Necessary backbone of superhighways for transport on geographical complex networks

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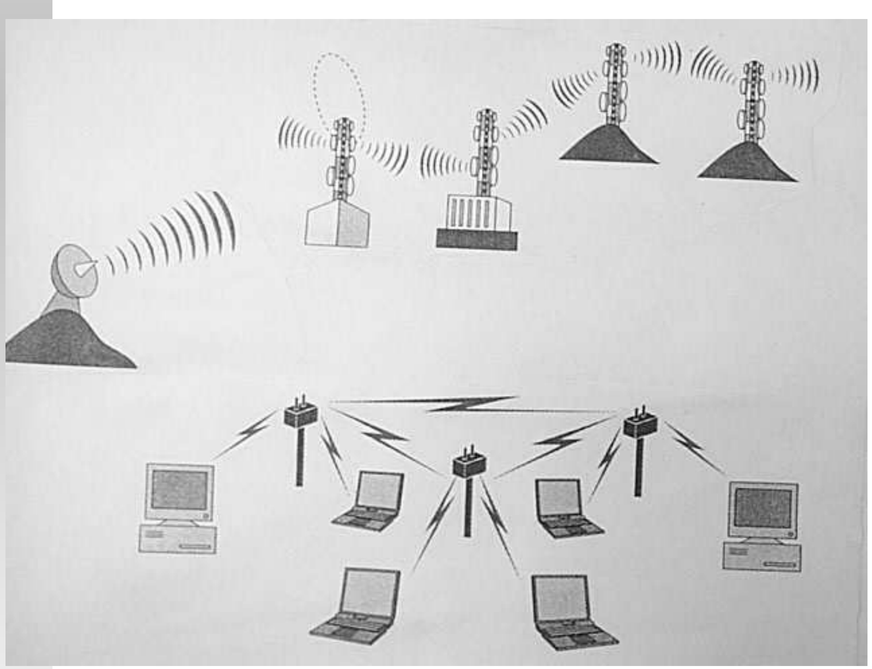
1. Introduction

Dynamic configuration of (backbone) networks in an emergent or self-organized manner

Problems for efficient communication such as in distributed sensor networks or P2P systems



dissipation of wireless beam-power or wired line-cost for long-range links

interference by crossing of links (non-planar)

to realize efficient transport of packets

1-1. Model of Ad-Hoc Nets

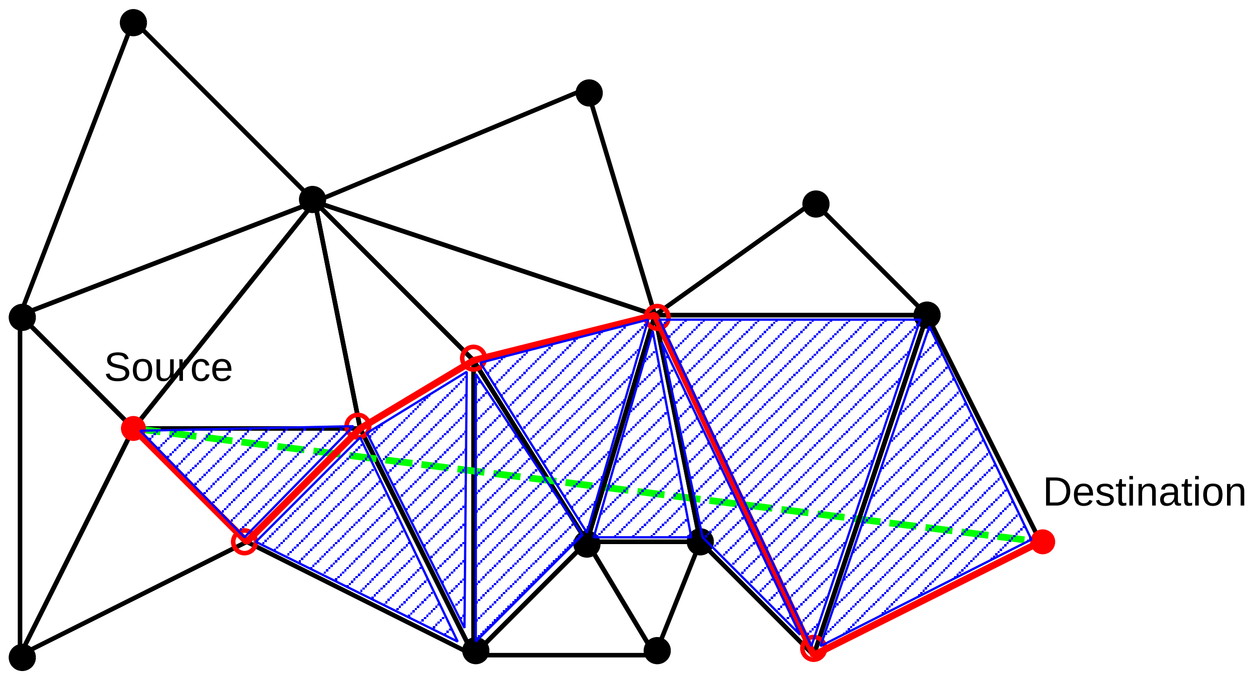
We assume that

- a network evolves with a new node
- the position of each node is fixed
- a few nodes have sufficient power to be hubs
- various transmission ranges and the orientation are controllable (for wireless links)
- It is better that only local information is used because of the topological change

These are reasonable in the current technology, but we don't care about the details in device or application

1-2. Efficient Routing

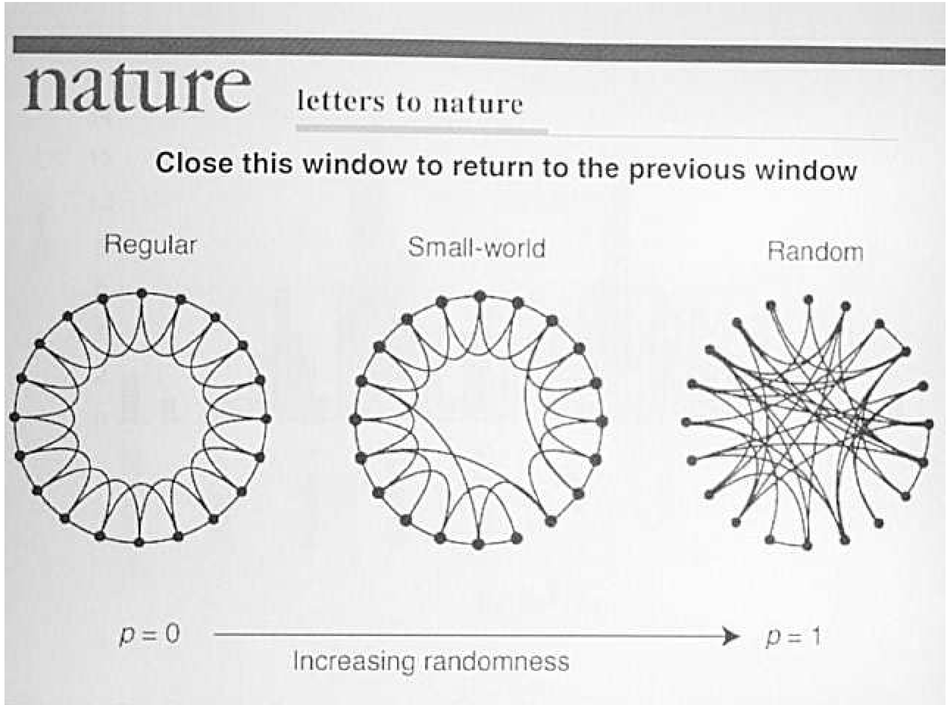
Planar triangulation: reasonable math. abstraction of ad hoc net. Moreover, a memoryless, no defeat, and competitive online routing algorithm has been developed for planar networks taking into account the face.



Bose and Morin, SIAM J. of Comp. 33(4), 2004

1-3. Motivation for Shortcuts

Shorten the path length remaining local clusters



SW model, D.J. Watts and S.H. Strogatz, Nature, 393, 1998

Moreover, the robustness can be improved by adding shortcuts to a one-dim. SW model

C. Moore, and M.E.J. Newman, PRE 61, 2000

1-4. Objective

To investigate the effect of shortcuts on geographical networks in the viewpoints:

the tolerance of connectivity to random failures and targeted attacks on hubs

the backbone for transport measured by the usage and centrality of links on the shortest paths

in typical planar network models:

- random Apollonian network in complex network science,
- Delaunay triangulation in computer science,
- our proposed models to bridge them.

