# *Optimal Topological Structure in Social, Technological, and Biological Networks*

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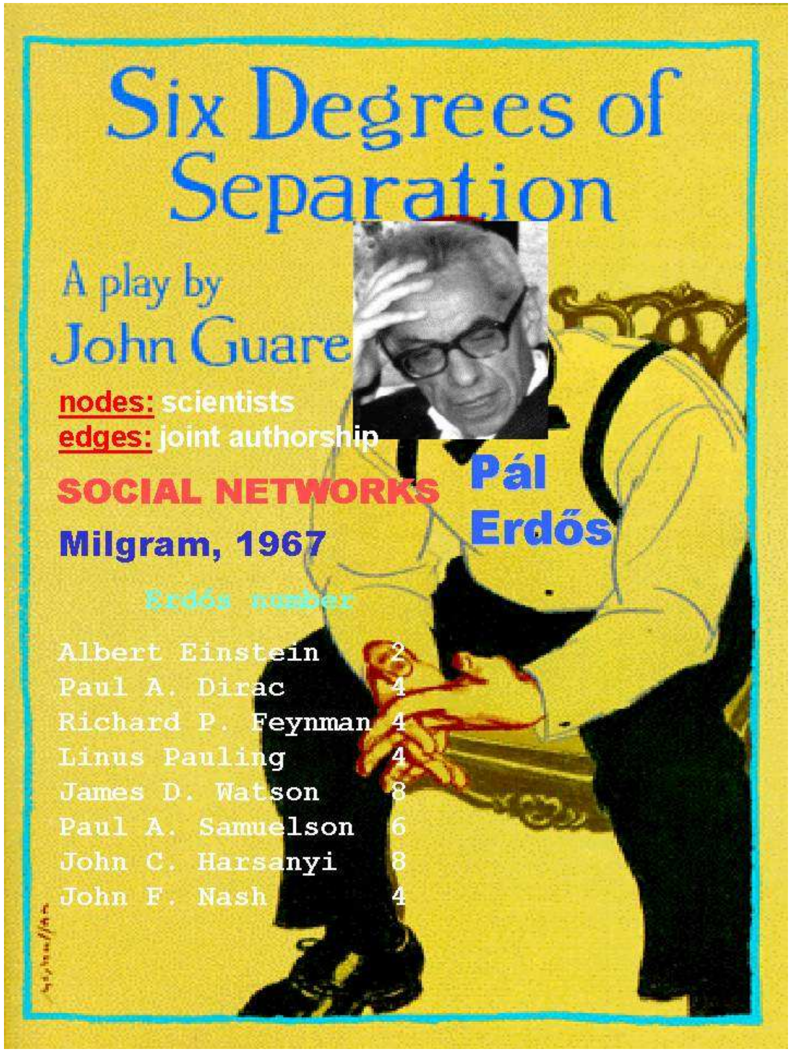
Many real nets $\neq$ regular(e.g lattice), uniformly random, positioned between them.

