Group divisible designs with two associate classes

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Abstract

A group divisible design GDD($v = v_1 + v_2 + \ldots + v_g, g, k, \lambda_1, \lambda_2$) is an ordered pair $(V, B)$ where $V$ is a $v$-set of symbols and $B$ is a collection of $k$-subsets (called blocks) of $V$ satisfying the following properties: the $v$-set is divided into $g$ groups of size $v_1, v_2, \ldots, v_g$; each pair of symbols from the same group occurs in exactly $\lambda_1$ blocks in $B$; and each pair symbols from different groups occurs in exactly $\lambda_2$ blocks in $B$. Pairs of symbols occurring in the same group are known to statisticians as \textit{first associates}, and pairs occurring in different groups are called \textit{second associates}. The existence of such GDDs has been of interest over the years, going back to at least the work of Bose and Shimamoto in 1952 who began classifying such designs [1]. More recently, much work has been done on existences of such designs when $\lambda_1 = 0$ (see [2] for a summary). More work intends to solve the existence problem of a GDD($v = m + n, 2, 3, \lambda_1, \lambda_2$) for some $m, n, \lambda_1$ and $\lambda_2$, for instance see [3], [9], [11], [12]. If $\lambda \geq 2$, the existence problem of GDD($v = m + n, 2, 3, \lambda, 2$) is completely solved in [14]. However, when $\lambda = 1$, the problem is much harder.

The necessary conditions can be easily obtained by describing it graphically. Let $\lambda K_v$ denote the graph on $v$ vertices in which each pair of vertices is joined by $\lambda$ edges. For graphs $G_1$ and $G_2$, the graph $G_1 \vee \lambda G_2$ is formed from the union of $G_1$ and $G_2$ by joining each vertex in $G_1$ to each vertex in $G_2$ with $\lambda$ edges. An existence of a GDD($v = m + n, 2, 3, \lambda_1, \lambda_2$) is equivalent to an existence of a $K_3$-decomposition of $\lambda_1 K_m \vee \lambda_2 \lambda_1 K_n$.

In this paper we give necessary conditions on $m$ and $n$ for an existence of a GDD($v = m + n, 2, 3, 1, 2$), along with sufficient conditions for all $m \leq \frac{n}{2}$. Furthermore, some construction techniques using graph labelings and latin squares to construct a GDD($v = 9 + 15, 2, 3, 1, 2$) and a GDD($v = 25 + 33, 2, 3, 1, 2$) are presented.

\textbf{Key Words:} Group Divisible Designs, Graph Decompositions, Graph Labelings, Latin Squares
References


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