Scale-free networks on a geographical planar space for efficient ad hoc communication

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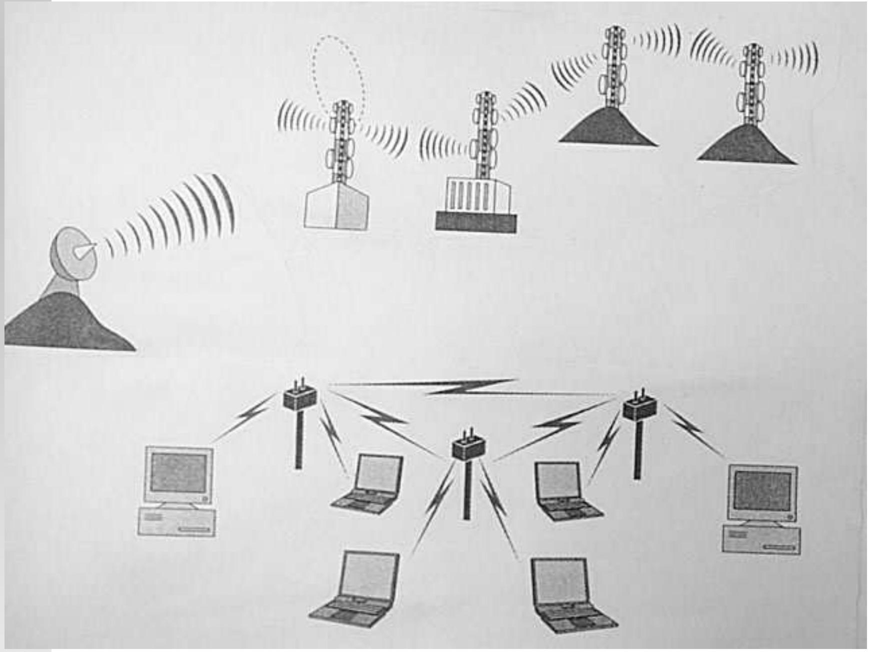
1. Introduction

Dynamic configuration of (backbone) networks

Problems for ad hoc communication such as in sensor networks or P2P systems

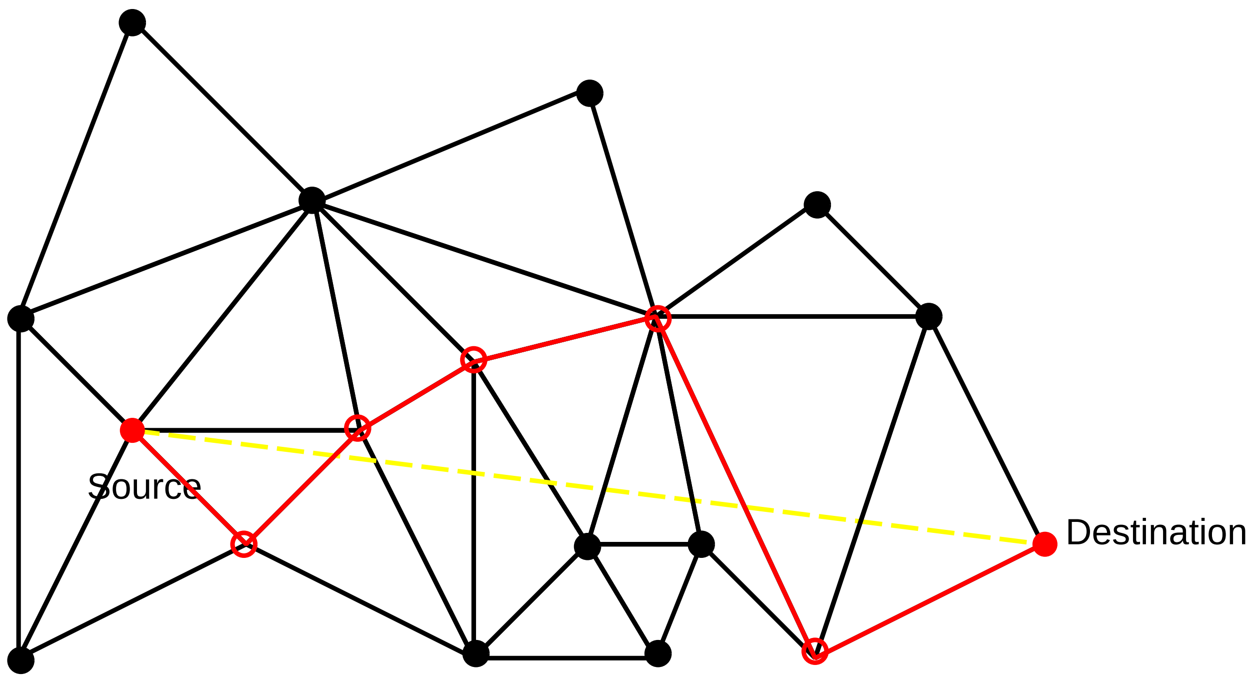


- **dissipation** of wireless beam-power or line-cost for long-range links
- **interference** by crossing links

a few nodes have sufficient power to be hubs

2. Efficient Routing

Planar triangulation: reasonable math. abstraction of ad hoc net. (each triangle forms a service region) Moreover, a memoryless, no defeat, and competitive online routing algorithm has been developed for networks on triangulation.