Two properties

**like regular:** highly clustered with triangles

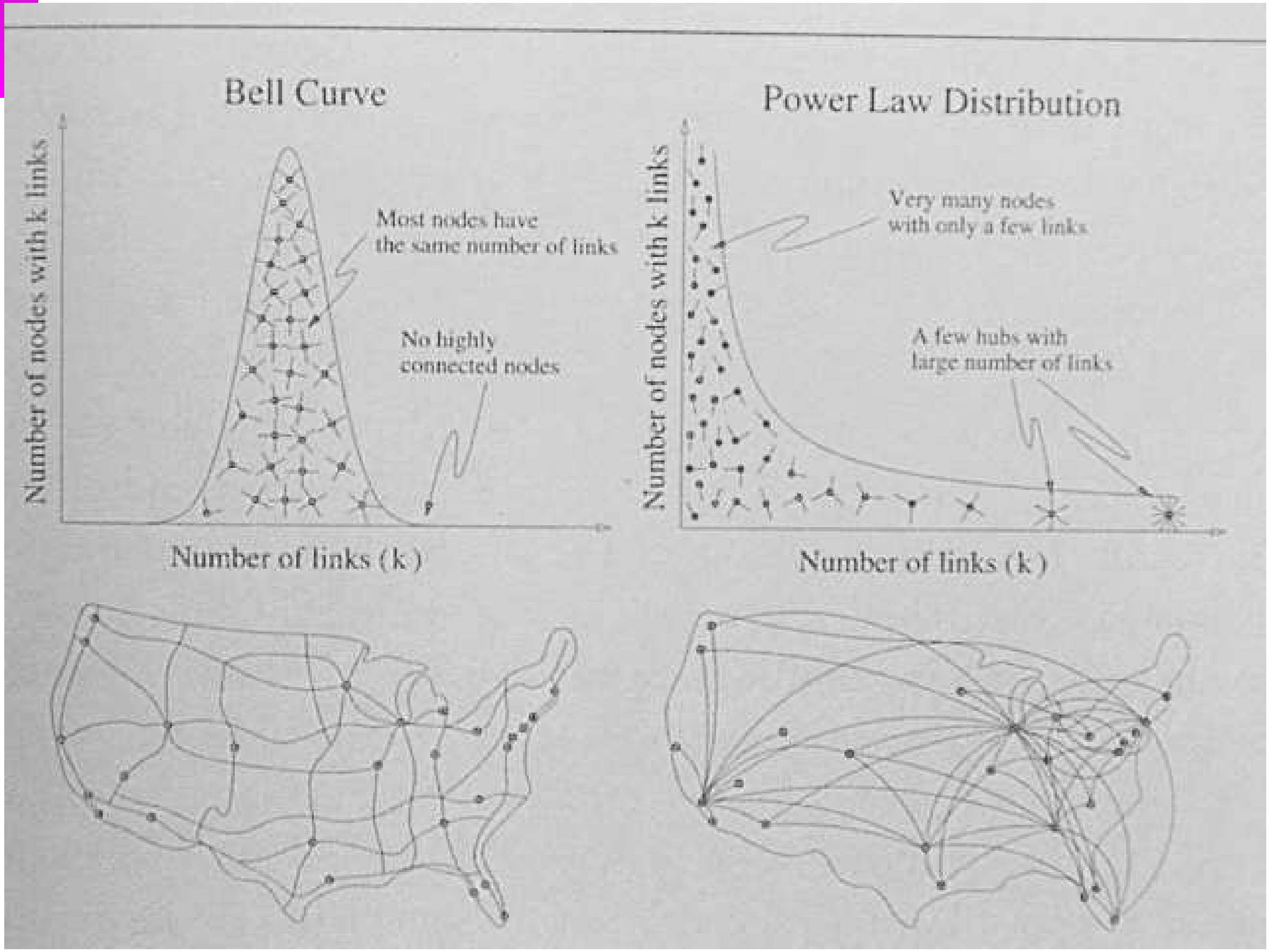
**like random:** average path length is short by 6-acquaintances, by only 20-clicks

D.J. Watts and S.H. Strogatz, nature, 393, 1998



# Scale-Free Network

There exists a surprisingly common structure: SF net.
the degree dist. exhibits $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$.




In A.L. Barabási, LINKED, Perseus, 2002

# *Universality*

Recently('98-'02), the surprisingly common structure has been found in many real nets

