Session: Animation
Thursday, 21 November
09:00 - 10:45

Stochastic Modeling of Immersed Rigid-body Dynamics

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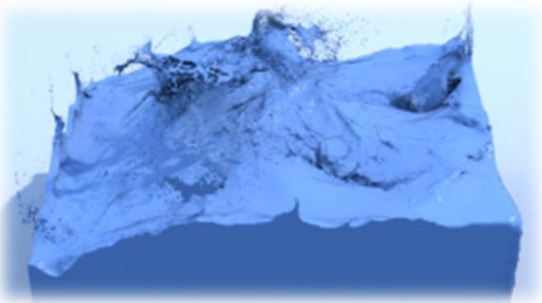
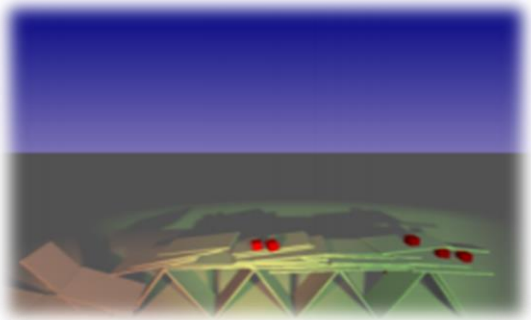
Immersed Body Dynamics

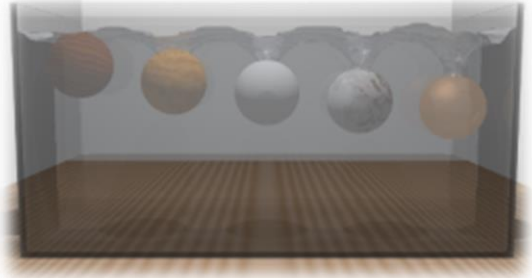
Challenges



Fluid Simulations(e.g. S2013)



Rigid-body Simulations(e.g. S2012)



Two-way Coupling(e.g. SA2012)

Dispersed Flows(e.g. S2010)

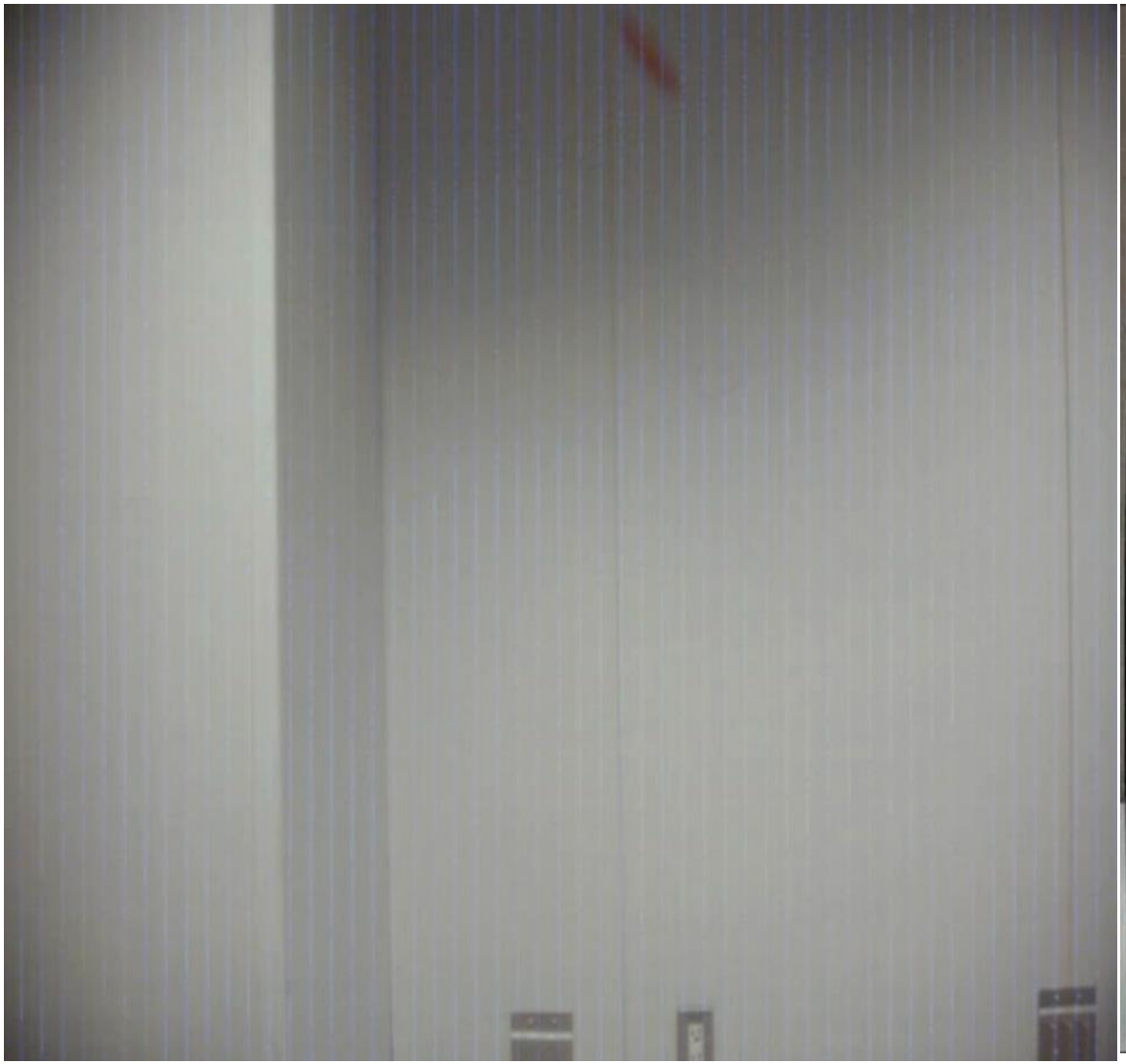
Turbulent Flows(e.g. SA2010)