3-1. Scale-free Nets with Hubs

Existing a surprisingly common structure: SF net.
the degree dist. exhibits $P(k) \sim k^{-\gamma}$, $2 < \gamma < 3$.

Social: acquaintance, world trading, actor-collabo., citation, language

Technological: Internet, WWW, email, power grid

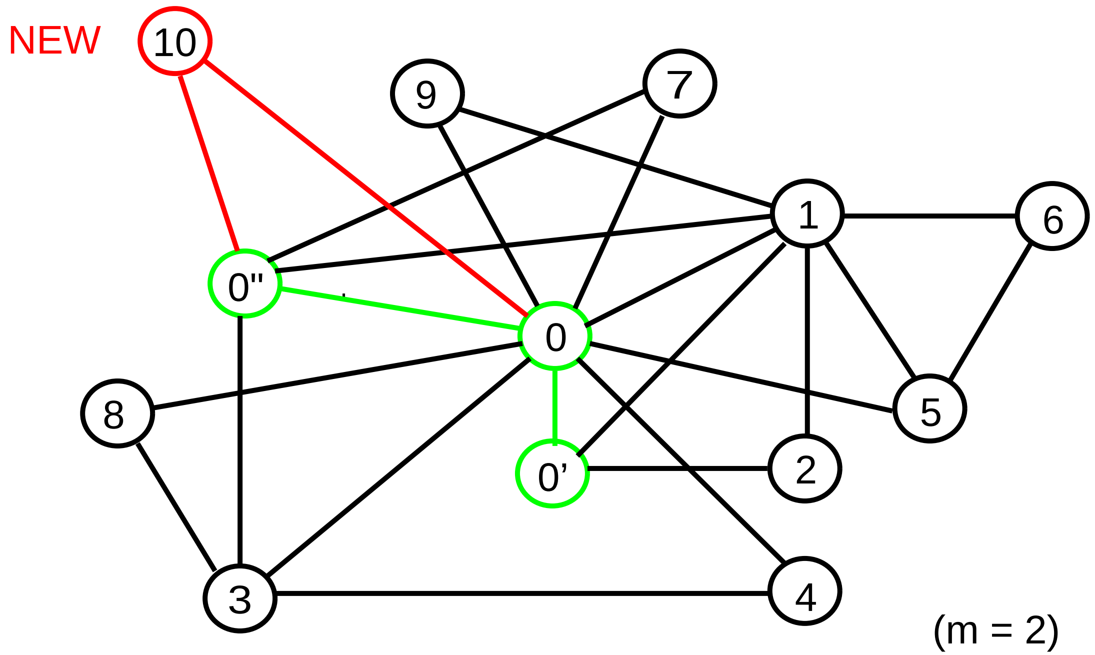
Biological: neural net, genom, metabolic pathway, foodweb

One of the fundamental generation mechanism has been proposed: Growth & Preferential Attachment

Barabási and Albert, Physica A, 272, 1999

3-2. BA model: $P(k) \sim k^{-3}$

- A network grows from initial N_0 nodes with $m < N_0$ links among them.
- At every time step t, a new node is introduced, and is randomly connected to m previous nodes i with an attachment probability $\Pi_i^{BA}(t) \sim k_i(t)$.



3-3. Properties of the SF Nets

- Efficient & Economy with short paths as the hop-count $O(\ln N)$ and the low cost of links as few as possible in the connectivity (Solé et al., Advances in Complex Systems 5, 2002), Robust connectivity for random failures (Albert et al., Nature 406, 2000)