1-5. Scale-Free Nets with Hubs

Existing a surprisingly common structure: SF net.

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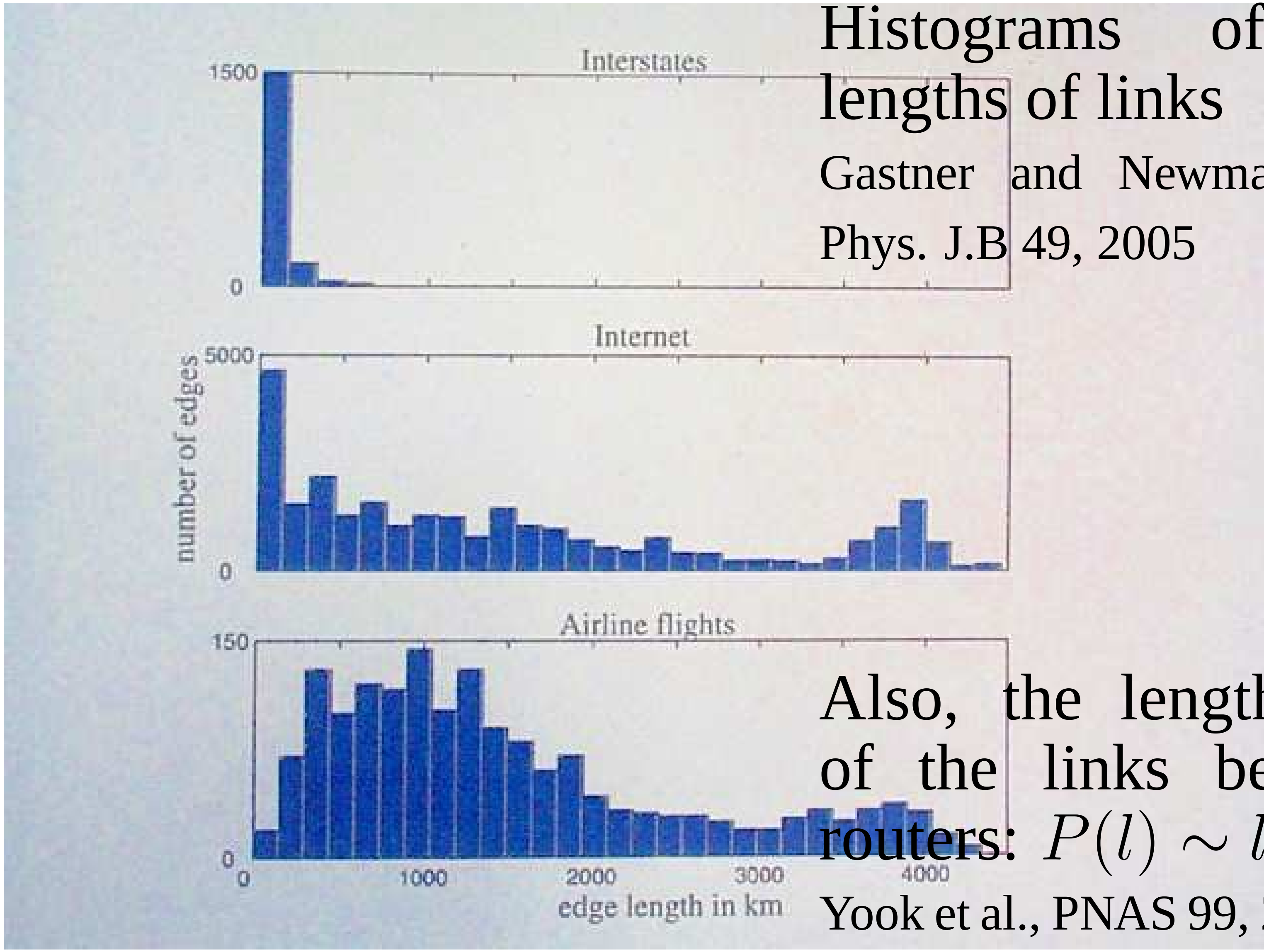
$\Rightarrow$ However, most of SF net models were irrelevant to a geographical space.

2. Geographical SF Nets

Many real nets have not only scale-free but also geographical heterogeneity:

- The Internet, power-grid, and airline-flight connections, etc. are embedded in a metric space
- Nodes are usually concentrated in a urban city (there are dense and sparse areas on the earth)
- Especially for wireless communication, planarity is suitable
- Majority of links are short due to economical reasons for the construction and maintenance costs

2-1. Rare Long-range Links



Histograms of the lengths of links

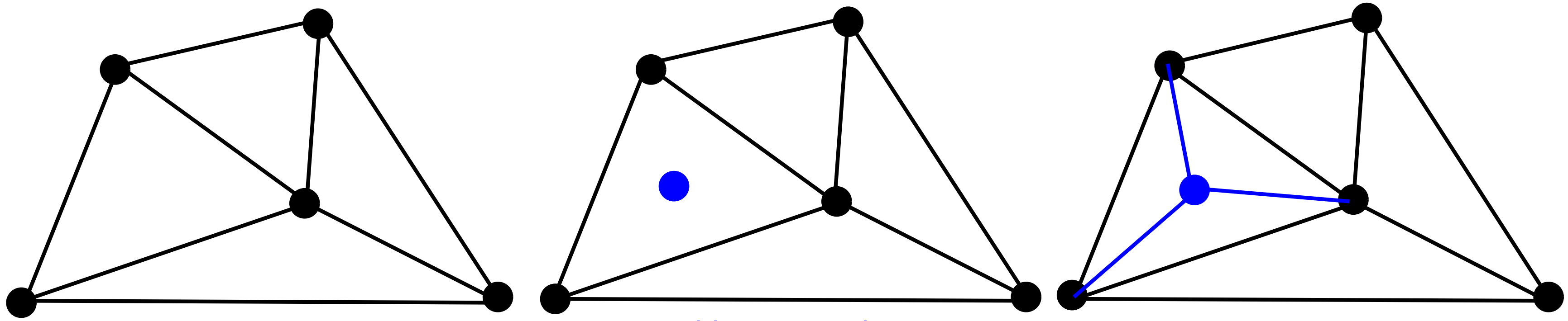
Gastner and Newman, Eur. Phys. J.B 49, 2005

Also, the length dist. of the links between routers: $P(l) \sim l^{-1}$

Yook et al., PNAS 99, 21, 2002

2-2. Random Apollonian Net

Configuration: iterative subdivision of a randomly chosen triangle from an initial triangulation

