I216 Computational Complexity and Discrete Mathematics Report (3)

2016, Term 2-1
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Propose(出題): November 17 (Thu)

Deadline(提出期限): November 24 (Thu), 12:30

Note(注意): Do not forget to write your name, student ID, problems, and answers on your report. The size of paper is A4, and staple them at the top left. You can use one side or both sides of paper. You can send your report by email in PDF format. （レポートには氏名，学生番号，問題，解答を，すべて書くこと．紙は A4 で左上をホチキス止めすること．片面使用でも両面使用でもよい．PDF ファイルをメールで送ってもよい.）

Answer two of the following five problems. (以下の 5 問から 2 問選んで答えよ．)

Problem 1 (5 points): Let $S = \mathbb{R} \setminus \{-1\}$ and consider an operation defined by $a \circ b = a + b + ab$. Then prove that "$\circ$" is an operation on S. Here, we suppose that arithmetic operations over $\mathbb{R}$ are defined.

Problem 2 (5 points): In the problem 1, prove that $(S, \circ)$ is a group. (Not need to prove "Closure.")

Problem 3 (5 points): Let G be an Abelian group and k be a positive integer. Prove that $G^{(k)} = \{x^k \in G \mid x \in G\}$ is a subgroup of G. Here, you can use $(a \cdot b)^n = a^n \cdot b^n$ for $a, b \in G$ when G is an Abelian group.

Problem 4 (5 points): Prove that the group whose order is a prime number is a cyclic group without a proper subgroup. (Hint: Prove it is a cyclic group, and it does not have a proper subgroup.)

Problem 5 (5 points): Let H be the subgroup of a group G. Prove that H is a normal subgroup, when H has the index 2. (Hint: It is better to divide into $a \in H$ and $a \notin H$.)