**Social:** acquaintance, world trading, actor-collabo., citation, language

**Technological:** Internet, WWW, email, power grid

**Biological:** neural net, genome, metabolic pathway, foodweb

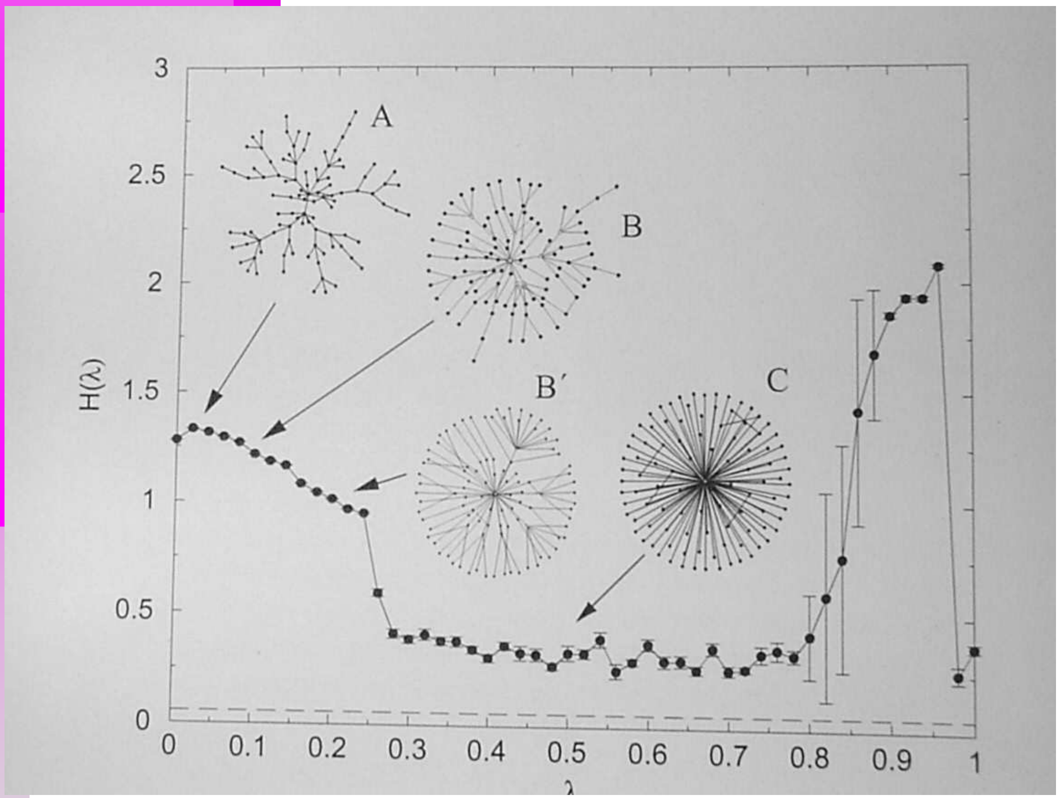
Universal evolution mechanism has been elucidated: Growth & Preferential Attachment

A.L. Barabási et al., Physica A, 272, 1999

# Optimal Topology for Communication

$\boxed{\text{economy, \# of links } \rho} \leftarrow 0 < \lambda < 1 \rightarrow \boxed{\text{efficiency, distance } d}$

Random (tree) - Pref. (SF) - Forced (star, clique)




SF appears in random generations for

$\min E(\lambda) = \lambda d + (1-\lambda)\rho$,

$d \stackrel{\text{def}}{=} \frac{\sum_{i<j} D_{ij}}{{}_nC_2} / D_{max}$,

$\rho \stackrel{\text{def}}{=} \frac{\sum_{i<j} a_{ij}}{{}_nC_2}$,

with a weight $\lambda$

entropy $H(\lambda)$ vs. weight $\lambda$

R.F. i Cancho and R.V. Solé, SantaFe Inst. working paper, 2001

# Connectivity Correlations

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**Assortative:** social

connections between similar peers

**Disassortative:** technological or biological

hub and peripheral nodes with low degrees

M.E.J Newman, PRE 67, 026126, 2003, A. Vázquez, PRE 67, 056104, 2003.