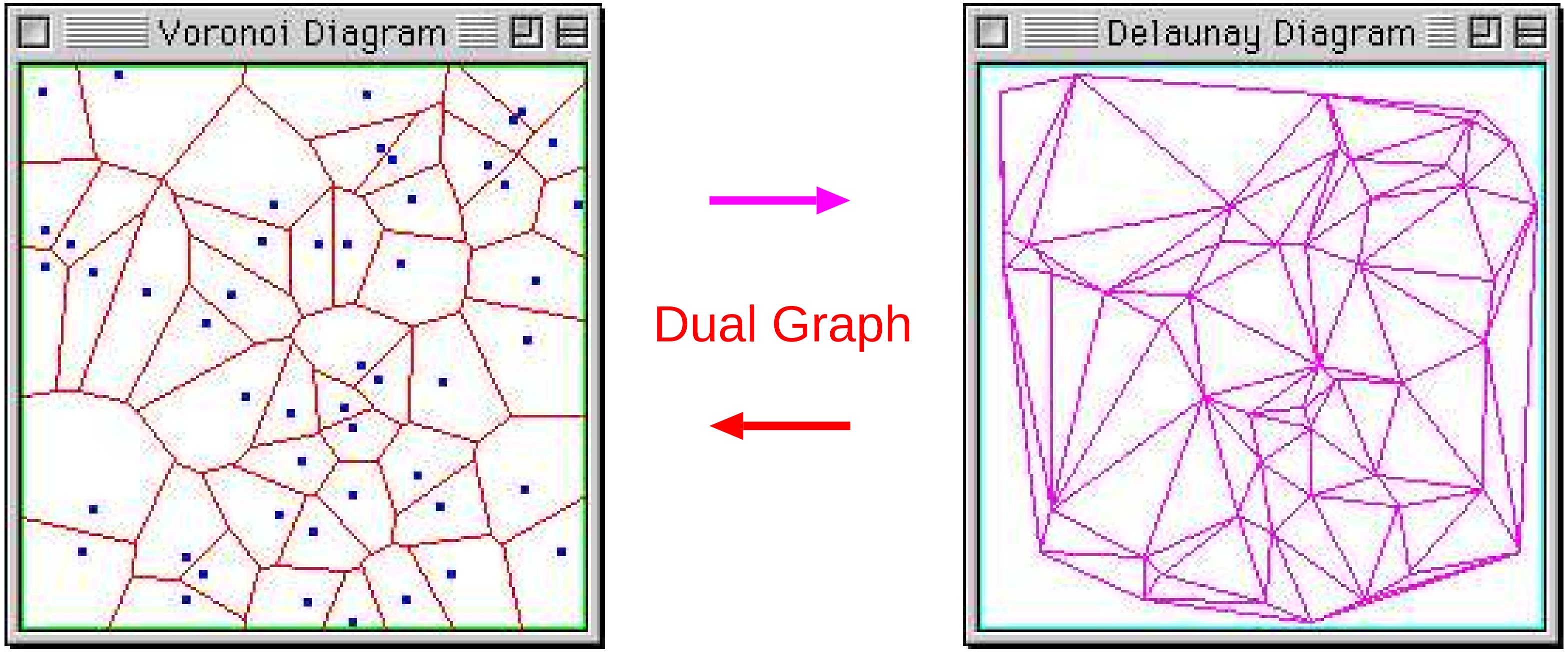


$\Rightarrow$ Some long-range links naturally appear in narrow collapsed triangles near the boundary edges

Zhou, Yan, Wang, Phys. Rev. E 71, 2005

2-3. Voronoi and Delaunay

Optimal triangulation in some geometric criteria: maximim angle, minimax circumcircle, short path length in the same order as the direct Euclidean dist.

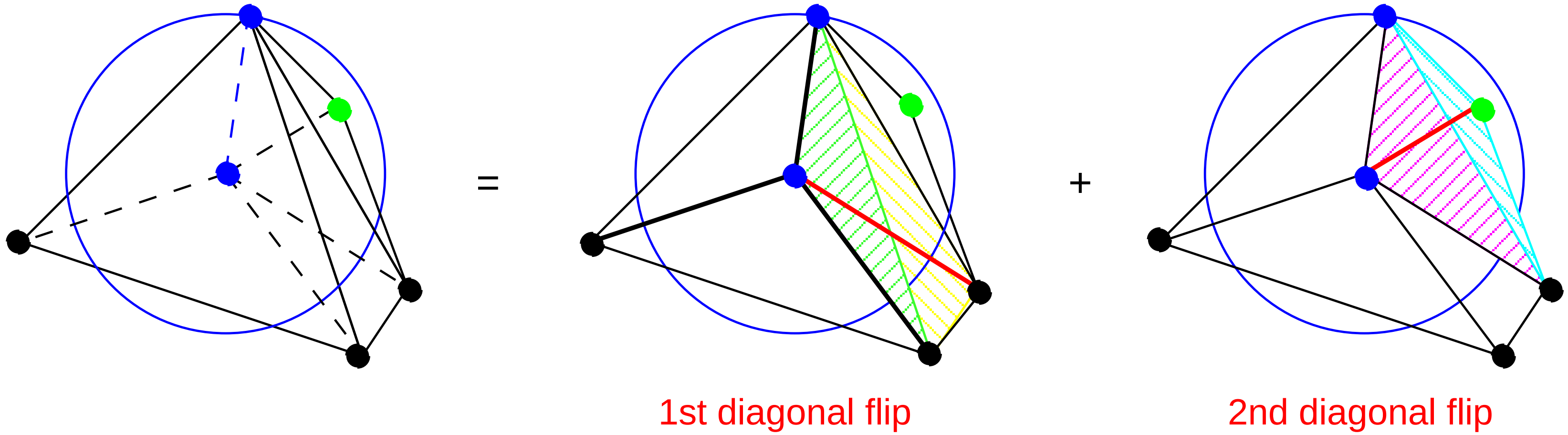


⇒ Consider the combination of RA (by triangulation on a plane) and DT to avoid the long-range links

2-4. Delaunay-like SF Net

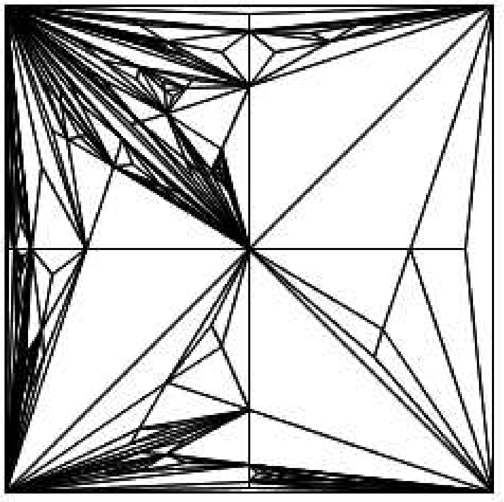
We propose DLSF:

- Set an initial planar triangulation.
- Select a triangle at random and add a new node at the barycenter. Then, connect the new node to its three nodes tentatively. By iteratively applying diagonal flips, connect it to the nearest node(s) within a radius as a localization.