- Unsteady dynamics of participated objects
- Vortical loads from the surrounding flow

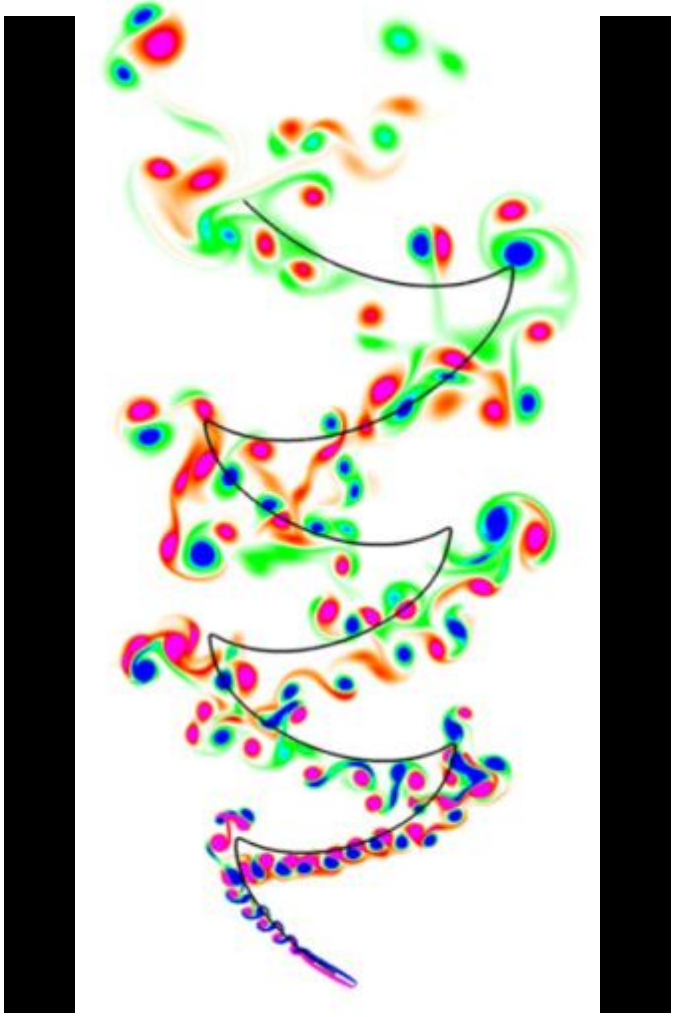
Immersed ***Rigid-body*** Dynamics

Daily-life, Beautiful, Chaotic, and Sensitive Motions

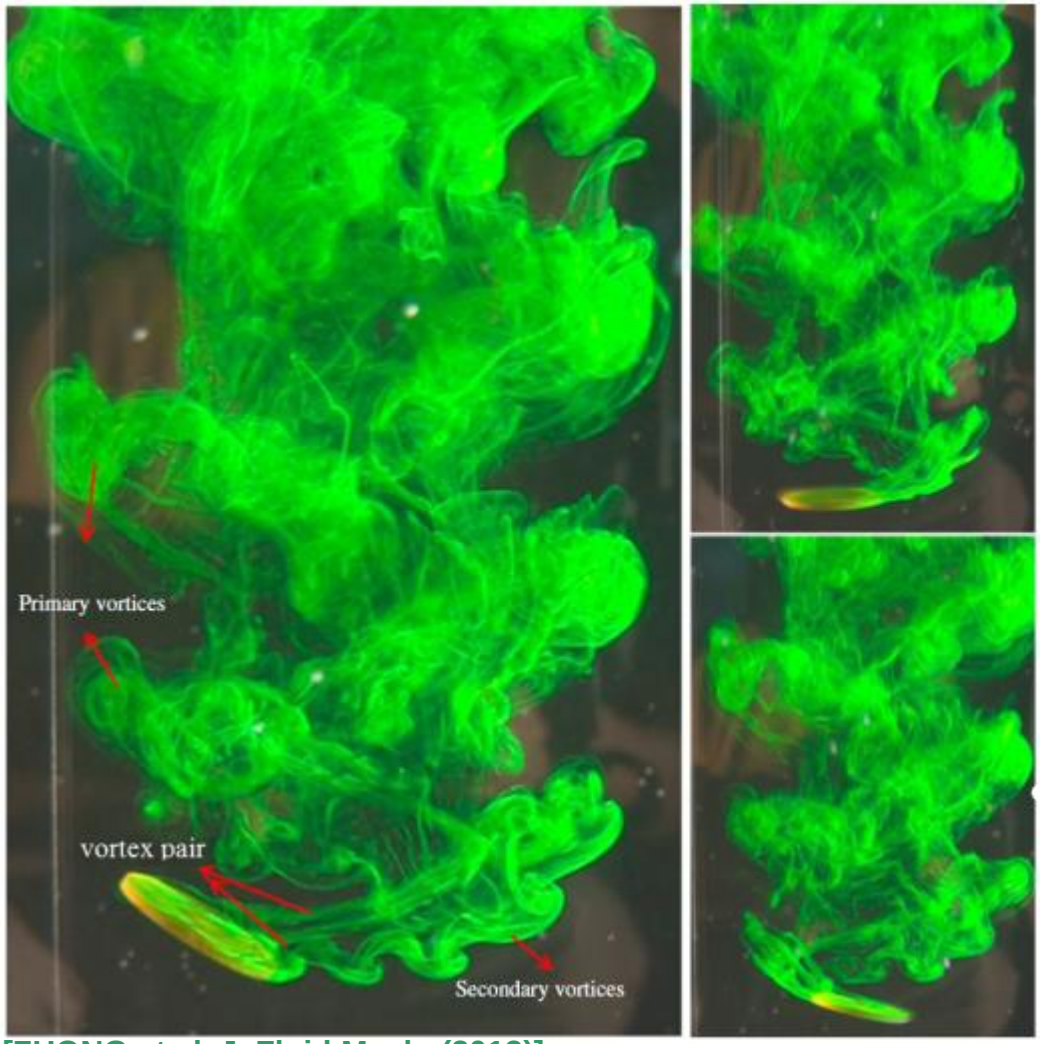
Phenomena



What happened? –from physics view



[YANG et al., J. Comput. Phys., 2012]



[ZHONG et al. J. Fluid Mech. (2013)]

In Computer Graphics,

How to achieve **Realistic** simulations in **Realtime**?

- ✓ **High-Reynolds-number Flow**
- ✓ **Turbulent Flows**
- ✓ **Unsteady Forces**

Related work

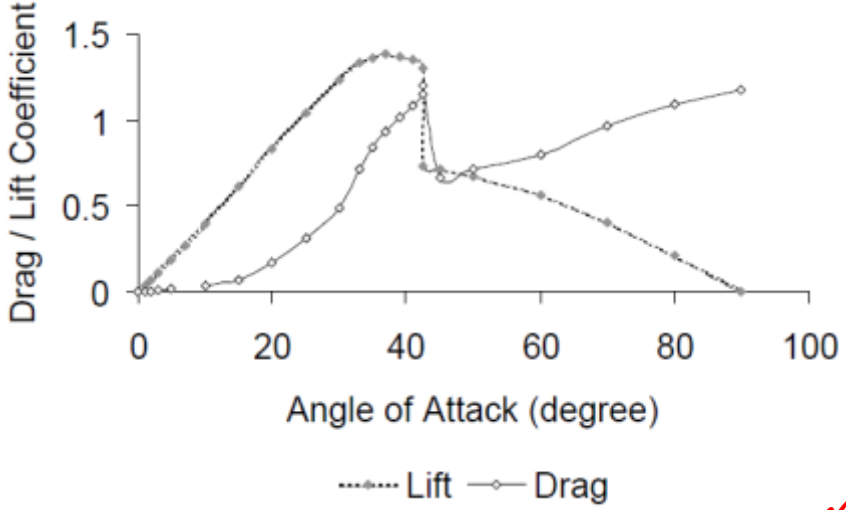
- ***Two-way Coupling***
  - Fluid(Euler) + Rigid Bodies (Lagrangian) [TOG04, 05, 06, 07, 11,SCA06, 08,09, 10]