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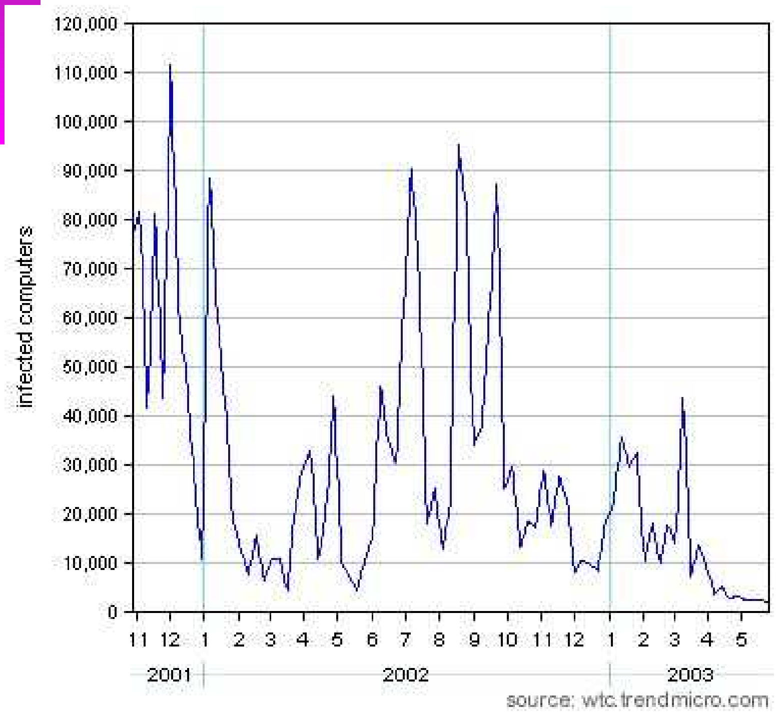
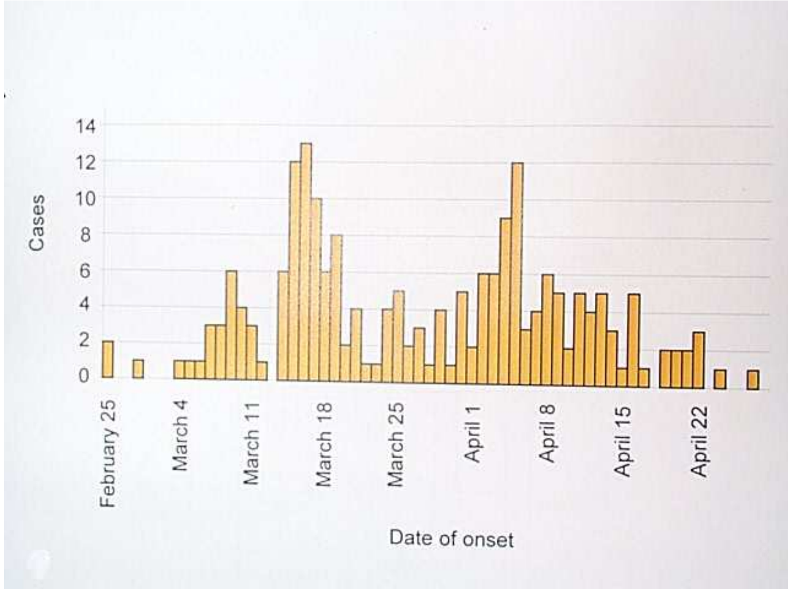
Let us consider the $\boxed{\text{conditional probability } P(k|l)}$ of connection of nodes with deg. $k, l$ for each type.

# *Oscillatory Epidemic Prevalence*

Typically observed, but unknown the mechanism




computer virus NIMDA




SARS in Singapore, Sciencexpress May 23, 2003

$\Rightarrow$ SIR (susceptible-infected-recovered/removed state transition) model <u>on SF nets</u> for epidemic spreading

Epidemic dynamics for the macro. eq. at the MF level

$$\frac{dS_k}{dt} = -bk \underbrace{S_k \Theta_k}_{contact} + a_k, \quad \frac{dI_k}{dt} = -\delta I_k + bk \underbrace{S_k \Theta_k}_{contact},$$

where $b$ and $\delta$ denote the infection and immune rate, $a_k = Ak^{-\gamma}$, $A > 0$, provides a constant increasing of $S_k$,

Mean-Field infection: $\Theta_k(t) \stackrel{\text{def}}{=} \sum_l \frac{l-1}{l} P(k|l) \frac{I_l(t)}{N_l(t)}$.

