. Pabhapote and N. Punnim, Group Divisible Designs with two associate classes and $\lambda_2 = 1$, submitted.

[12] N. Punnim and D. G. Sarvate, Some Results on Group Divisible Designs with two groups and block size 3, submitted.

[13] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris (1967), 349-355.

[14] C. Uiyyasathian and W. Lapchinda, Group Divisible Designs with two associate classes and $\lambda_2 = 2$, submitted.