  - Fully Lagrangian meshless method [SCA05, TVCG09, SIGGRPAH12, CGF12, SA12]

- ***Turbulent Flows***
  - Wavelet Noise[SIGGRAPH08]
  - Synthetic turbulence[SA2009]
    - Anisotropic particles[SA2010]
    - Stochastic particles[EG2011]

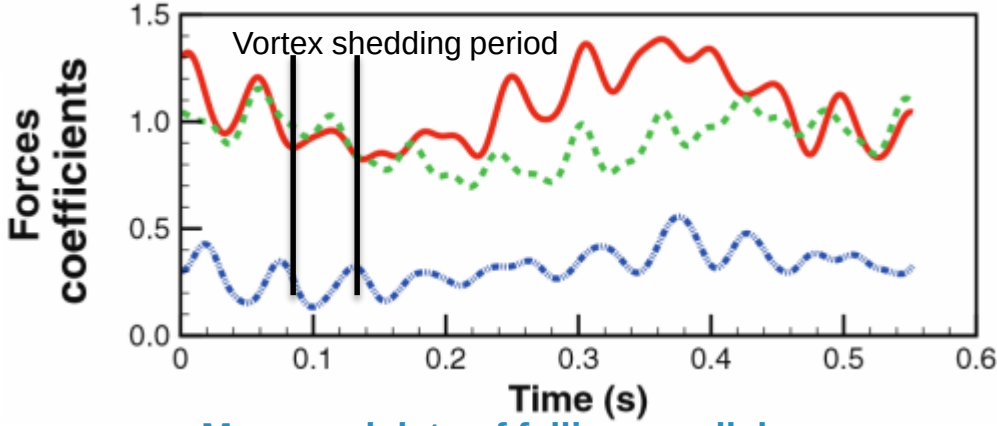
✓ **No work about freely moving bodies in turbulent flows**

Related work

- ***Motions inside Flow***
  - Swimming motions[SCA04, TVCG11, SIGGRAPH11]
  - Flying motions[SCA03,SIGGRAPH03,09]
  - Bubble dynamics[EG09,SIGGRAPH13]



Steady coefficients in CG (e.g. SIGGRAPH03)



Measured data of falling parallelograms
[Varshney et al. Physical Review E(2013)]

