Stochastic Modeling of Immersed Rigid-body Dynamics

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Immersed Body Dynamics
Challenges

- **Fluid Simulations** *(e.g. S2013)*
- **Rigid-body Simulations** *(e.g. S2012)*
- **Dispersed Flows** *(e.g. S2010)*
- **Turbulent Flows** *(e.g. SA2010)*

- Unsteady dynamics of participated objects
- Vortical loads from the surrounding flow
Immersed *Rigid-body* Dynamics

Daily-life, Beautiful, Chaotic, and Sensitive Motions
Phenomena
What happened? –from physics view


How to achieve **Realistic** simulations in **Realtime**?

- High-Reynolds-number Flow
- Turbulent Flows
- Unsteady Forces
Related work

- **Two-way Coupling**
  - Fluid (Euler) + Rigid Bodies (Lagrangian) \([\text{TOG04, 05, 06, 07, 11, SCA06, 08, 09, 10}]\)
  - Fully Lagrangian meshless method \([\text{SCA05, TVCG09, SIGGRPAH12, CGF12, SA12}]\)

- **Turbulent Flows**
  - Wavelet Noise \([\text{SIGGRAPH08}]\)
  - Synthetic turbulence \([\text{SA2009}]\)
    - Anisotropic particles \([\text{SA2010}]\)
    - Stochastic particles \([\text{EG2011}]\)

✓ No work about freely moving bodies in turbulent flows
Related work

**Motions inside Flow**
- Swimming motions [SCA04, TVCG11, SIGGRAPH11]
- Flying motions [SCA03, SIGGRAPH03,09]
- Bubble dynamics [EG09, SIGGRAPH13]

Steady coefficients in CG (e.g. SIGGRAPH03)

- Cannot explain motion’s unsteady nature
- A sensitive motion

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Vortex shedding period
Related work

**Underwater Dynamics**

- Underwater rigid-body dynamics[SIGGRPAH12]
  - Kirchhoff tensor due to added-mass effects
- Underwater cloth dynamics[SIGGRPAH10]
  - Fractional derivatives due to Basset forces

✓ For inviscid flows
✓ For low-Reynolds-number flows
Previous work

- **Motion Planning**
  - Rapidly-Exploring Random Trees [TVC2005]
  - Motion Graphs [TVC2013]
  - For simple geometries
  - Miss surrounding flow info
Immersed *Rigid-body* Dynamics

A Stochastic Model
Rigid-body Simulator

- Kinematic Equations

\[ \dot{R} = R\hat{\omega}, \]
\[ \dot{x} = Rv \]

Skew Matrix \( \hat{\omega} = \)

\[
\begin{pmatrix}
0 & -\omega(3) & \omega(2) \\
\omega(3) & 0 & -\omega(1) \\
-\omega(2) & \omega(1) & 0
\end{pmatrix}
\]
Rigid-body Simulator

- **Dynamic Equations**
  - Newton-Euler Equations
    \[
    M \cdot \dot{v} = F_g \\
    J \cdot \dot{\omega} = \Gamma_g
    \]
  - **Kirchhoff Equations** [Lamb, 1945]
    \[
    \begin{pmatrix}
      \dot{i} \\
      \dot{p}
    \end{pmatrix} =
    \begin{pmatrix}
      l \times \omega + p \times v \\
      p \times \omega
    \end{pmatrix}
    \]
  - **Generalized Kirchhoff Equations**
    \[
    \begin{pmatrix}
      \dot{i} \\
      \dot{p}
    \end{pmatrix} =
    \begin{pmatrix}
      l \times \omega + p \times v \\
      p \times \omega
    \end{pmatrix} +
    \begin{pmatrix}
      \tau_g \\
      f_g
    \end{pmatrix} +
    \begin{pmatrix}
      \tau_t \\
      f_t
    \end{pmatrix}
    \]
  - **Voritical Loads**
  - **Buoyancy-corrected Gravity**
  - **Kirchhoff Tensors**
Kirchhoff Tensors [Weissmann et al. SIGGRAPH2012]

Neumann Condition

\[
\frac{\partial (\nabla \phi(p_i) - (\omega \times p_i + v))}{\partial n} = 0
\]

Potential

\[
\phi(p_i) = \sum_j \frac{\sigma_j}{\|p_i - s_j\|}, p_i \in \partial B
\]

Velocity

\[
\nabla \phi(p_i) = -\sum_j \frac{\sigma_j (p_i - s_j)}{\|p_i - s_j\|^3}
\]

One point quadrature

\[
\int_{\partial B} f(z) \, dA \approx \sum_i f(z_i) \, \text{vol}(\Gamma_i)
\]

where

\[
\begin{pmatrix} J_f \\ M_f \end{pmatrix} = \rho_f \sum_i \sum_j \frac{\sigma_j}{\|p_i - s_j\|} \begin{pmatrix} p_i \times n(p_i) \\ n(p_i) \end{pmatrix} \, \text{vol}(\Gamma_i)
\]

Normal flux

\[
\sigma = M^{-1} \sum_i \langle \omega \times p_i + v, n(p_i) \rangle \, \text{vol}(\Gamma_i)
\]

K is independent of body’s dynamical states
Rigid-body Simulator

- Other Functional Forces
  - Buoyancy-corrected Gravity
    \[ f_g = R^T (m - \rho_f V) g \]
    \[ \tau_g = \rho_f V \vec{r} \times R^T g \]
    center of buoyancy
  - Vortical Loads
    \[ \tau_t = \vec{p} \times f_t \]
    center of pressure
    \[ f_t = f_{drag} + f_{lift} \]
    \[ = -\frac{1}{2} \rho_f A |U| (C_d U + C_l U \times \frac{N \times U}{|N \times U|}) \]
    intermediate velocity
Framework

