以下に置いて、φ, ψ は論理式、$\Gamma, \Delta, \Pi, \Sigma$ は論理式の有限列 (空でも良い) とする。

LK の始式

$\varphi \vdash \varphi$

(等号のための始式)

$\vdash t = t \qquad t = s \vdash s = t \qquad t_1 = t_2, t_2 = t_3 \vdash t_1 = t_3$

$t_1 = s_1, \ldots t_n = s_n \vdash f(t_1, \ldots, t_n) = f(s_1, \ldots, s_n)$

$t_1 = s_1, \ldots t_n = s_n, R(t_1, \ldots, t_n) \vdash R(s_1, \ldots, s_n)$

LK の推論規則

(構造規則)

$$\frac{\Gamma \vdash \Delta}{\varphi, \Gamma \vdash \Delta}\ (\mathrm{WL}) \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \varphi}\ (\mathrm{WR})$$

$$\frac{\varphi, \varphi, \Gamma \vdash \Delta}{\varphi, \Gamma \vdash \Delta}\ (\mathrm{CL}) \qquad \frac{\Gamma \vdash \Delta, \varphi, \varphi}{\Gamma \vdash \Delta, \varphi}\ (\mathrm{CR})$$

$$\frac{\Pi, \varphi, \psi, \Gamma \vdash \Delta}{\Pi, \psi, \varphi, \Gamma \vdash \Delta}\ (\mathrm{EL}) \qquad \frac{\Gamma \vdash \Delta, \varphi, \psi, \Sigma}{\Gamma \vdash \Delta, \psi, \varphi, \Sigma}\ (\mathrm{ER})$$

$$\frac{\Gamma \vdash \Delta, \varphi \qquad \varphi, \Gamma \vdash \Delta}{\Gamma \vdash \Delta}\ (\mathrm{cut})$$

(論理結合子の規則)

$$\frac{\varphi, \Gamma \vdash \Delta}{\varphi \wedge \psi, \Gamma \vdash \Delta}\ (\wedge\mathrm{L}) \quad \frac{\varphi, \Gamma \vdash \Delta}{\psi \wedge \varphi, \Gamma \vdash \Delta}\ (\wedge\mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, \varphi \qquad \Gamma \vdash \Delta, \psi}{\Gamma \vdash \Delta, \varphi \wedge \psi}\ (\wedge\mathrm{R})$$

$$\frac{\varphi, \Gamma \vdash \Delta \qquad \psi, \Gamma \vdash \Delta}{\varphi \vee \psi, \Gamma \vdash \Delta}\ (\vee\mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, \varphi}{\Gamma \vdash \Delta, \varphi \vee \psi}\ (\vee\mathrm{R}) \quad \frac{\Gamma \vdash \Delta, \varphi}{\Gamma \vdash \Delta, \psi \vee \varphi}\ (\vee\mathrm{R})$$

$$\frac{\Gamma \vdash \Delta, \varphi \qquad \psi, \Gamma \vdash \Delta}{\varphi \to \psi, \Gamma \vdash \Delta}\ (\to\mathrm{L}) \qquad \frac{\varphi, \Gamma \vdash \Delta, \psi}{\Gamma \vdash \Delta, \varphi \to \psi}\ (\to\mathrm{R})$$

$$\frac{\Gamma \vdash \Delta, \varphi}{\neg\varphi, \Gamma \vdash \Delta}\ (\neg\mathrm{L}) \qquad \frac{\varphi, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg\varphi}\ (\neg\mathrm{R})$$

(量化記号の規則)

$$\frac{\varphi[t/x], \Gamma \vdash \Delta}{\forall x\varphi, \Gamma \vdash \Delta}\ (\forall\mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, \varphi[y/x]}{\Gamma \vdash \Delta, \forall x\varphi}\ (\forall\mathrm{R})$$

ただし $\Gamma, \Delta, \forall x\varphi$ は y を自由変数に持たない

$$\frac{\varphi[y/x], \Gamma \vdash \Delta}{\exists x\varphi, \Gamma \vdash \Delta}\ (\exists\mathrm{L}) \qquad \frac{\Gamma \vdash \Delta, \varphi[t/x]}{\Gamma \vdash \Delta, \exists x\varphi}\ (\exists\mathrm{R})$$

ただし $\Gamma, \Delta, \exists x\varphi$ は y を自由変数に持たない

Weakening, Contraction, Exchange を適用した変形規則

$$\frac{\Gamma \vdash \Delta, \varphi \qquad \Pi, \varphi \vdash \Sigma}{\Gamma, \Pi \vdash \Delta, \Sigma}\ (\text{cut'})^{\dagger}$$

$$\frac{\varphi, \psi, \Gamma \vdash \Delta}{\varphi \wedge \psi, \Gamma \vdash \Delta}\ (\wedge\text{L'}) \qquad \frac{\Gamma \vdash \Delta, \varphi \qquad \Pi \vdash \Sigma, \psi}{\Gamma, \Pi \vdash \Delta, \Sigma, \varphi \wedge \psi}\ (\wedge\text{R'})$$

$$\frac{\varphi, \Gamma \vdash \Delta \qquad \psi, \Pi \vdash \Sigma}{\varphi \vee \psi, \Gamma, \Pi \vdash \Delta, \Sigma}\ (\vee\text{L'}) \qquad \frac{\Gamma \vdash \Delta, \varphi, \psi}{\Gamma \vdash \Delta, \varphi \vee \psi}\ (\vee\text{R'})$$

$$\frac{\Gamma \vdash \Delta, \varphi \qquad \psi, \Pi \vdash \Sigma}{\varphi \to \psi, \Gamma, \Pi \vdash \Delta, \Sigma}\ (\to\text{L'})^{\dagger}$$

† こちらが原規則

等号のための推論規則

等号の始式は以下の推論規則に置き換えることができる.

$$\frac{t = t, \Gamma \vdash \Delta}{\Gamma \vdash \Delta}\ (\text{eq1})$$

$$\frac{s = t, \Gamma \vdash \Delta}{t = s, \Gamma \vdash \Delta}\ (\text{eq2})$$

$$\frac{t_1 = t_3, \Gamma \vdash \Delta}{t_1 = t_2, t_2 = t_3, \Gamma \vdash \Delta}\ (\text{eq3})$$

$$\frac{f(t_1, \ldots, t_n) = f(s_1, \ldots, s_n), \Gamma \vdash \Delta}{t_1 = s_1, \ldots, t_n = s_n, \Gamma \vdash \Delta}\ (\text{eq4})$$

$$\frac{R(s_1, \ldots, s_n), \Gamma \vdash \Delta}{t_1 = s_1, \ldots, t_n = s_n, R(t_1, \ldots, t_n), \Gamma \vdash \Delta}\ (\text{eq5})$$