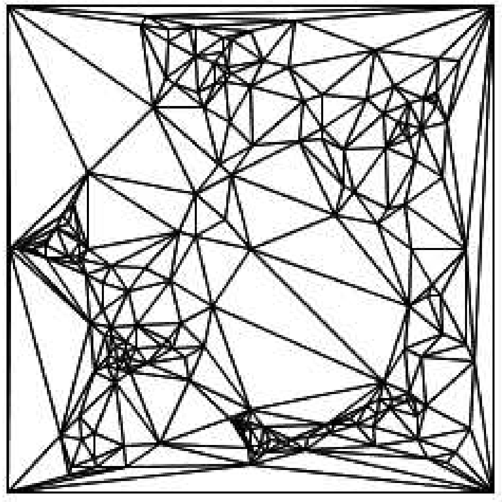


2-5. Topological Structure

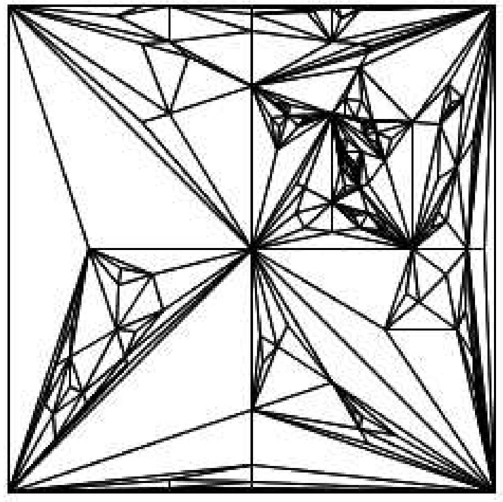
DLSF has the intermediate structure



RA



DT

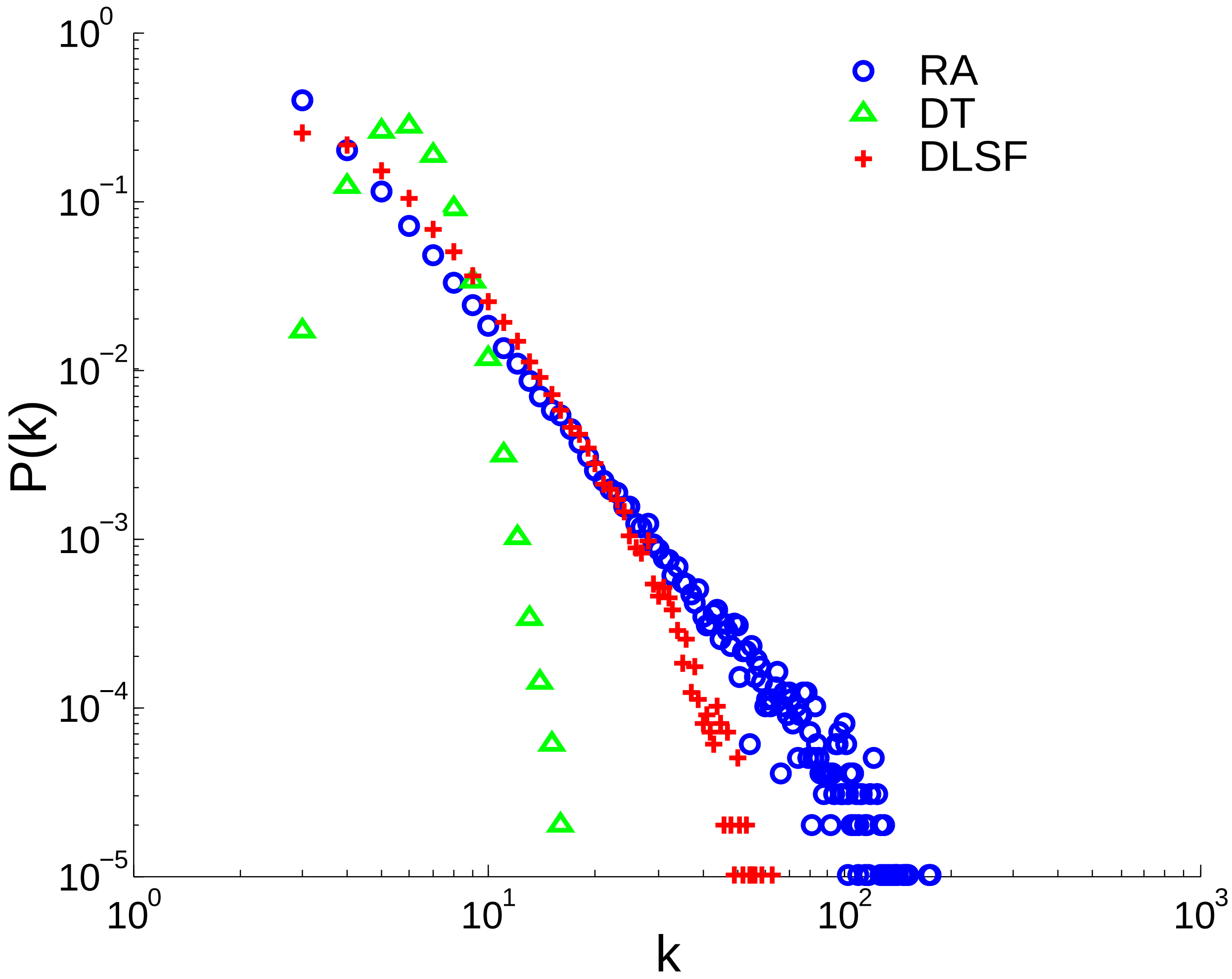


DLSF

We should remark that there exists

- mixing of dense and sparse areas as similar to a population density
- star-like stubs at four corner and the center nodes

2-6. Degree Distribution



RA: power law, DT: lognormal, DLSF: power law with exponential cutoff (the deg. of hubs are reduced)

3. Improvement of Robustness

SF nets are extremely vulnerable for the intentional attacks on hubs

R. Albert, and A.-L. Barabási, Nature 406, 378, 2000

⇒ It's further affected by geographical constraints, such as local cycles.

3-1. Theoretical Prediction

The breaking of connectivity for random failures becomes more serious by small-order cycles

On the assumpt. of a tree $L_c = 0$, the percolation threshold $q_c^* = \langle k \rangle / (\langle k^2 \rangle - \langle k \rangle)$ is well-known.

$$\Downarrow$$

It is generalized to any cycle length e.g. $L_c = 3$,

$$q_c = \frac{\langle k \rangle}{\langle k(k-1) \rangle - \left(1 - q_c^{\frac{\langle k(k-2) \rangle}{\langle k \rangle}}\right) \langle C(k) k (k-1)^2 \rangle},$$

$1 - f_c = q_c > q_c^*$ predicts decreasing of robustness.

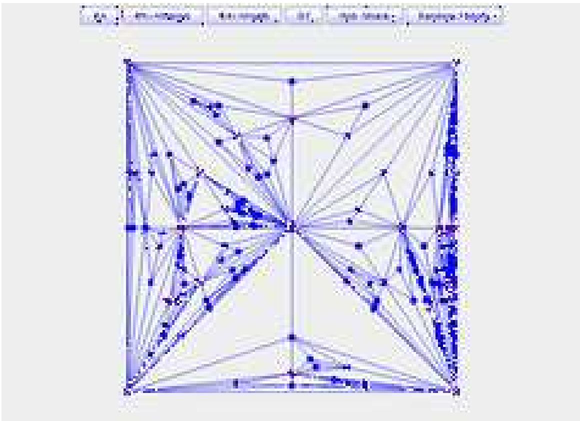
Huang et al., Europhy. Lett. 72, 2005 & PRE 75, 2007