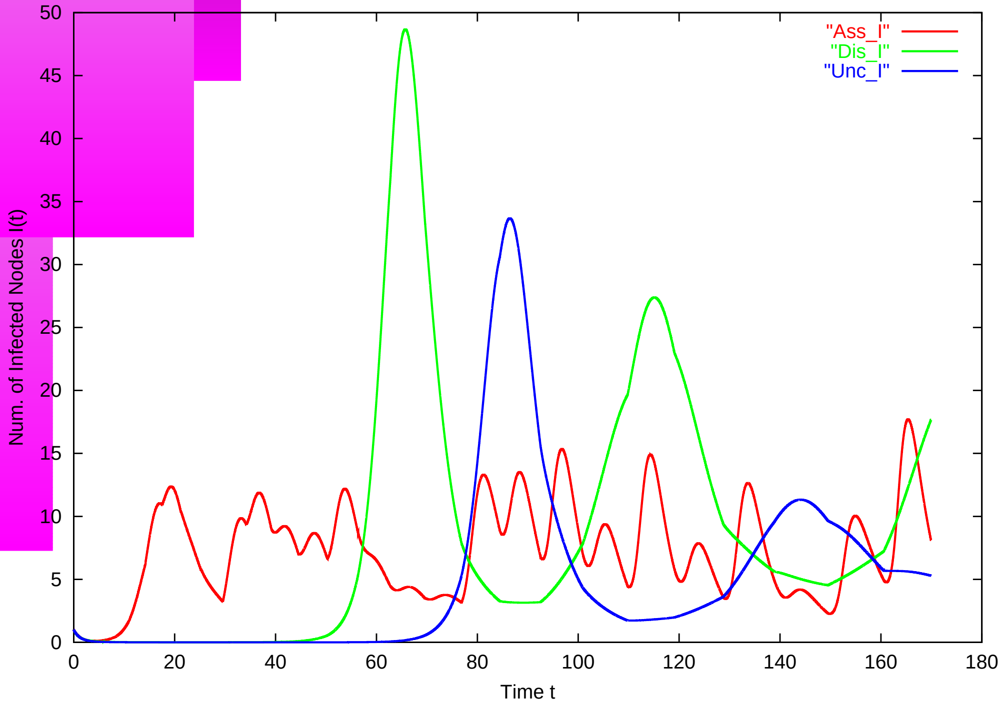
From $N_k = S_k + I_k + R_k$ and $\frac{dR_k}{dt} = \delta I_k$, $\frac{dN_k}{dt} = a_k$, the growing $N_k(t) \sim a_k t$ gives asymptotic $P(k) \sim k^{-\gamma}$,

$$N(t) = \sum_k N_k \sim \left( \sum_k a_k \right) t.$$

$\Rightarrow$ linearly growing SF net, and simultaneously progress of epidemic spreading

Different behavior depend on the correlation types

Trade-off: persistency and breaking size




Time evolution of $I(t) = \sum_k I_k(t)$

Ass: persistently survived with fluctuation

Dis: later outbreaks

Unc: corresponded to the conventional SF models without correlations

$\Rightarrow$ the behavior on Dis or Unc is also consistent with a stochastic SIR model, but it on Ass has’nt been found

