

Initial sequents

$$\varphi \vdash \varphi$$

(initial sequents for equation)

$$\vdash t = t \quad t = s \vdash s = t \quad t_1 = t_2, t_2 = t_3 \vdash t_1 = t_3$$

$$t_1 = s_1, \dots, t_n = s_n \vdash f(t_1, \dots, t_n) = f(s_1, \dots, s_n)$$

$$t_1 = s_1, \dots, t_n = s_n, R(t_1, \dots, t_n) \vdash R(s_1, \dots, s_n)$$

Inference rules

(structural rules)

$$\frac{\Gamma \vdash \Delta}{\varphi, \Gamma \vdash \Delta} \text{ (WL)}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, \varphi} \text{ (WR)}$$

$$\frac{\varphi, \varphi, \Gamma \vdash \Delta}{\varphi, \Gamma \vdash \Delta} \text{ (CL)}$$

$$\frac{\Gamma \vdash \Delta, \varphi, \varphi}{\Gamma \vdash \Delta, \varphi} \text{ (CR)}$$

$$\frac{\Pi, \varphi, \psi, \Gamma \vdash \Delta}{\Pi, \psi, \varphi, \Gamma \vdash \Delta} \text{ (EL)}$$

$$\frac{\Gamma \vdash \Delta, \varphi, \psi, \Sigma}{\Gamma \vdash \Delta, \psi, \varphi, \Sigma} \text{ (ER)}$$

$$\frac{\Gamma \vdash \Delta, \varphi \quad \varphi, \Gamma \vdash \Delta}{\Gamma \vdash \Delta} \text{ (cut)}$$

(logical connectives)

$$\frac{\varphi, \Gamma \vdash \Delta}{\varphi \wedge \psi, \Gamma \vdash \Delta} \text{ (\wedge L)} \quad \frac{\varphi, \Gamma \vdash \Delta}{\psi \wedge \varphi, \Gamma \vdash \Delta} \text{ (\wedge L)}$$

$$\frac{\Gamma \vdash \Delta, \varphi \quad \Gamma \vdash \Delta, \psi}{\Gamma \vdash \Delta, \varphi \wedge \psi} \text{ (\wedge R)}$$

$$\frac{\varphi, \Gamma \vdash \Delta \quad \psi, \Gamma \vdash \Delta}{\varphi \vee \psi, \Gamma \vdash \Delta} \text{ (\vee L)}$$

$$\frac{\Gamma \vdash \Delta, \varphi}{\Gamma \vdash \Delta, \varphi \vee \psi} \text{ (\vee R)} \quad \frac{\Gamma \vdash \Delta, \varphi}{\Gamma \vdash \Delta, \psi \vee \varphi} \text{ (\vee R)}$$

$$\frac{\Gamma \vdash \Delta, \varphi \quad \psi, \Gamma \vdash \Delta}{\varphi \rightarrow \psi, \Gamma \vdash \Delta} \text{ (\rightarrow L)}$$

$$\frac{\varphi, \Gamma \vdash \Delta, \psi}{\Gamma \vdash \Delta, \varphi \rightarrow \psi} \text{ (\rightarrow R)}$$

$$\frac{\Gamma \vdash \Delta, \varphi}{\neg \varphi, \Gamma \vdash \Delta} \text{ (\neg L)}$$

$$\frac{\varphi, \Gamma \vdash \Delta}{\Gamma \vdash \Delta, \neg \varphi} \text{ (\neg R)}$$

(quantifiers)

$$\frac{\varphi[t/x], \Gamma \vdash \Delta}{\forall x \varphi, \Gamma \vdash \Delta} \text{ (\forall L)}$$

$$\frac{\Gamma \vdash \Delta, \varphi[y/x]}{\Gamma \vdash \Delta, \forall x \varphi} \text{ (\forall R)}$$

where y does not appear freely in $\Gamma, \Delta, \forall x \varphi$

$$\frac{\varphi[y/x], \Gamma \vdash \Delta}{\exists x \varphi, \Gamma \vdash \Delta} \text{ (\exists L)}$$

$$\frac{\Gamma \vdash \Delta, \varphi[t/x]}{\Gamma \vdash \Delta, \exists x \varphi} \text{ (\exists R)}$$

where y does not appear freely in $\Gamma, \Delta, \exists x \varphi$

Extra rules induced by Weakening, Contraction, Exchange

$$\frac{\Gamma \vdash \Delta, \varphi \quad \Pi, \varphi \vdash \Sigma}{\Gamma, \Pi \vdash \Delta, \Sigma} (\text{cut}')^\dagger$$

$$\frac{\varphi, \psi, \Gamma \vdash \Delta}{\varphi \wedge \psi, \Gamma \vdash \Delta} (\wedge L')$$

$$\frac{\Gamma \vdash \Delta, \varphi \quad \Pi \vdash \Sigma, \psi}{\Gamma, \Pi \vdash \Delta, \Sigma, \varphi \wedge \psi} (\wedge R')$$

$$\frac{\varphi, \Gamma \vdash \Delta \quad \psi, \Pi \vdash \Sigma}{\varphi \vee \psi, \Gamma, \Pi \vdash \Delta, \Sigma} (\vee L')$$

$$\frac{\Gamma \vdash \Delta, \varphi, \psi}{\Gamma \vdash \Delta, \varphi \vee \psi} (\vee R')$$

$$\frac{\Gamma \vdash \Delta, \varphi \quad \psi, \Pi \vdash \Sigma}{\varphi \rightarrow \psi, \Gamma, \Pi \vdash \Delta, \Sigma} (\rightarrow L)^\dagger$$

† original rules of LK