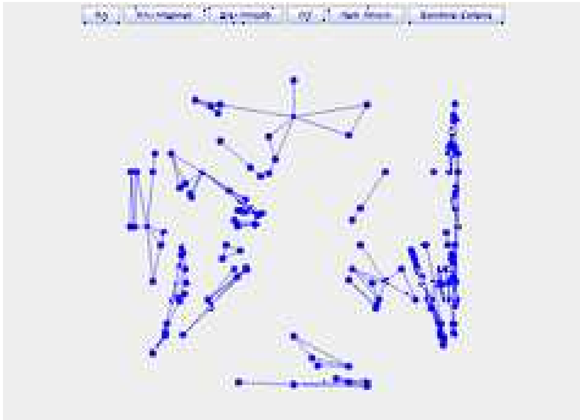
3-2. Damages by Attacks

Initial N=200

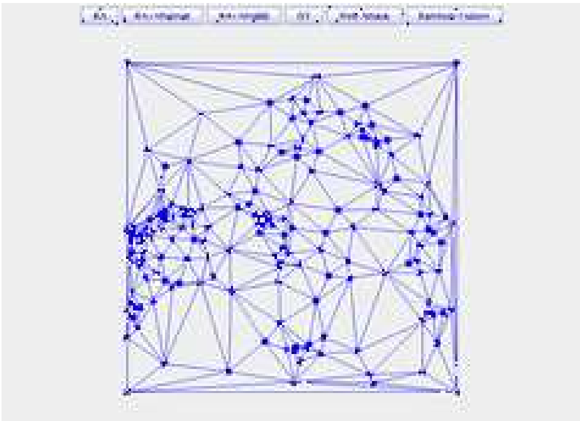
targeted attacks on 16 hubs

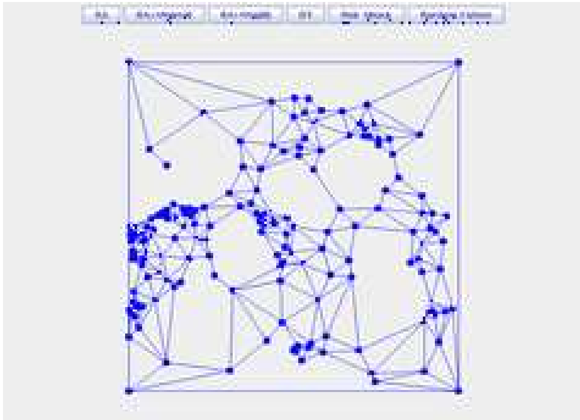
RA





DT

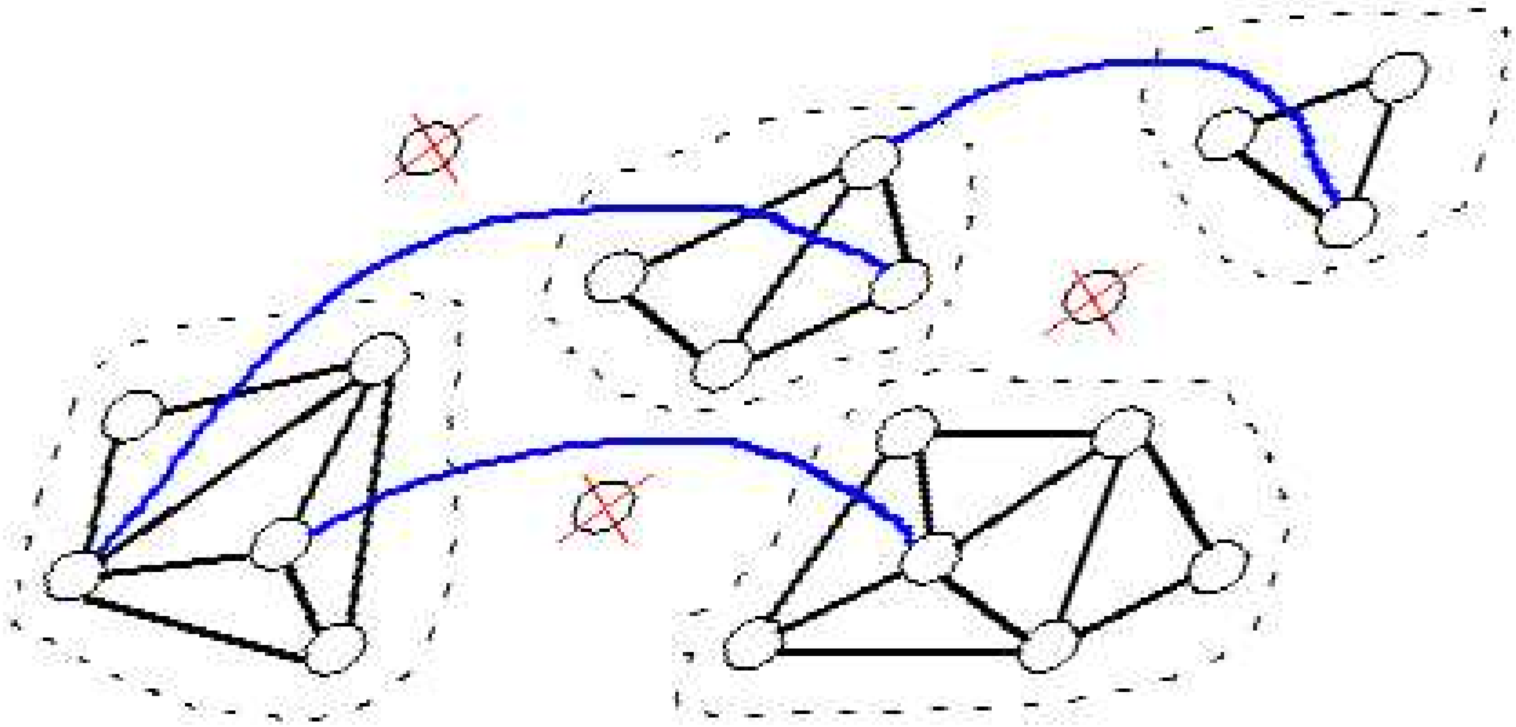




Vulnerable RA broken to many isolated clusters

3-3. Overhead Bridge

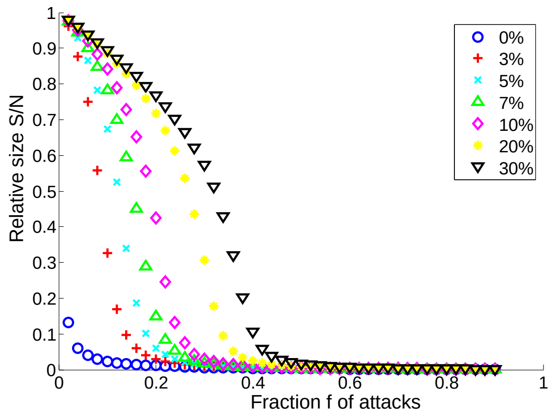
The effect of adding shortcuts on the robustness



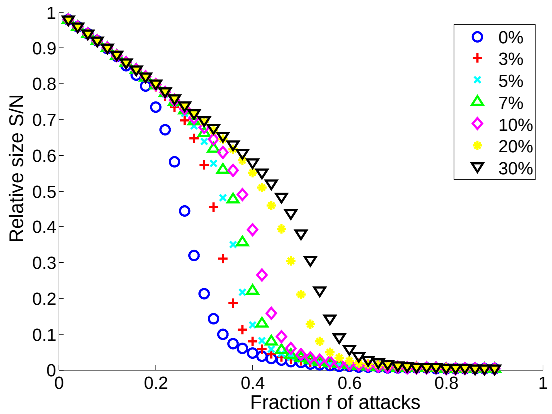
⇒ To bridge the isolated areas on the plain

3-4. Giant Component

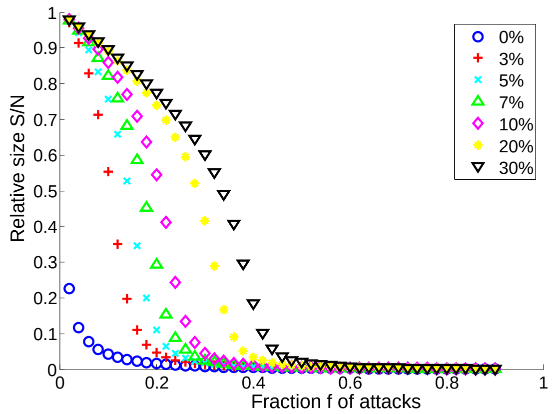
Relative size S/N of the GC vs. fraction f of attacks



RA



DT



DLSF

Hayashi et. al., Physica A 380, 2007

$\Rightarrow$ The robustness is improved remaining with a bigger GC from ◯ 0 % to ▽ 30 % of the shortcuts rate

3-5. Hetero. Communication

Spatially heterogeneous communication

Assumed that both source and terminal are chosen in uniformly random from all nodes at every time step.

However the spatial distribution of packet generation and receiving is remarkably heterogeneous

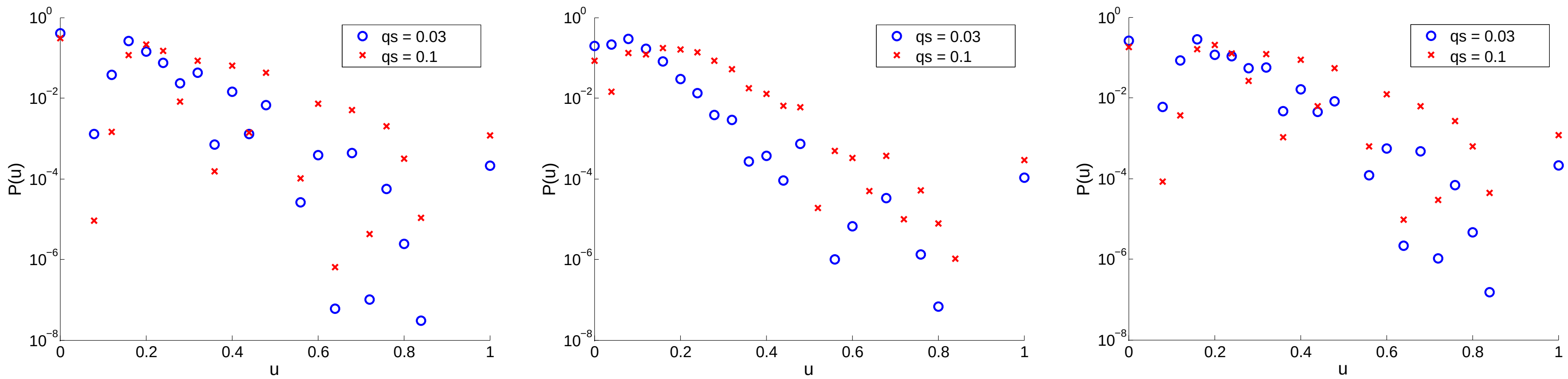
- Dense area $\leftrightarrow$ much more communication
- Sparse area $\leftrightarrow$ few frequent one

according to the geographical configuration of nodes.