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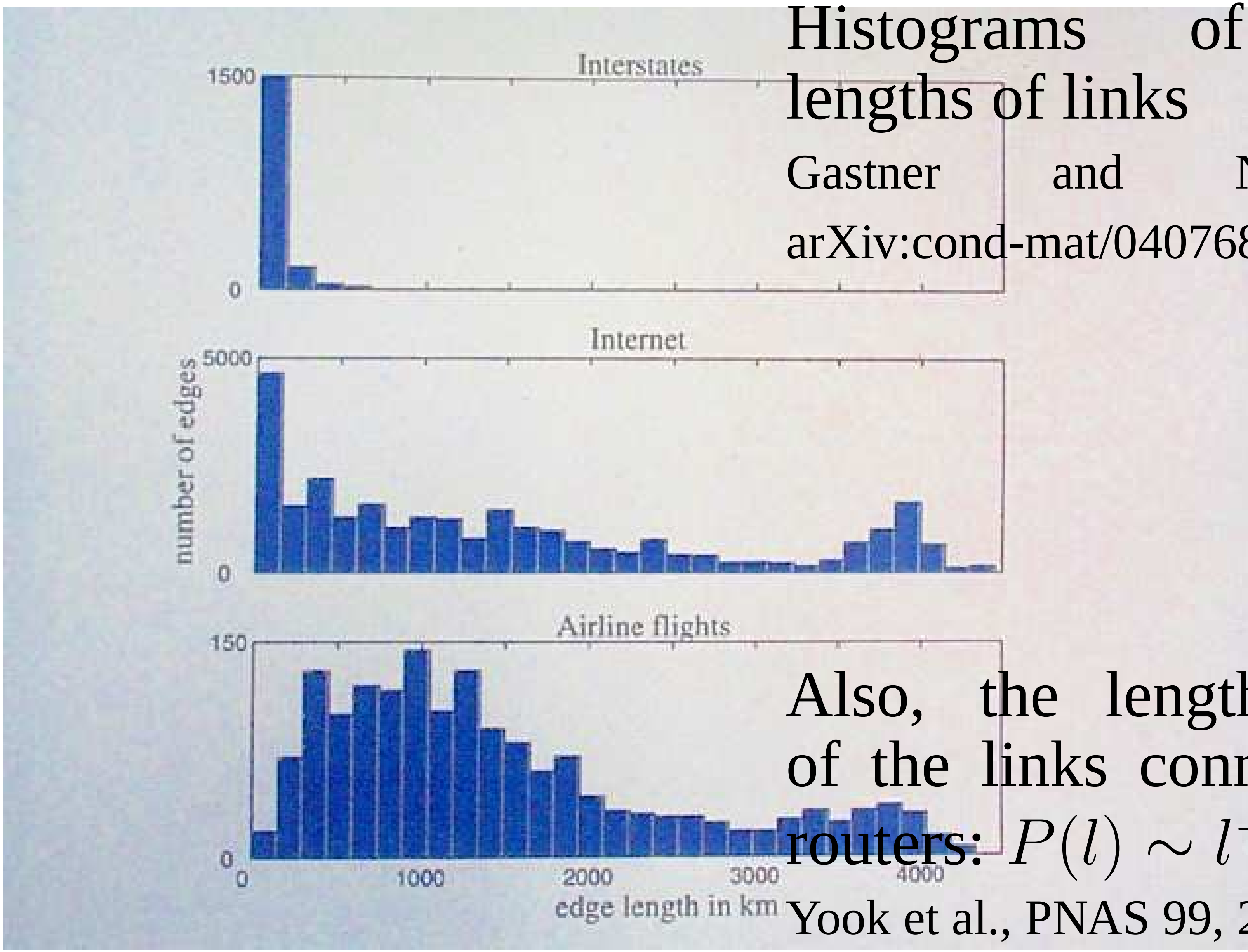
$$\Downarrow$$

- Therefore, we consider geographical SF nets, especially as planner graphs without crossing links to avoid interference of wireless beams.

4-1. Rare Long-range Links



Histograms of the lengths of links

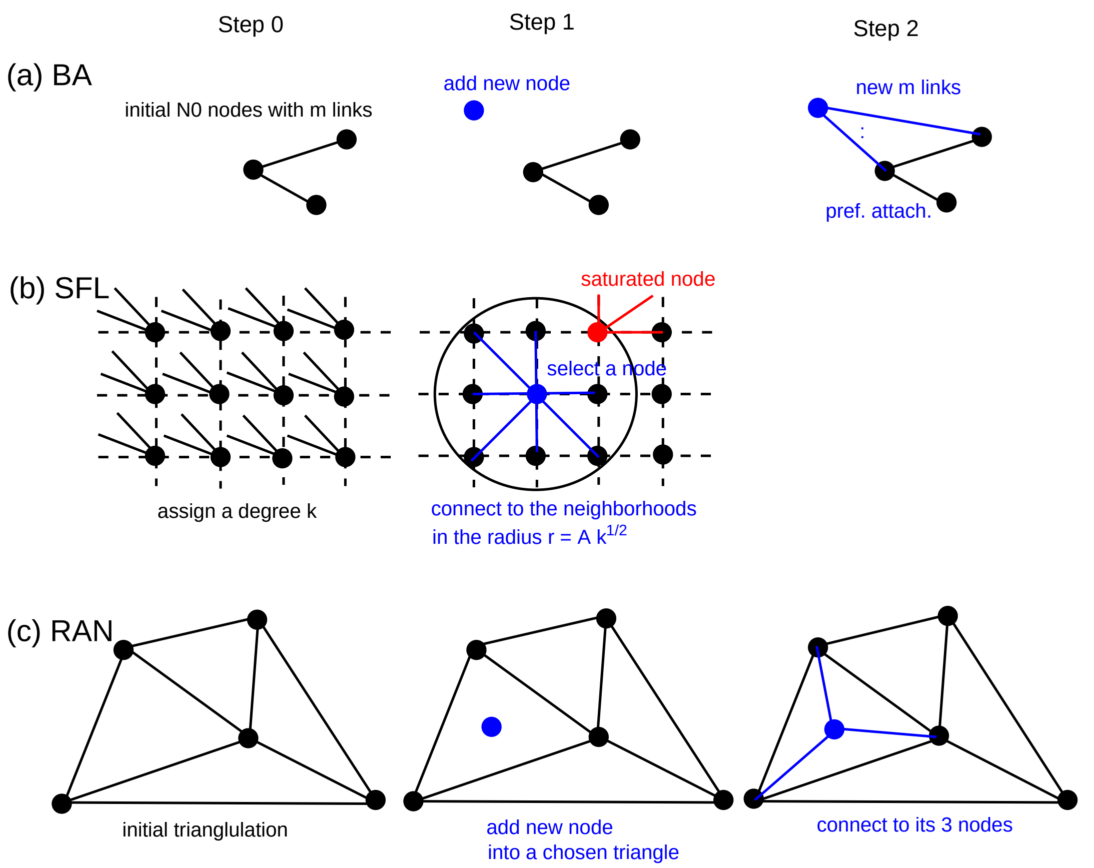
Gastner and Newman, arXiv:cond-mat/0407680, 2004