✓ Cannot explain motion's unsteady nature
✓ A sensitive motion

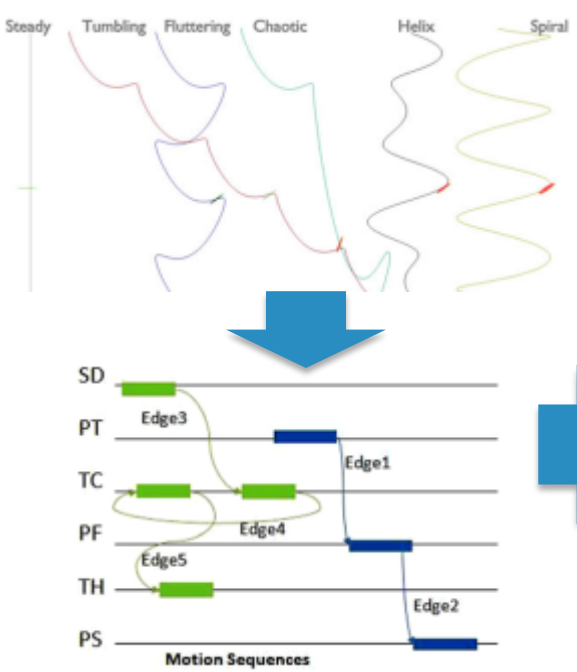
Related work

- ***Underwater Dynamics***
  - Underwater rigid-body dynamics[SIGGRPAH12]
    - Kirchhoff tensor due to added-mass effects
  - Underwater cloth dynamics[SIGGRPAH10]
    - Fractional derivatives due to Basset forces

✓ **For inviscid flows**
✓ **For low-Reynolds-number flows**

Previous work

- ***Motion Planning***
  - Rapidly-Exploring Random Trees[TVC2005]
  - Motion Graphs[TVC2013]

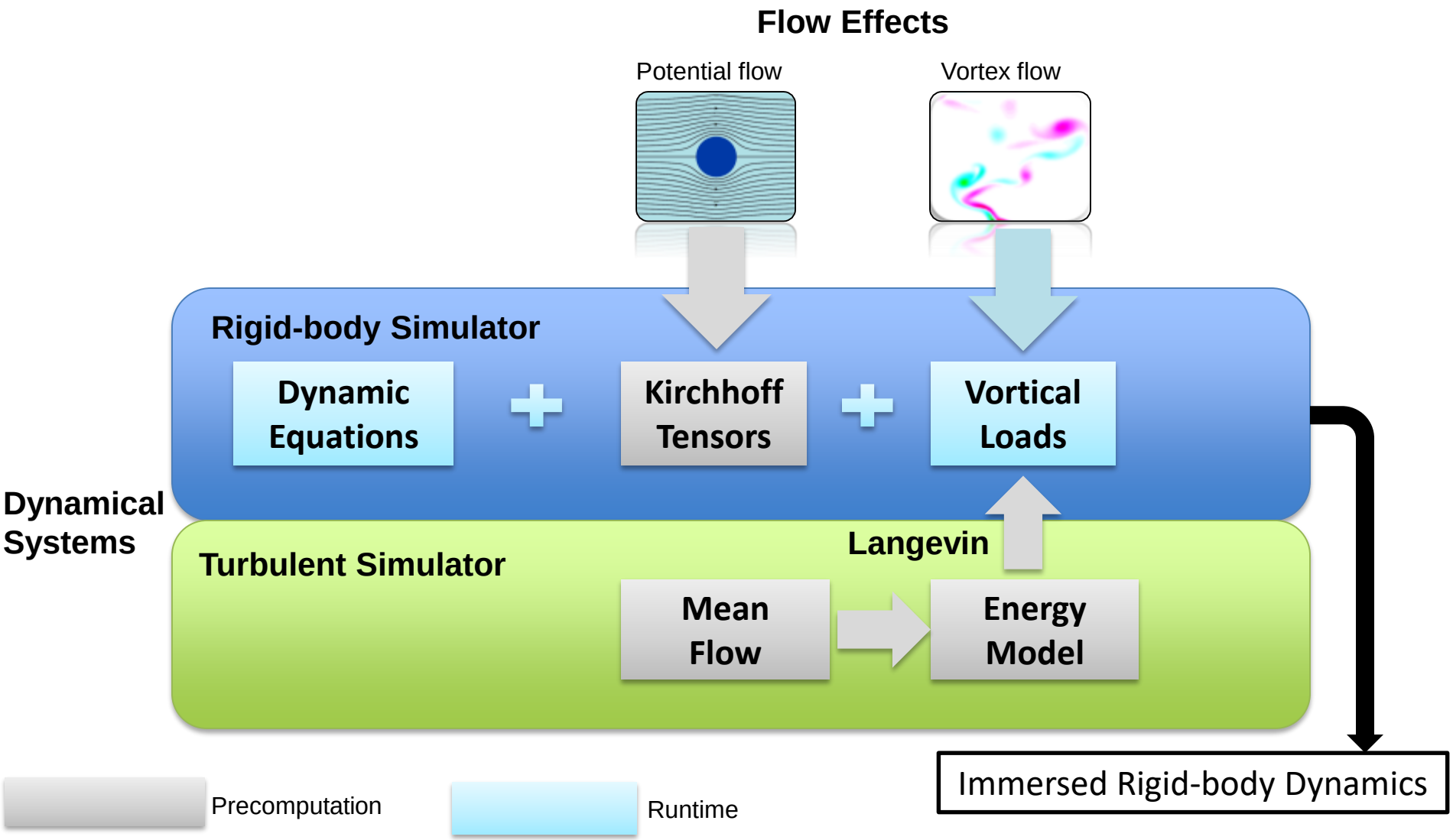


- ✓ **For simple geometries**
- ✓ **Miss surrounding flow info**

Immersed ***Rigid-body*** Dynamics

A Stochastic Model

Framework


Flow Effects
Potential flow
Vortex flow
Rigid-body Simulator
Dynamic Equations
Kirchhoff Tensors
Vortical Loads
Dynamical Systems
Turbulent Simulator
Langevin
Mean Flow
Energy Model
Immersed Rigid-body Dynamics
Precomputation
Runtime