Flow Effects

Rigid-body Simulator

Dynamical Systems

Turbulent Simulator

Langevin

Mean Flow

Energy Model

Precomputation

Runtime

Immersed Rigid-body Dynamics
**Turbulent-viscosity Model**

- **Navier-Stokes Equations**
  
  \[ u = \langle u \rangle + u' \]

- **Reynolds-Averaged Navier-Stokes Equations**

  \[
  \begin{align*}
  D_t\langle u \rangle &= \frac{1}{\rho} \nabla P + \nabla \cdot (\nu + \nu_T)(\nabla \langle u \rangle + \nabla^T \langle u \rangle) \\
  \nabla \cdot \langle u \rangle &= 0
  \end{align*}
  \]

  **Turbulent Viscosity**

  **Energy Transport Equations**

  \[
  \begin{align*}
  D_t k &= \nabla \cdot \left( (\nu + \frac{\nu_T}{\sigma_k}) \nabla k \right) + G - \varepsilon \\
  D_t \varepsilon &= \nabla \cdot \left( (\nu + \frac{\nu_T}{\sigma_\varepsilon}) \nabla \varepsilon \right) + \chi (C_1 G - C_2 \varepsilon)
  \end{align*}
  \]

  \[ \sigma_k = 1.0, \sigma_\varepsilon = 1.3, C_1 = 1.44 \text{ and } C_2 = 1.92 \]
Turbulent-viscosity Model

- Turbulent Viscosity

\[ v_T = C_\mu \frac{k^2}{\varepsilon} \quad C_\mu = 0.09 \]

- Energy Production

\[ G = 2v_T \sum_{ij} S_{ij}^2 \]

\[ S_{ij} = \frac{1}{2} \left( \frac{\partial \langle u \rangle_i}{\partial x_j} + \frac{\partial \langle u \rangle_j}{\partial x_i} \right) \]

\[
\begin{align*}
D_t k &= G - \varepsilon \\
D_t \varepsilon &= \chi (C_1 G - C_2 \varepsilon) \\
\chi &= \varepsilon / k
\end{align*}
\]

[\text{Pfaff et al. SA2010}]

Initial Conditions

\[ k_0 = \frac{3}{2} U_0^2; \quad \varepsilon_0 = C_\mu \frac{3}{2} k_0^{\frac{3}{2}} / \Delta l \]

\[ U_0 = \sqrt{(\rho_s / \rho_f - 1) g t} \]
Lagrangian Stochastic Model

Generalized Langevin Equations

\[ dX(t) = D_1 X(t) \, dt + \sqrt{D_2} \, dW \]

\[ D_1 = -\left( \frac{1}{2} + \frac{3}{4} C_k \right) \frac{\varepsilon}{k} \]

\[ D_2 = C_k \varepsilon \]

\[ W = N(0, \delta t) \]

Kolmogorov coefficient \( C_k = 2.1 \)

※For rotational velocities, No drift function.
Implementation
Implementation

- Intel Core i7 CPU with 3.20 GHz and 12.0 GB RAM
- Rigid-body simulator
  - A geometric Lie group integrator
  - Kirchhoff tensors computation: 53 ms (1280 triangles)
- Turbulent simulator
  - Stable fluid solver
  - Computation cost: 182 ms (32 × 32 × 8 MAC)
- Runtime simulation cost: around **20 ms** per time step
Implementation

Fractional-step Method

\[ \mathbf{u} \xrightarrow{(f_g, \tau_g)} \dot{\mathbf{u}} \]

1. Calculate Kirchhoff tensor
2. Integrate Buoyancy-corrected Gravity

\[ \dot{\mathbf{u}} \xrightarrow{\text{Langevin}} \ddot{\mathbf{u}} \]

3. Calculate base flow
4. Solve energy transport
5. Combine stochastic model

\[ \ddot{\mathbf{u}} = \dot{\mathbf{u}} - \left( -\sqrt{C_k \varepsilon \Delta t} \dot{\xi}_1 \right) \]
\[ \left( \chi \left( \frac{1}{2} + \frac{3}{4} C_k \right) \dot{v} \Delta t - \sqrt{C_k \varepsilon \Delta t} \dot{\xi}_2 \right) \]

6. Calculate vortical loads
7. Update new velocity

\[ \ddot{\mathbf{u}} \xrightarrow{(f_t, \tau_t)} \mathbf{u}^{\text{new}} \]
Stochastic Modeling of Immersed Rigid-body Dynamics

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Gliding paper airplane

Captured Motion

Simulation result
Falling Rubber Ellipsoid

Captured Motion
[SIGGRAPH2012]  Our approach
Contributions

- **First step** towards Immersed Body Dynamics

- Proposed a **stochastic model** based on the generalized Langevin equations of both translational and rotational velocities

- Proposed a **fractional-step** method to solve GKE with calculated vortical loads due to the viscous effect of the surrounding flow.

- An **efficient** approach using multi-precomputation steps to compute Kirchhoff tensor and turbulent energy model

Achieved **Realistic** simulations in **Realtime**
Limitations and Future work

- Sensitive to control parameters
  - Controllable simulations is challenging

- Hard to analysis motion patterns
  - Combine with motion planning

- Accuracy of coupling between flow and bodies
  - Handle unsteady forces explicitly

- For immersed deformable/articulated bodies
Thank You For Your Attention!

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