Also, the length dist. of the links connecting routers: $P(l) \sim l^{-1}$

Yook et al., PNAS 99, 21, 2002

4-2. Geographical SF Nets

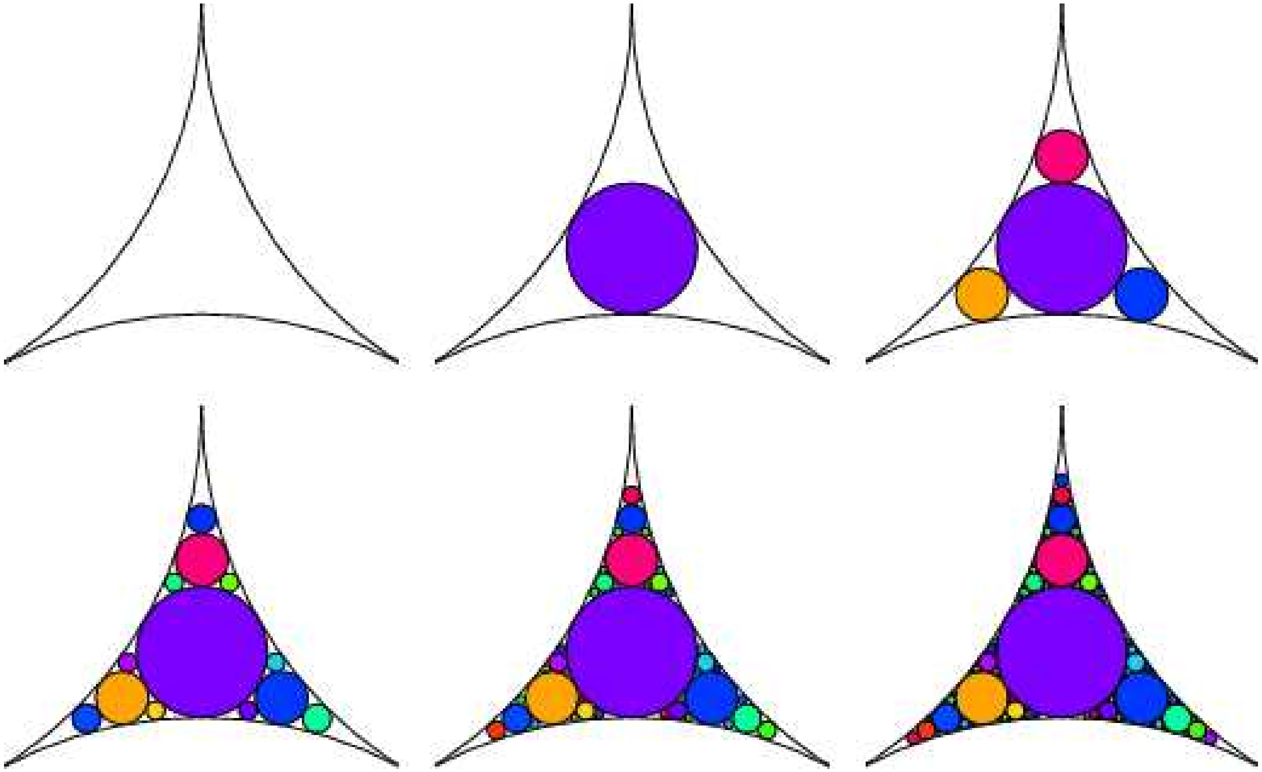
- Modulated BA: $\Pi_i \sim k_i \times l^{\alpha}$, rand. position of node
- SF on lattices: connect within $r = A \times k_i^{1/d}$
- Space-filling: subdivision of a region (heterogeneous dist. of nodes)



4-3. Planarity and Shortness

class	planarity of net	shortness of links
Modulated BA Manna'02, Xulvi-Brunet'02	$\times$ $\exists$ crossing links (not prohibited)	$\bigcirc$ with disadvantaged long-range links
SF on lattices ben-Avraham'03, Warren'02	$\times$ cross of regular links and shortcuts	$\triangle$ $\exists$ long shortcuts from hubs
Space-filling Apollonian nets. Doye'05, Zhou'04	$\bigcirc$ by subdivision of a selected region	$\triangle$ $\exists$ long-range links in narrow triangles

4-4. Space-filling Packing



The dual by connecting the centers of touched circles is nothing but the (deterministic) Apollonian network.