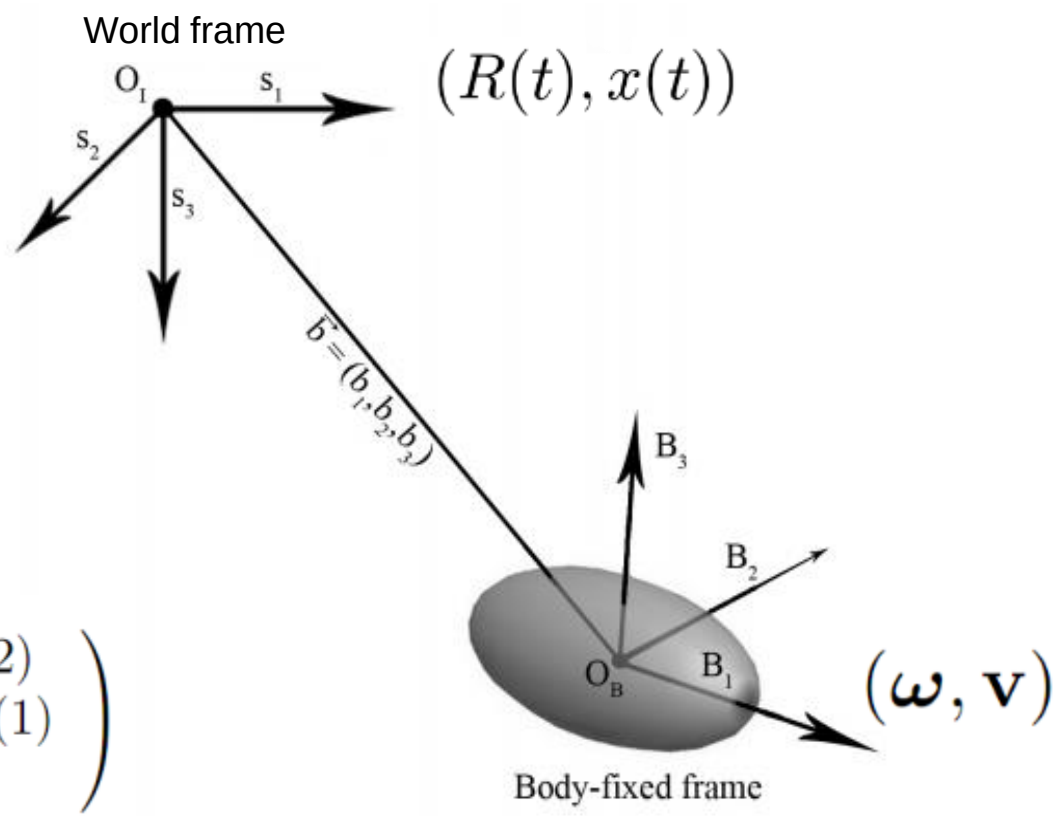
Rigid-body Simulator

- **Kinematic Equations**

$$\dot{R} = R\hat{\omega},$$
$$\dot{x} = Rv$$

Skew Matrix $\hat{\omega} =$

$$\begin{pmatrix} 0 & -\omega(3) & \omega(2) \\ \omega(3) & 0 & -\omega(1) \\ -\omega(2) & \omega(1) & 0 \end{pmatrix}$$



Rigid-body Simulator

- **Dynamic Equations**
  - Newton-Euler Equations

$$M \cdot \dot{v} = F_g$$
$$J \cdot \dot{\omega} = \Gamma_g$$

  - Kirchhoff Equations[Lamb,1945]

$$\begin{pmatrix} \dot{l} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} l \times \omega + p \times v \\ p \times \omega \end{pmatrix}$$

  - Generalized Kirchhoff Equations

Voritical Loads

$$\begin{pmatrix} \dot{l} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} l \times \omega + p \times v \\ p \times \omega \end{pmatrix} + \begin{pmatrix} \tau_g \\ f_g \end{pmatrix} + \begin{pmatrix} \tau_t \\ f_t \end{pmatrix}$$

Buoyancy-corrected Gravity

$$(l, p)^T = K \cdot (\omega, v)^T$$

Kirchhoff Tensors

Rigid-body Simulator

- Kirchhoff Tensors[Weissmann et al. SIGGRPAH2012]

