GRADIENT FLOWS ON A SPACE OF POWER-FUNCTIONS
FOR MINIMIZING A CONVEX FUNCTIONAL

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We study a class of nonlinear dynamical systems to develop efficient algorithms. As an efficient algorithm, interior point method based on Newton’s method is well known for solving convex programming problems which include linear, quadratic, semidefinite, $l_p$-programming problems [2] [3]. On the other hand, the geodesic of information geometry [1] is represented by a continuous Newton’s method for minimizing a convex functional called divergence. Thus, we discuss a relation between information geometry and convex programming in a related family of continuous Newton’s method. In particular, as the optimization of parameter values, we consider the $\alpha$-projection problem from a given data onto an information geometric submanifold spanned with power-functions such as a weighted $l_p$-norm.

First, we present there exists a structural similarity between the $\alpha$-projection and semidefinite programming problems [4]. The geometric structure is based on the autoparallelisms [1] or linear property in the function space over finite discrete variables or the space of positive definite matrices, respectively. The property is used to derive an approximation algorithm as predictor-corrector method for solving the problem through the geodesic. The maximum step-length is determined by geometric quantities with respect to the Riemannian metric and the dual-connection. We also show that the proposed method is effective in simulation.

Next, we reformulate the $\alpha$-projection problem into a form of convex programming as a $l_p$-programming and the related ones. From the reformulated problems, we derive self-concordant barrier functions [2] [3] according to the real values of $\alpha$. Thus, it means the existence of a polynomial time interior-point algorithm for our problem. Furthermore, we present the coincidence with the gradient directions on the geodesic for the divergence and on the affine-scaling trajectory for a modified barrier function. These results connect part of nonlinear and algorithmic analyses with the discreteness of variables.

References