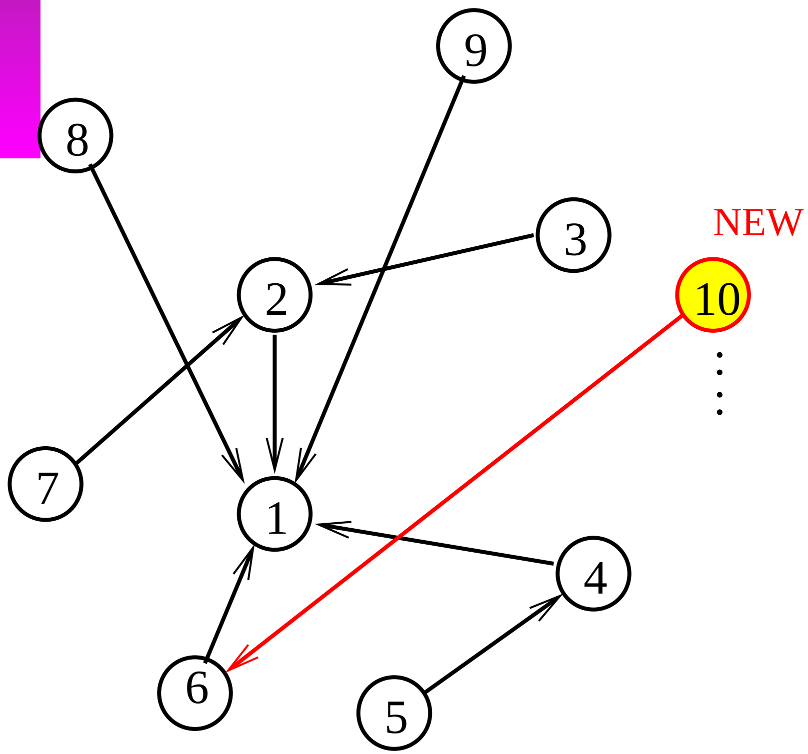
# *Summary*

- We've briefly reviewed recent studies inspired from a commonly existing SF structure in social, technological, and biological networks.
- The topology ($\neq$ regular, random) is the optimal for minimizing both the # of links and distance as economy and efficiency of communication.
- However, besides the SF, there exist Ass (social, between peers) and Dis (tech. or bio., hub-periph.) connectivity correlations.
  In our simulation of the SIR dynamics, the correlations cause quite different behavior for epidemic spreading.

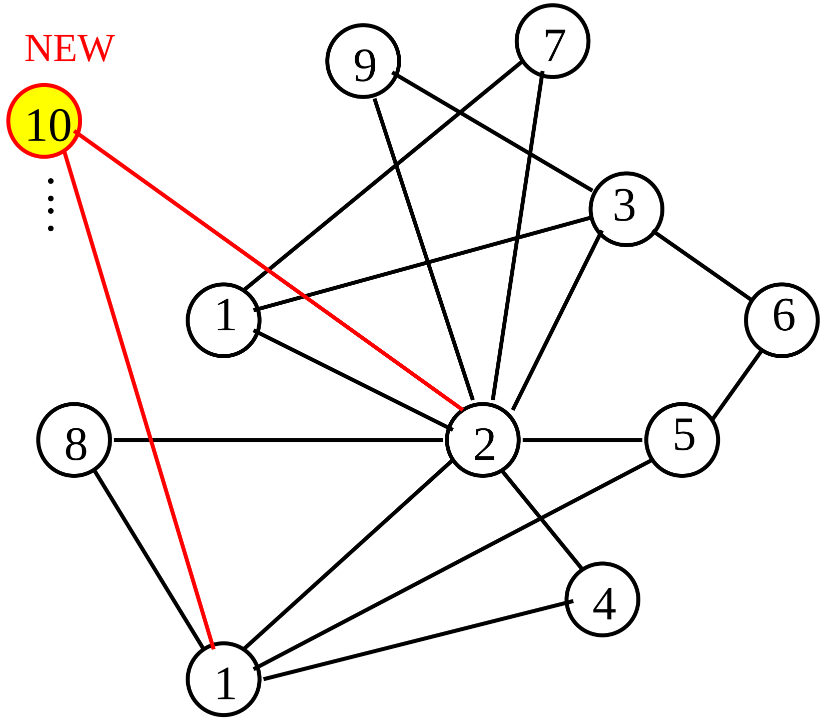
$\Rightarrow$ a good struct. will be used for future net. design.