Neumann Condition

$$\frac{\partial(\nabla\phi(p_i) - (\omega \times p_i + v))}{\partial n} = 0$$

Potential

$$\phi(p_i) = \sum_j \frac{\sigma_j}{\|p_i - s_j\|}, p_i \in \partial B$$

Velocity

$$\nabla\phi(p_i) = -\sum_j \sigma_j \frac{p_i - s_j}{\|p_i - s_j\|^3}$$

One point quadrature

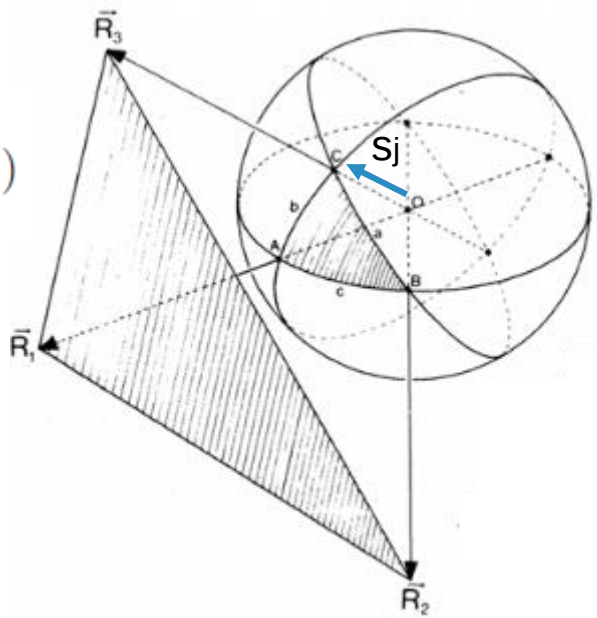
$$\int_{\partial B} f(\mathbf{z})\, dA \approx \sum_i f(\mathbf{z}_i)\, \mathrm{vol}(\Gamma_i)$$

$$\begin{pmatrix} J_f \\ M_f \end{pmatrix} = \rho_f \sum_i \sum_j \frac{\sigma_j}{\|p_i - s_j\|} \begin{pmatrix} p_i \times n(p_i) \\ n(p_i) \end{pmatrix} vol(\Gamma_i)$$

where

$$\sigma = M^{-1} \boxed{\sum_i \langle \omega \times p_i + v, n(p_i) \rangle vol(\Gamma_i)}$$