When a triangle is randomly selected for subdivision, it's called RAN.

4-5. Analysis of $P(k)$ in RAN

Evolution eq. by iterative triangulation:

$$n(k+1, N+1) = \frac{k}{N_\triangle} n(k, N) + \left(1 - \frac{k+1}{N_\triangle}\right) n(k+1, N),$$

where $N_\triangle$ denotes the number of triangles.
In the $P(k) \approx n(k, N)/N$, it can be rewritten as

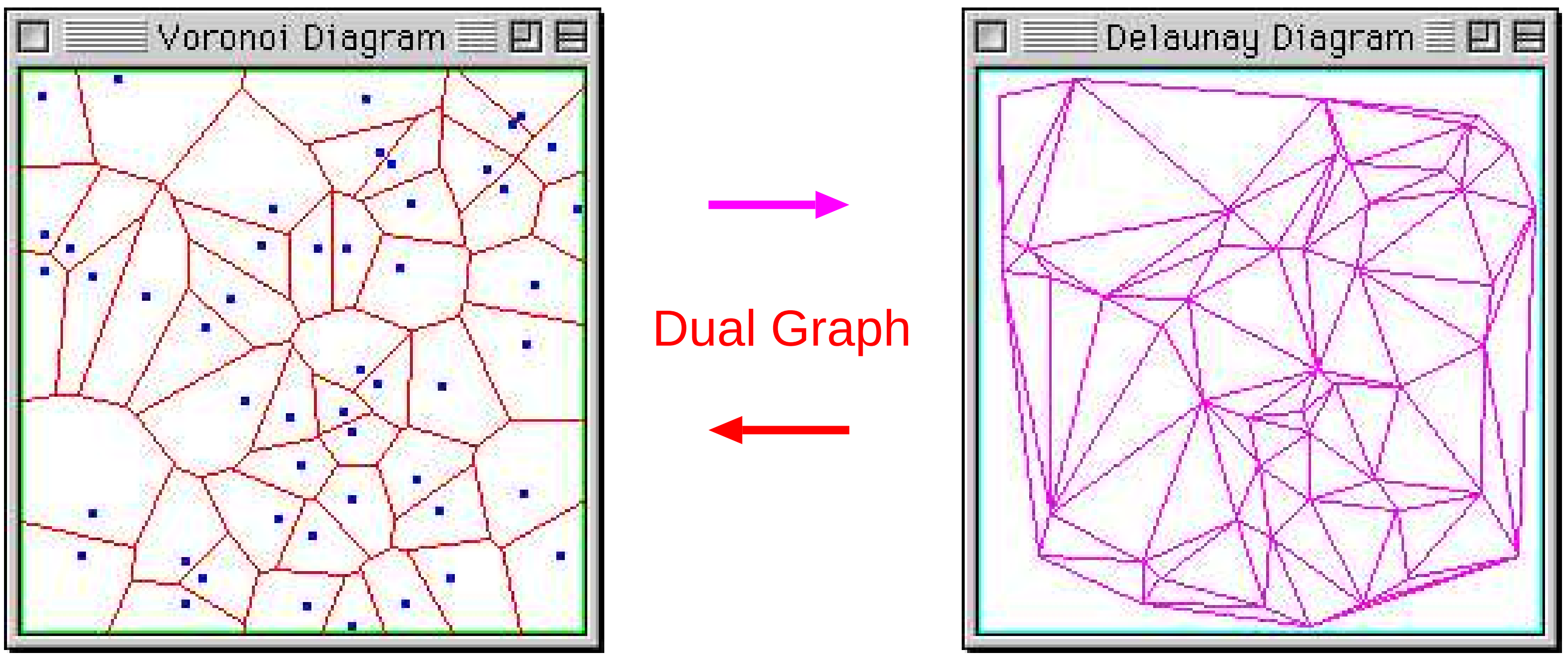
$$\begin{aligned}(N+1)P(k+1) &= NkP(k)/N_\triangle + NP(k+1) \\ &\quad -N(k+1)P(k+1)/N_\triangle.\end{aligned}$$

By the continuous approx., we obtain $P(k) \sim k^{-\gamma_{RA}}$
with $\gamma_{RA} = (N_\triangle + N)/N \approx 3$ for large N.