3-6. Superhighways

Consider the distribution $P(u)$ of frequency of superhighways $\quad u \stackrel{\text{def}}{=} l_{super}/l_{short}$

where l_{super} is the number of shortcuts in a given shortest path of length l_{short}



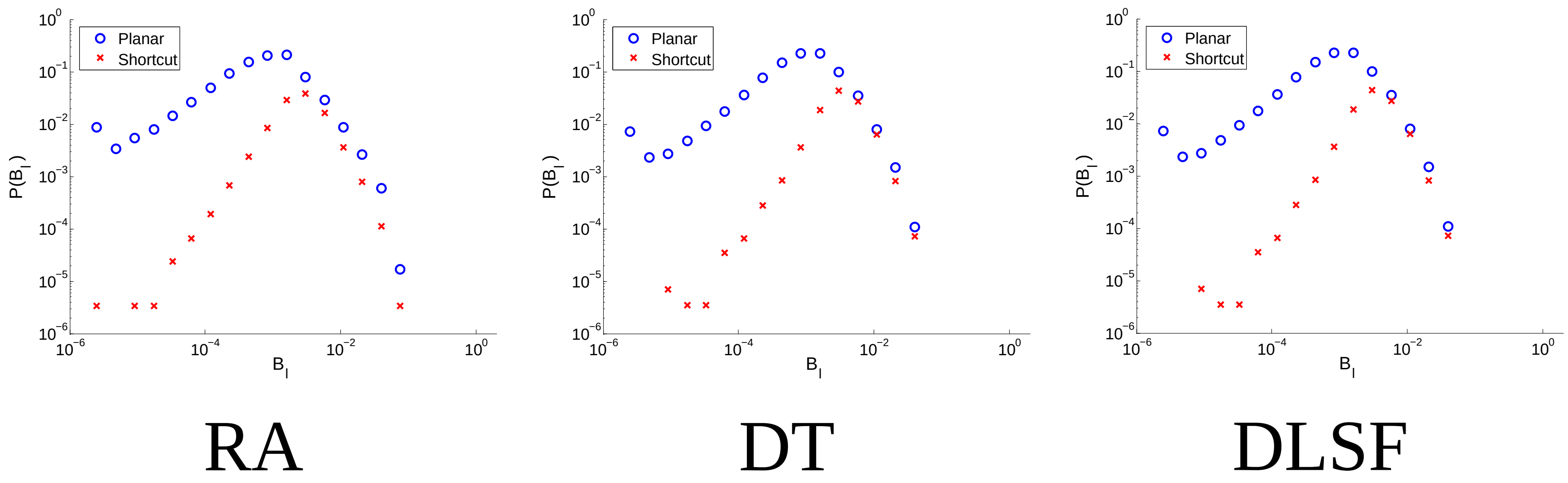
RA DT DLSF

$\Rightarrow$ High frequency in spite of only 3 or 10 % of the total

3-7. $P(B_l)$ of Link Centrality

Normalized betweenness centrality of link l on the shortest distant path between nodes j and k

$$B_l \stackrel{\text{def}}{=} \frac{2}{(N-2)(N-3)} \sum_{k \neq j \neq l1,l2} \frac{b_k^j(l)}{b_k^j},$$



$\Rightarrow$ The peak is righter for the shortcuts with higher B_l

4. Summary: shortcut effects

Based on the realistic SF properties,

- we've proposed a modified model from RA in complex net. science and DT in computer science to reduce long-range links on a planar space for ad hoc networks.
- For the vulnerability to attacks caused by geographical constraints, it has been shown that adding shortcuts of only about 10 % of the total improve it.
- They also construct a necessary backbone such as superhighways to bridge isolated clusters (more contribute in the measures of frequency and centrality of links used on the shortest distant paths.)

Thanks

Thank you for your kind attention !

Related papers: (see Ref. in the Proc.)

- Geographical effects on the path length and the robustness in complex networks,
  Physical Review E 73, 066113, (2006).
- Improvement of the robustness on geographical networks by adding shortcuts,
  Physica A 380, 552-562, (2007).