Normal flux



K is independent of body's dynamical states

Rigid-body Simulator

- Other Functional Forces
  - Buoyancy-corrected Gravity

$$f_g = R^T(m - \rho_f V)g$$

$$\tau_g = \rho_f V \vec{r} \times R^T g$$

center of buoyancy

  - Vortical Loads

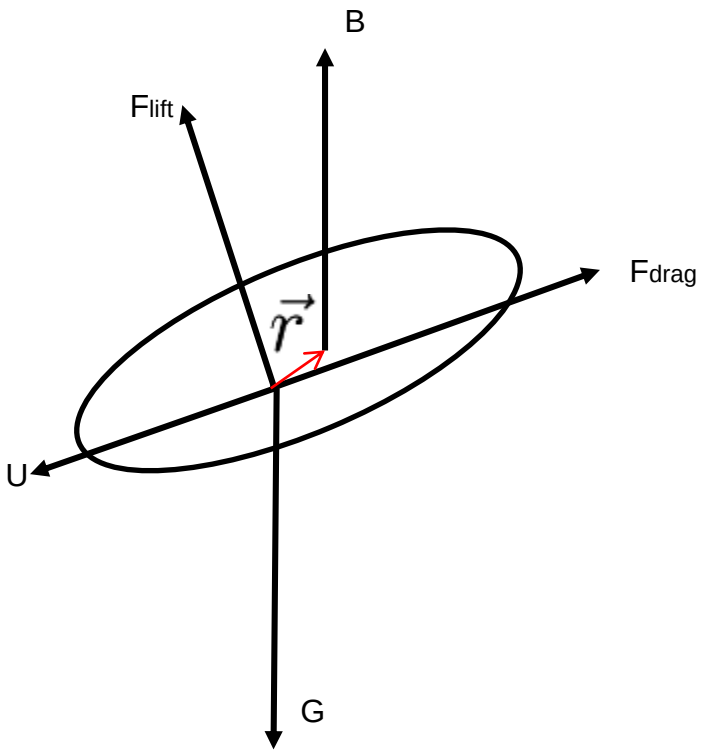
$$\tau_t = \vec{p} \times f_t$$

center of pressure

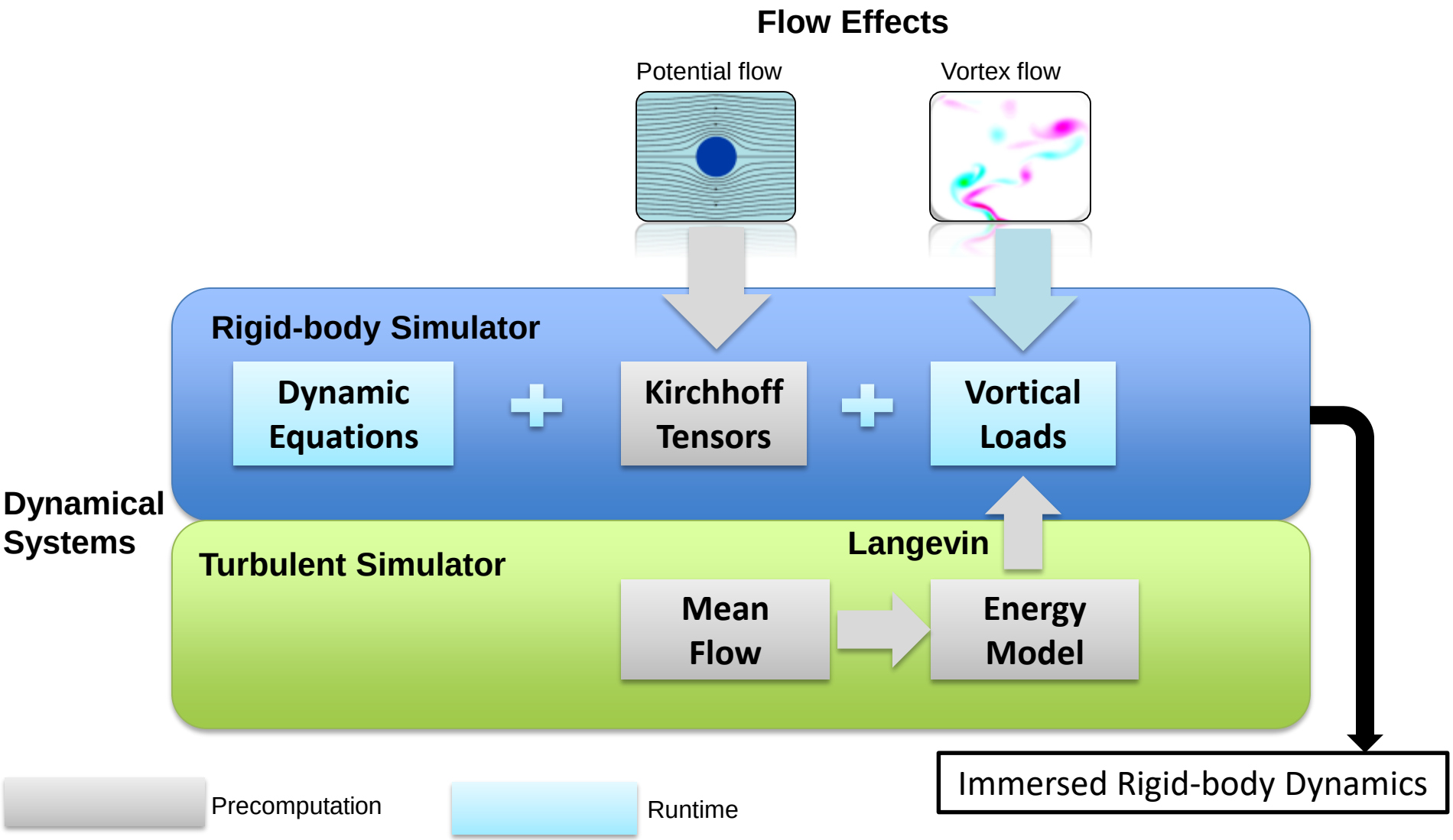
$$f_t = f_{drag} + f_{lift}$$

$$= -\frac{1}{2}\rho_f A|U|(C_d U + C_l U \times \frac{N \times U}{|N \times U|})$$

intermediate velocity



Framework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Turbulent-viscosity Model

- Navier-Stokes Equations

$$(u = \langle u \rangle + u')$$

- Reynolds-Averaged Navier-Stokes Equations

$$\begin{cases} D_t\langle u\rangle = \frac{1}{\rho}\nabla P + \nabla\cdot(\nu + v_T)(\nabla\langle u\rangle + \nabla^T\langle u\rangle) \\ \nabla\cdot\langle u\rangle = 0 \end{cases}$$

Turbulent Viscosity

Energy Transport Equations

$$\begin{cases} D_t k = \nabla\cdot((\nu + \frac{v_T}{\sigma_k})\nabla k) + G - \varepsilon \\ D_t\varepsilon = \nabla\cdot((\nu + \frac{v_T}{\sigma_\varepsilon})\nabla\varepsilon) + \chi(C_1 G - C_2\varepsilon) \end{cases}$$

$$\sigma_k = 1.0,\ \sigma_\varepsilon = 1.3,\ C_1 = 1.44 \text{ and } C_2 = 1.92$$

Turbulent-viscosity Model

- Turbulent Viscosity

$$v_T = C_\mu \frac{k^2}{\varepsilon} \qquad C_\mu = 0.09$$

- Energy Production

$$G = 2v_T \sum_{ij} S_{ij}^2 \qquad S_{ij} = \frac{1}{2}\left(\frac{\partial \langle u \rangle_i}{\partial x_j} + \frac{\partial \langle u \rangle_j}{\partial x_i}\right)$$

[Pfaff et al. SA2010]

$$\begin{cases} D_t k = G - \varepsilon \\ D_t \varepsilon = \chi(C_1 G - C_2 \varepsilon) \\ \chi = \varepsilon / k \end{cases}$$

Initial Conditions

$$k_0 = \frac{3}{2} U_0^2; \qquad \varepsilon_0 = C_\mu^{\frac{3}{4}} k_0^{\frac{3}{2}} / \Delta l$$

$$U_0 = \sqrt{(\rho_s / \rho_f - 1) g t}$$

Lagrangian Stochastic Model

- Generalized Langevin Equations

$$dX(t) = D_1 X(t)dt + \sqrt{D_2}dW$$

Drift Function Diffusion Function Wiener process

$$\begin{cases} D_1 = -(\frac{1}{2} + \frac{3}{4}C_k)\frac{\varepsilon}{k} \\ D_2 = C_k \varepsilon \\ W = N(0, \delta t) \end{cases}$$

Kolmogorov coefficient $C_k = 2.1$

※For rotational velocities, No drift function.