Zhou et al., arXiv:cond-mat/0409414, 2004

5-1. Voronoi and Delaunay

Consider the combination of RA (by triangulation on a plane) and DT to avoid the long-range links

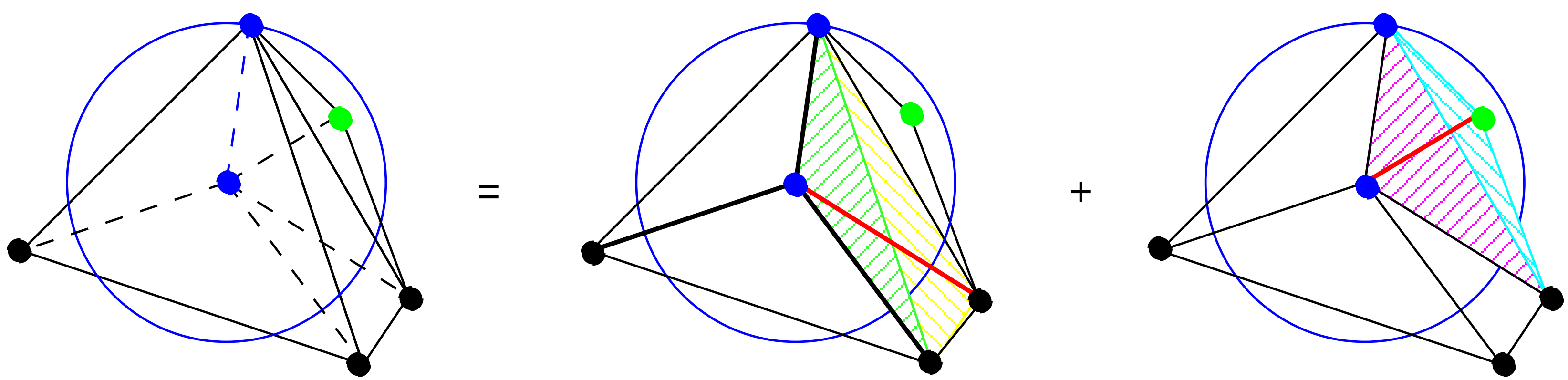


⇒ optimal triangulation in some criteria: maximim angle, minimax circumscribed circle, short path length close to the direct Euclidean dist., etc.

5-2. Delaunay-like SF Net

We propose RA+NN:

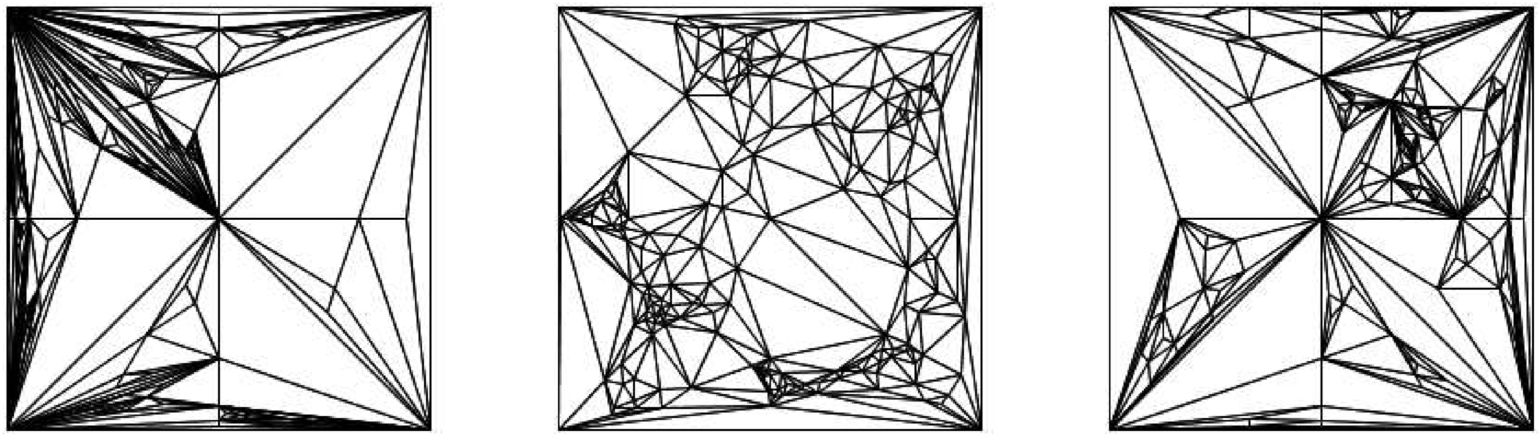
- Set an initial planar triangulation.
- Select a triangle at random and add a new node at the barycenter. Then, connect the new node to the three nodes of its triangle. By iteratively applying diagonal flips, connect it to the nearest node(s) within a radius.