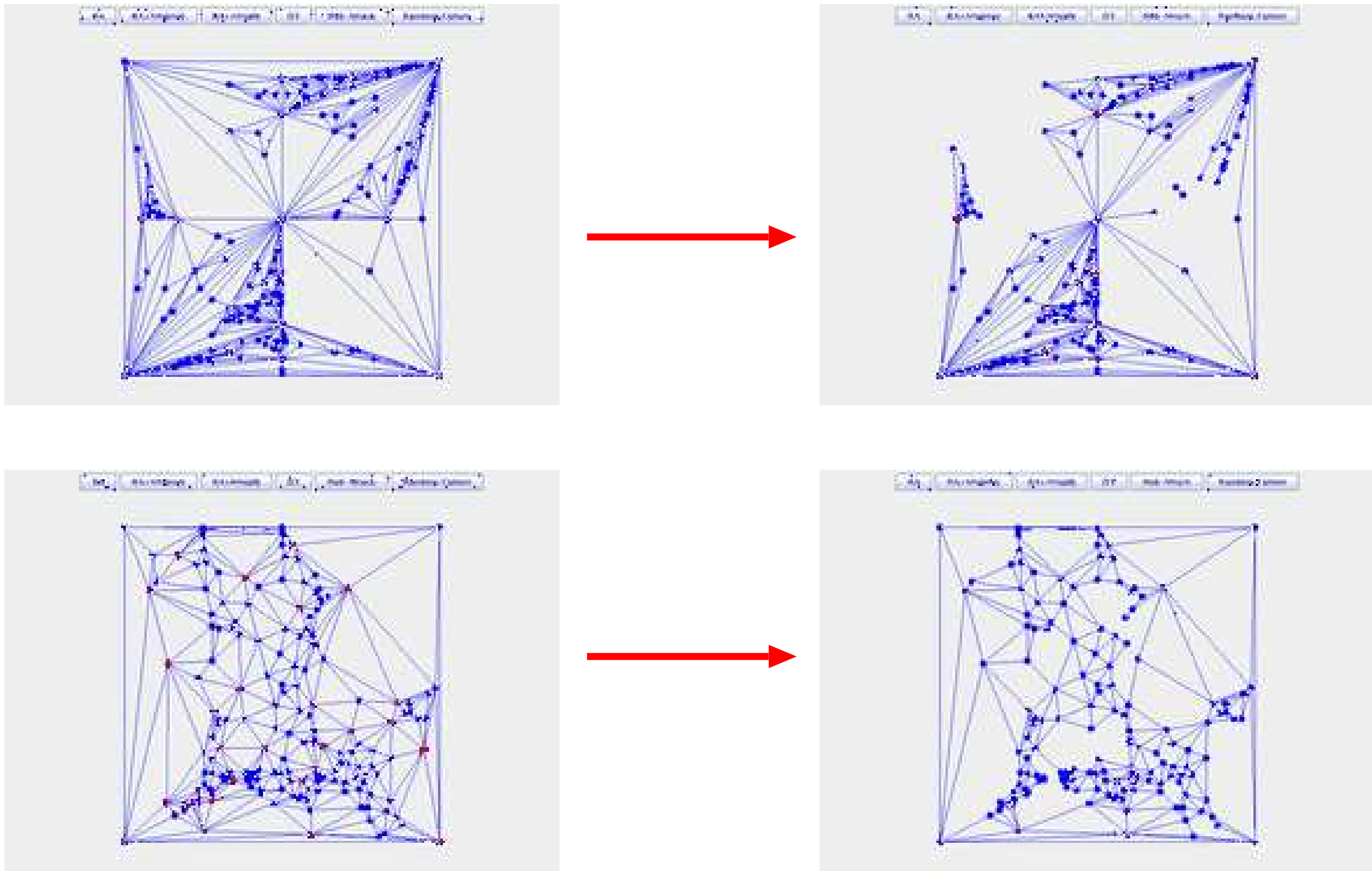
A1. Damages by Failures

Initial N=200

randomly removed
32 nodes

RA

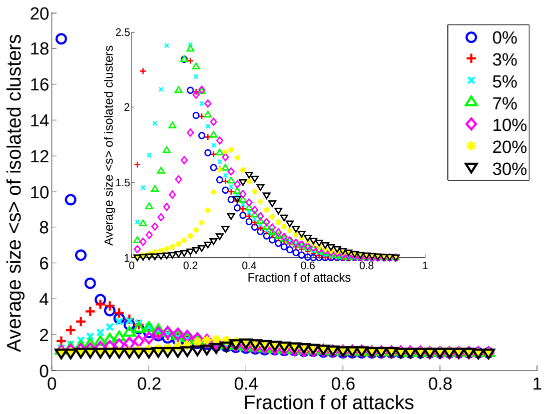
DT



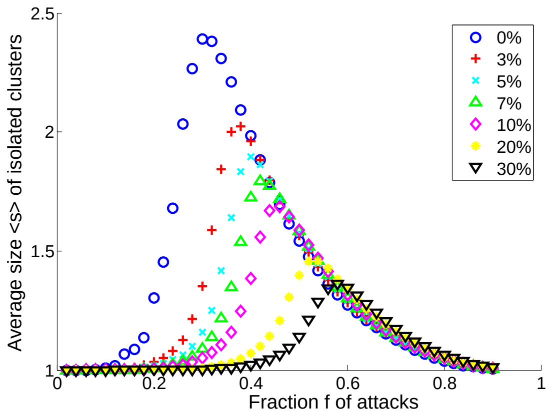
Similarly remained connectivity in the GC

A2. Breaking of GC at the Peak

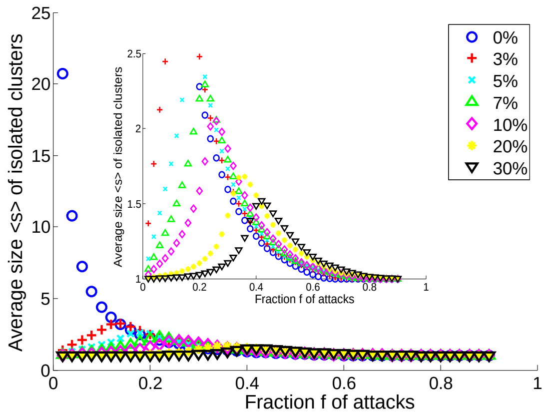
Average size $\langle s \rangle$ of isolated clusters except of the GC vs. fraction f of attacks



RA



DT

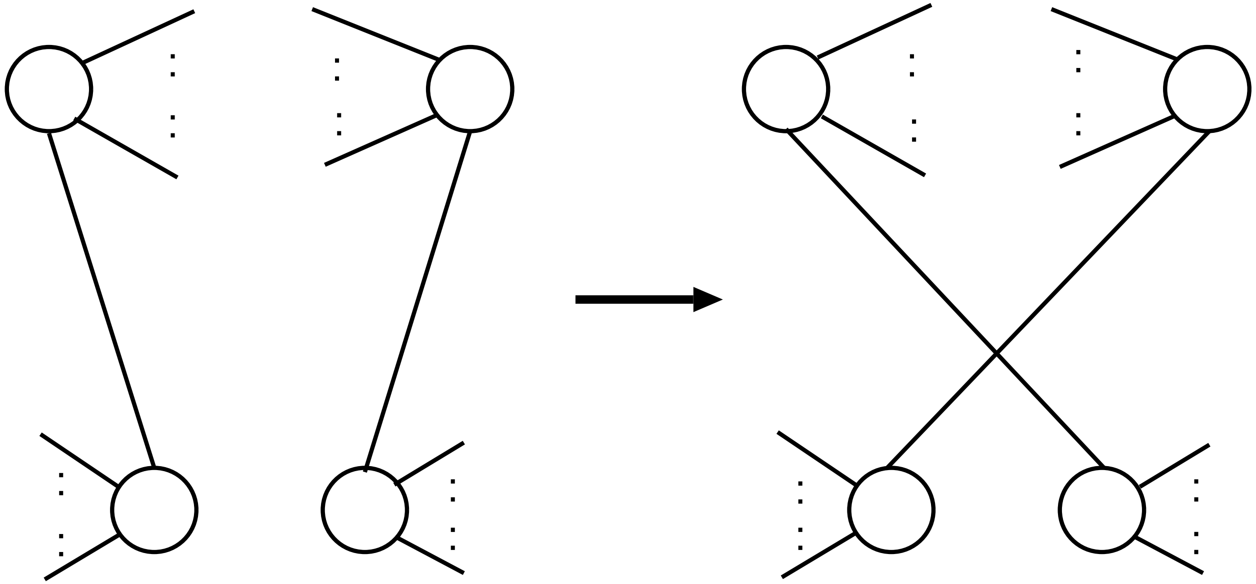


DLSF

Inset shows the peaks enlarged by other scale of the vertical axis. The robustness is more improved by larger shortcut rate (from ○ 0 % to ▽ 30 %).

A3. Randomly Rewired Nets

We compare the tolerance to random failures of nodes and targeted attacks on hubs in the geographical and non-geographical randomly rewired networks, when a fraction f of nodes is removed.

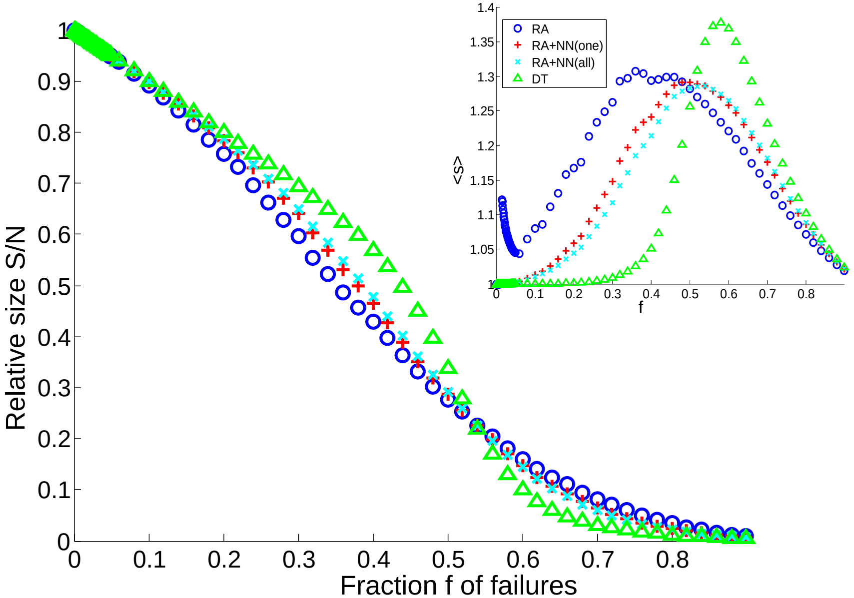


Rewiring a pair of links with the same degree at each node

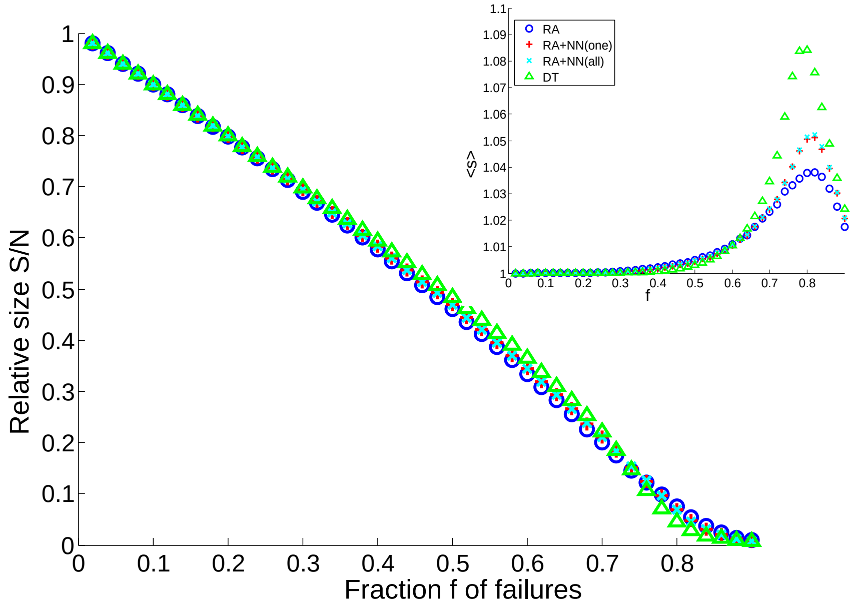
Maslov et al., Physica A 333, 2004

A4. Tolerance to Failures

Relative size S/N of the giant component
Inset: the average size $\langle s \rangle$ of isolated clusters



