

I211 Mathematical Logic, final report

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Answer one of the following.

You may consult the literature (including the web) or talk with others when you prepare the report. In that case, the reference or collaborators should be indicated, and you need to understand the contents and explain with your words.

Q 101 (keyword: compactness theorem). Let $\mathcal{L} = \{e, \cdot\}$. Show that there is no theory for finite groups, in other words, there is no \mathcal{L} -theory T which satisfies the following:

- (i) if M is a model of T , then M is a finite group, and,
- (ii) if M is a finite group, then M is a model of T .

Hint: Assume that there is T satisfying (i),(ii), and consider \mathcal{L} -sentences φ_n saying that “there is more than n many elements.” Then, is $T + \{\varphi_n : n \in \mathbb{N}\}$ satisfiable?

Q 102 (keywords: decidability, undecidability). It is known that the provability in LK for propositional logic is decidable, and the provability in LK for predicate logic is undecidable. Here, what do ‘decidable’ and ‘undecidable’ mean? Give the mathematical definitions of these words and explain the reason why LK for predicate logic is undecidable (with mathematical proof, if possible).

Q 103 (keywords: completeness theorem, cut elimination theorem). Answer the following.

1. Prove the completeness theorem and the cut elimination theorem for LK for predicate logic. (You may complete the proof explained in the lecture.)
2. Is the sequent “ \vdash ” (both sides are empty) provable in LK? What does this sequent mean?