Proof of Commuting Confluence Lemma

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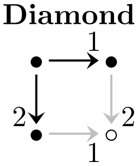
Abstract

This is an erratum to Vestergaard: "The Primitive Proof Theory of the lambda-Calculus"; it is not self-contained.

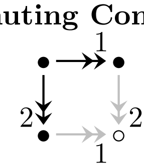
Commuting Diamond Lemma



$\wedge \diamond(\longrightarrow_1) \wedge \diamond(\longrightarrow_2) \Rightarrow \diamond(\longrightarrow_{1,2})$

Proof Assume the left-hand side of the implication and case-split on whether we have $\longrightarrow_1$ or $\longrightarrow_2$ in the $\longrightarrow_{1,2}$-divergence. In each case, we are done by an assumption and (the definitional) $\longrightarrow_i \subseteq \longrightarrow_{1,2}$, for $i \in \{1, 2\}$. $\square$

Commuting Confluence Lemma



$\wedge\ \mathrm{Confl}(\longrightarrow_1) \wedge \mathrm{Confl}(\longrightarrow_2) \Rightarrow \mathrm{Confl}(\longrightarrow_{1,2})$

Proof According to the Commuting Diamond Lemma, the property's assumptions imply $\diamond(\twoheadrightarrow_1 \cup \twoheadrightarrow_2)$. By the Hindley-Rosen Lemma, we therefore have $\mathrm{Confl}(\twoheadrightarrow_1 \cup \twoheadrightarrow_2)$ and only the following remains to be seen.

$$(\twoheadrightarrow_1 \cup \twoheadrightarrow_2)^\star \quad = \quad \twoheadrightarrow_{1,2} \tag{1}$$

We first observe that it is a simple proof that, for any relation, $\longrightarrow$, we have $\twoheadrightarrow = (\twoheadrightarrow)^\star$. Now, in order to prove (1), we must establish two inclusions.

"$\subseteq$": By our initial observation and Proposition 1.11, it suffices to prove that $\twoheadrightarrow_1 \cup \twoheadrightarrow_2 \subseteq \twoheadrightarrow_{1,2}$, which also follows from Proposition 1.11, when case-splitting on set-union.

"$\supseteq$": By Proposition 1.11 (again), it suffices to prove $\longrightarrow_{1,2} \subseteq \twoheadrightarrow_1 \cup \twoheadrightarrow_2$, which follows by definition, when case-splitting on $\longrightarrow_{1,2}$. $\square$