# Appendix 1: GN and BA models

$P(k) \sim k^{-\gamma}$, $\gamma \sim 3$, $m = 1$ $\qquad$ $\gamma = 3$, $m = 2$



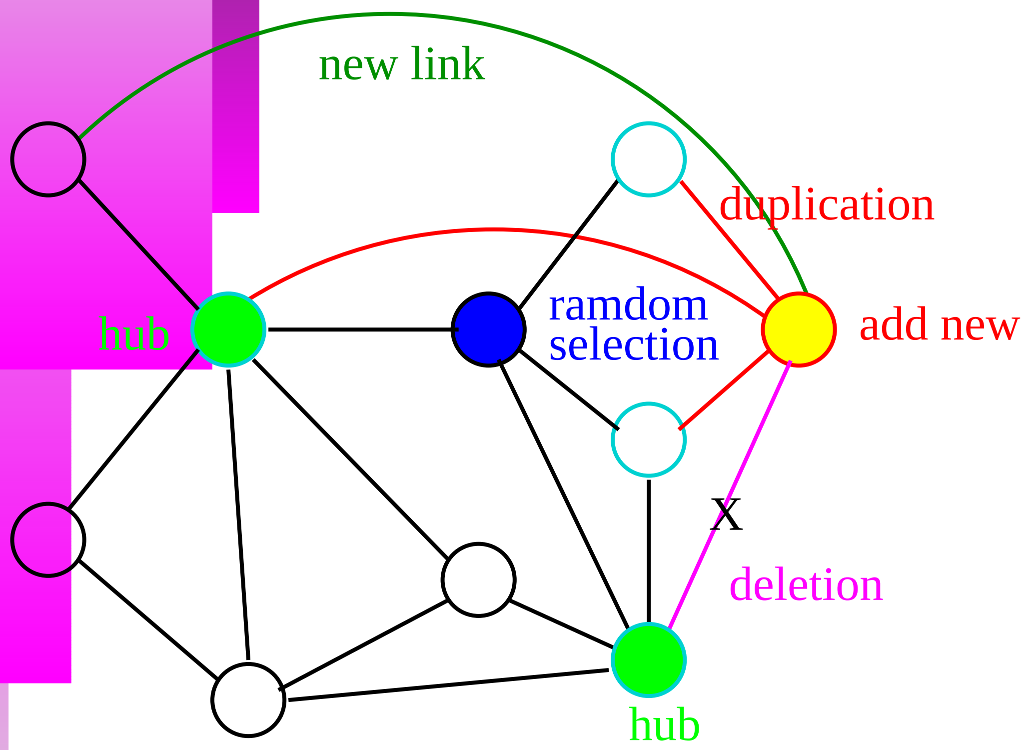

Krapivsky-Render's GN model




BA model

$\Rightarrow$ age-effect (S.N. Dorogovtsev et al. PRL 85, 2000), rewire(A. Albert, PRL 85, 2000), fi tness(G. Bianconi, PRL 86, 2001)