1st diagonal flip   2nd diagonal flip

6-1. Simulation

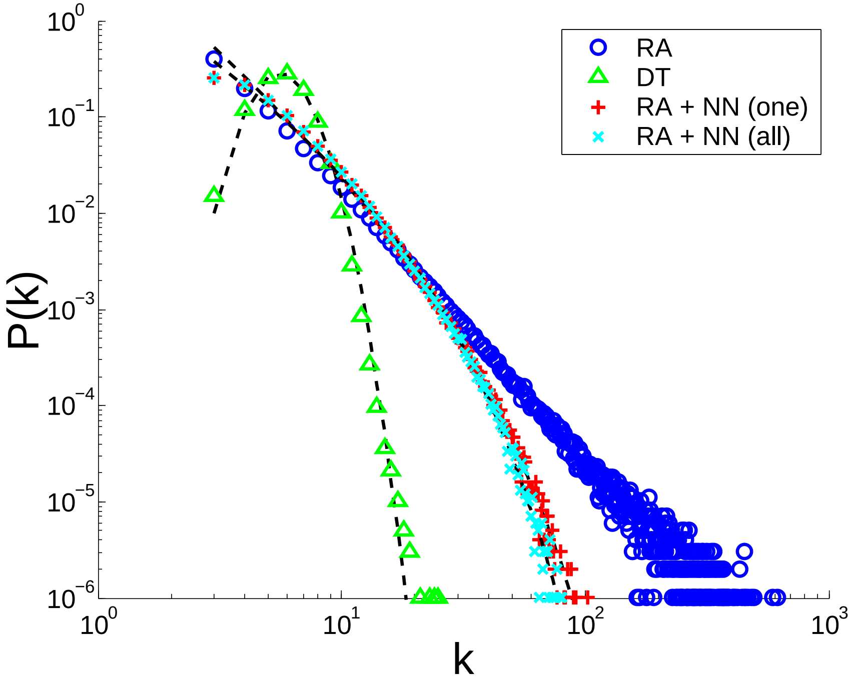
We compare the topological properties of the models: RA, DT, and our proposed RA+NN in the averaging of 100 realizations at size $N = 10,000$.

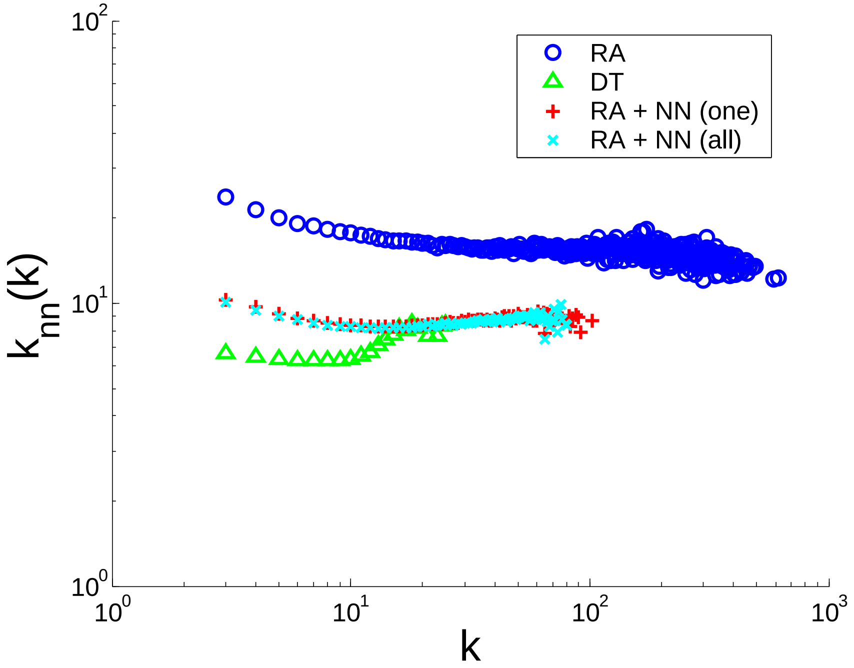


RA DT RA+NN(one)

$\Rightarrow$ RA+NN has the intermediate structure.