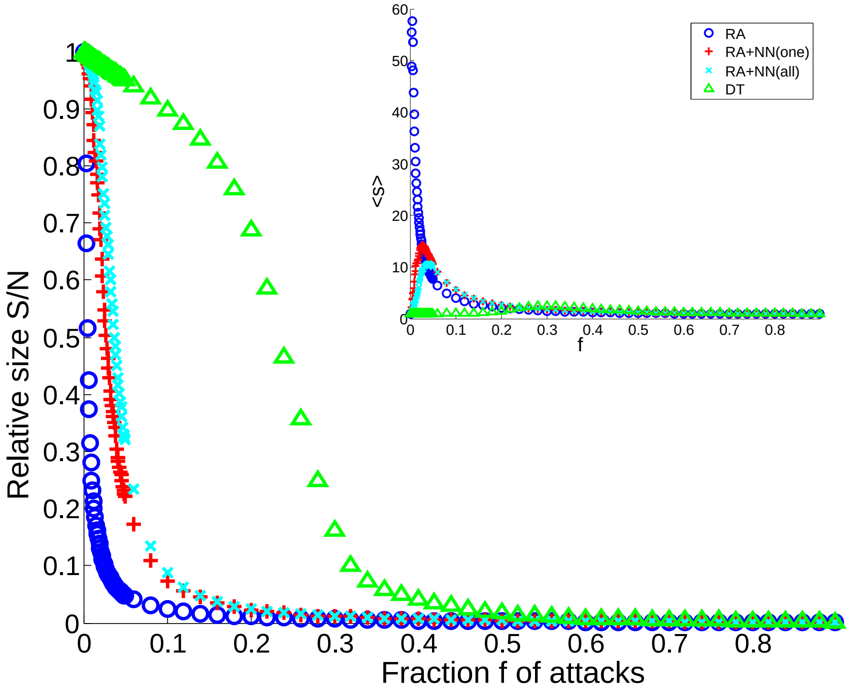
(a) geographical nets



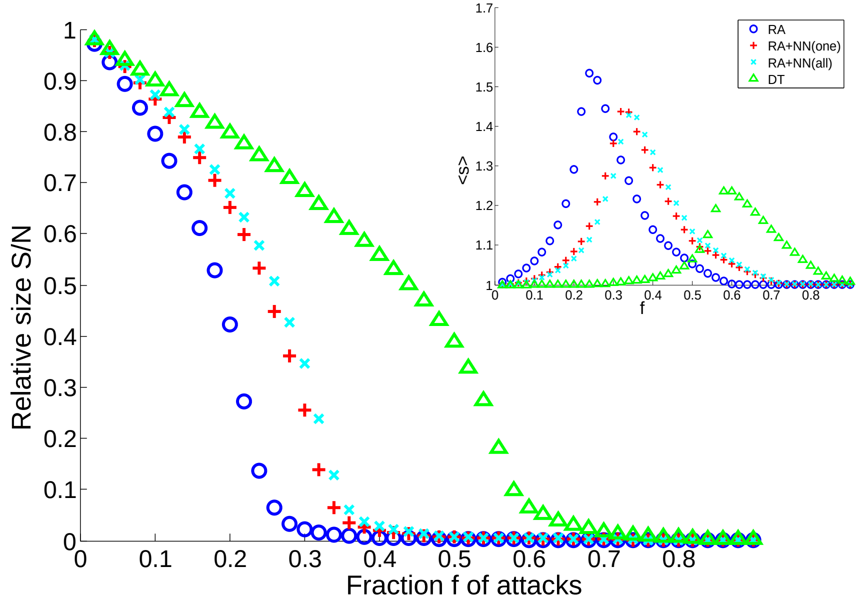
(b) rewired nets

$\Rightarrow$ Similar robustness in RA: $\bigcirc$, DT: $\triangle$, DLSF: $+$

A5. Tolerance to Attacks



(a) geographical nets



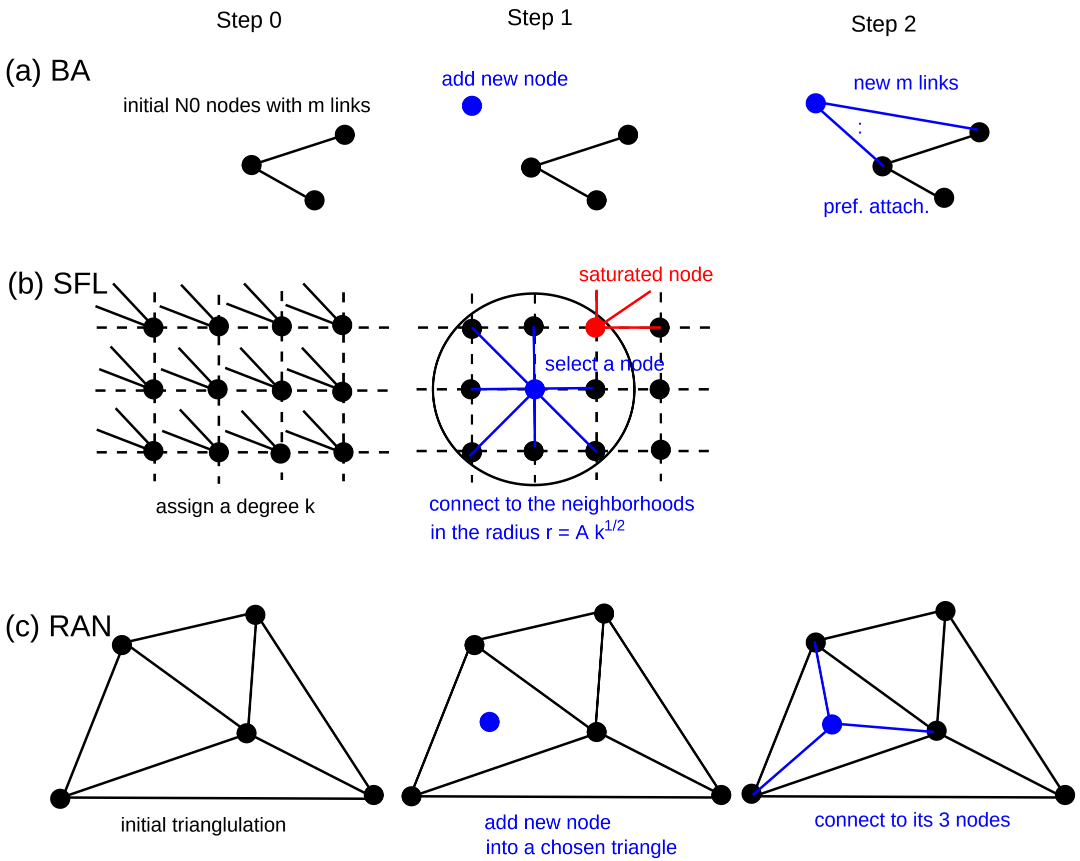
(b) rewired nets

$\Rightarrow$ Improvement from the extremely vulnerable RA

Hayashi et. al., PRE 73, 2006

A6. Classes of Geo. SF Nets

- Modulated BA: $\Pi_i \sim k_i \times l^\alpha$, rand. position of node
- SF on lattices: connect within $r = A \times k_i^{1/d}$
- Space-filling: subdivision of a region (heterogeneous dist. of nodes)



A7. Planarity and Shortness

class	planarity of net	shortness of links
Modulated BA Manna'02, Xulvi-Brunet'02	$\times$ $\exists$ crossing links (not prohibited)	$\bigcirc$ with disadvantaged long-range links
SF on lattices ben-Avraham'03, Warren'02	$\times$ cross of regular links and shortcuts	$\triangle$ $\exists$ long shortcuts from hubs
Space-filling Apollonian nets. Doye'05, Zhou'04	$\bigcirc$ by subdivision of a selected region	$\triangle$ $\exists$ long-range links in narrow triangles