Implementation

Implementation

- Intel Core i7 CPU with 3.20 GHz and 12.0 GB RAM
- Rigid-body simulator
  - A geometric Lie group integrator
  - Kirchhoff tensors computation: 53 ms (1280 triangles)
- Turbulent simulator
  - Stable fluid solver
  - Computation cost: 182 ms ($32 \times 32 \times 8$ MAC)
- Runtime simulation cost: around **20 ms** per time step

Implementation

- ***Fractional-step Method***

$$u \xrightarrow{(f_g, \tau_g)} \dot{u}$$

1. Calculate Kirchhoff tensor
2. Integrate Buoyancy-corrected Gravity

$$\dot{u} \xrightarrow{Langevin} \ddot{u}$$

3. Calculate base flow
4. Solve energy transport
5. Combine stochastic model

$$\ddot{u} = \dot{u} - \begin{pmatrix} -\sqrt{C_k \varepsilon \Delta t}\vec{\xi_1} \\ \chi(\frac{1}{2} + \frac{3}{4}C_k)\dot{v}\Delta t - \sqrt{C_k \varepsilon \Delta t}\vec{\xi_2} \end{pmatrix}$$

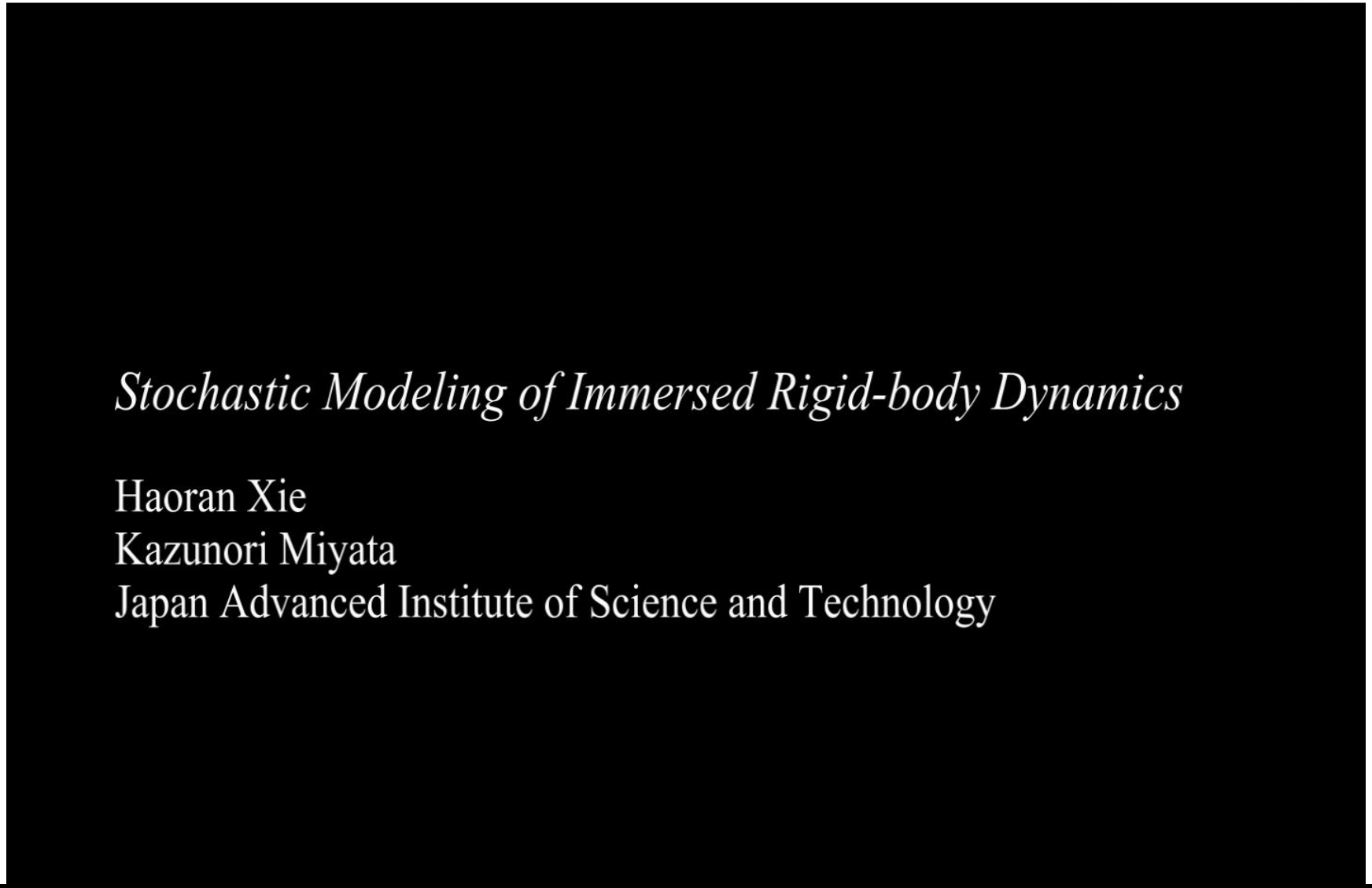
$Norm(0,1)$

$$\ddot{u} \xrightarrow{(f_t, \tau_t)} u^{new}$$

6. Calculate vortical loads
7. Update new velocity

Results

Video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Gliding paper airplane

Captured Motion

Simulation result

Falling Rubber Ellipsoid

Captured Motion

[SIGGRAPH2012]

Our approach

Contributions

- First step towards Immersed Body Dynamics
- Proposed a stochastic model based on the generalized Langevin equations of both translational and rotational velocities
- Proposed a fractional-step method to solve GKE with calculated vortical loads due to the viscous effect of the surrounding flow.
- An efficient approach using multi-precompuation steps to compute Kirchhoff tensor and turbulent energy model

Achieved **Realistic** simulations in **Realtime**

Limitations and Future work

- Sensitive to control parameters
  - Controllable simulations is challenging
- Hard to analysis motion patterns
  - Combine with motion planning
- Accuracy of coupling between flow and bodies
  - Handle unsteady forces explicitly
- For immersed deformable/articulated bodies

Thank You For Your Attention!

Stochastic Modeling of Immersed Rigid-body Dynamics

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