# *Appendix 2: Duplication model*



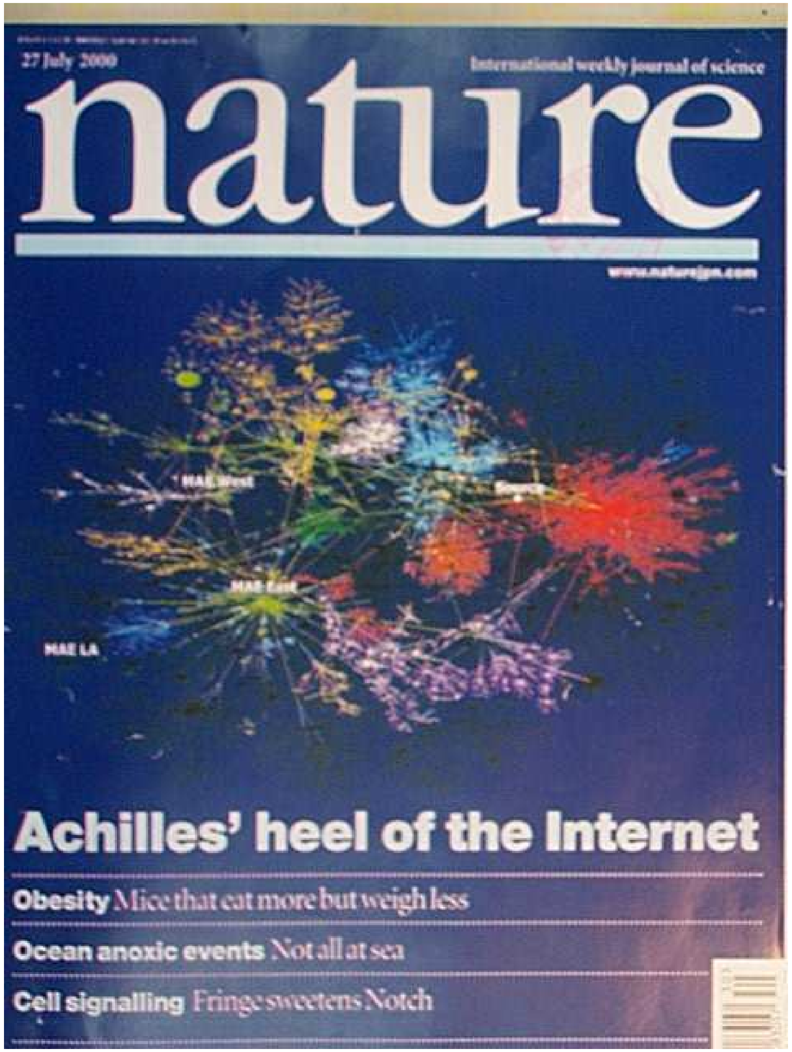

In spite of random node selection, the neighbor hub node has many chance to get duplicate connections (proportional to the degree).

$\Rightarrow$ Biologically plausible networks realize Preferential Attachment in a local rule !

# Appendix 3: Robust and Vulnerable Connectivity

**Robust:** for random failure, remaining the connectivity

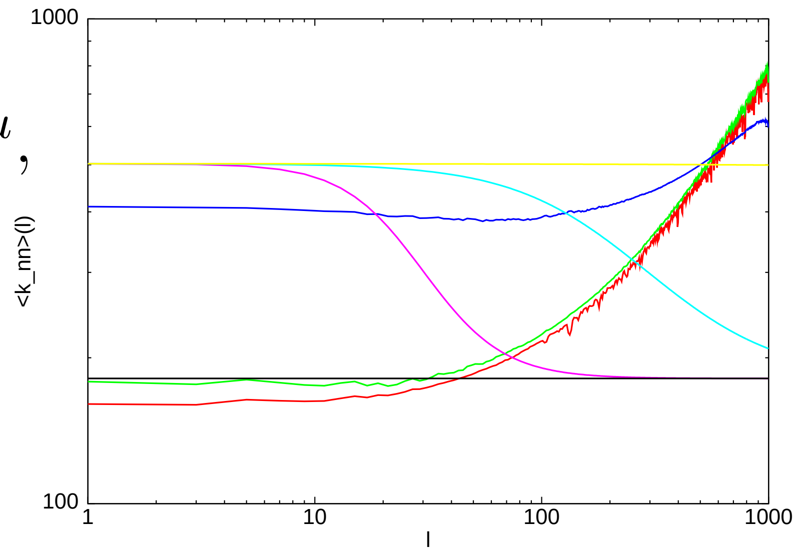
**Vulnerable:** for targeted attack against hubs, disconnecting into isolated parts



# Appendix 4: Variety of Correlations

$$P(k|l) \sim \alpha k^{1-\gamma} + \frac{1-\alpha}{|k-l|^{\nu}+1},$$
$$\sim \alpha k^{1-\gamma} + (1-\alpha) l^{-\mu},$$

and adding random noise as arbitrary correlations




$$< k_{nn} > (l) \stackrel{\text{def}}{=} \sum_{k=1}^{K} k P(k|l)$$

ave. correl. $< k_{nn} >$ vs. degree $l$

$\alpha = 0.2$, $\gamma = 2.5$, Ass: $\nu = 2.5, 1.5, 0.5$, Dis: $\mu = 2.5, 1.5, 0.5$

estimated from real data of actor-collabo. (Ass), Internet-AS level-, and email (Dis).