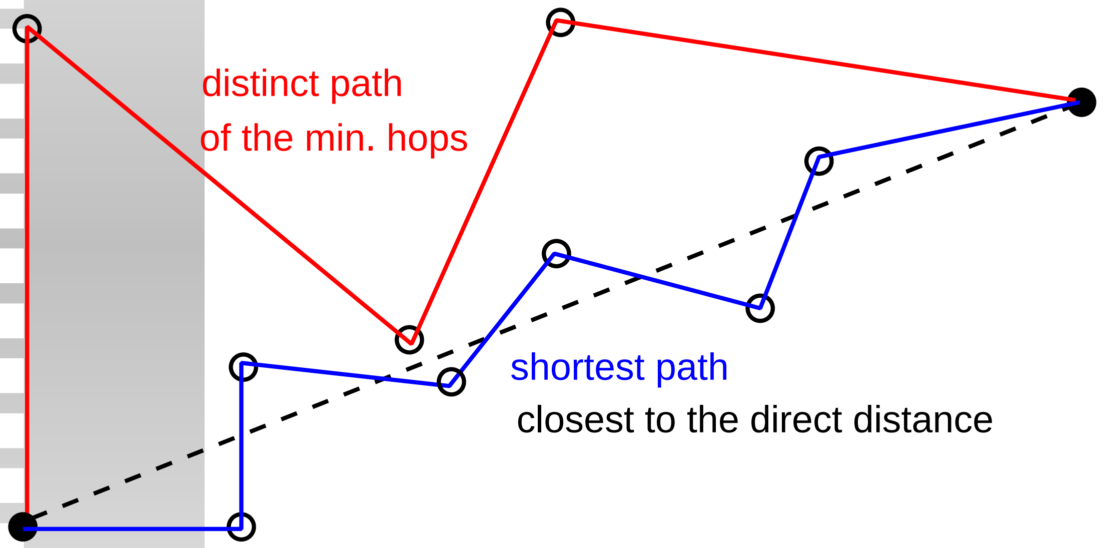
6-2. Degree Dist. & Correlation





$\Rightarrow$ RA+NN has a power law dist. with an exp. cutoff, and a weak negative deg.-deg. correlation.

6-3. Movement and Transfer



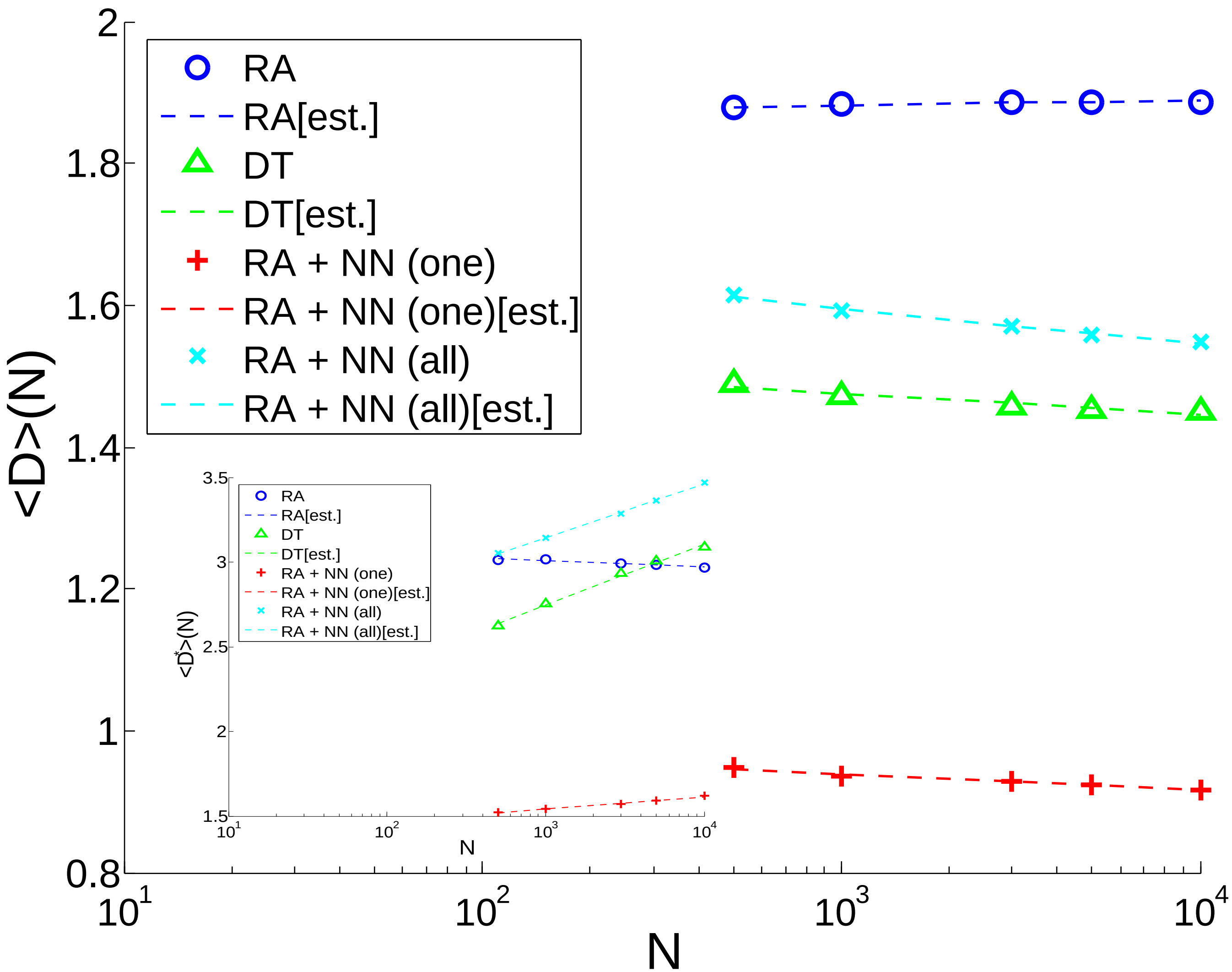
$\langle D \rangle$**:** average distance on the shortest paths

$\langle D' \rangle$**:** average distance on the paths of the min. hops

$\langle L \rangle$**:** average number of hops on them

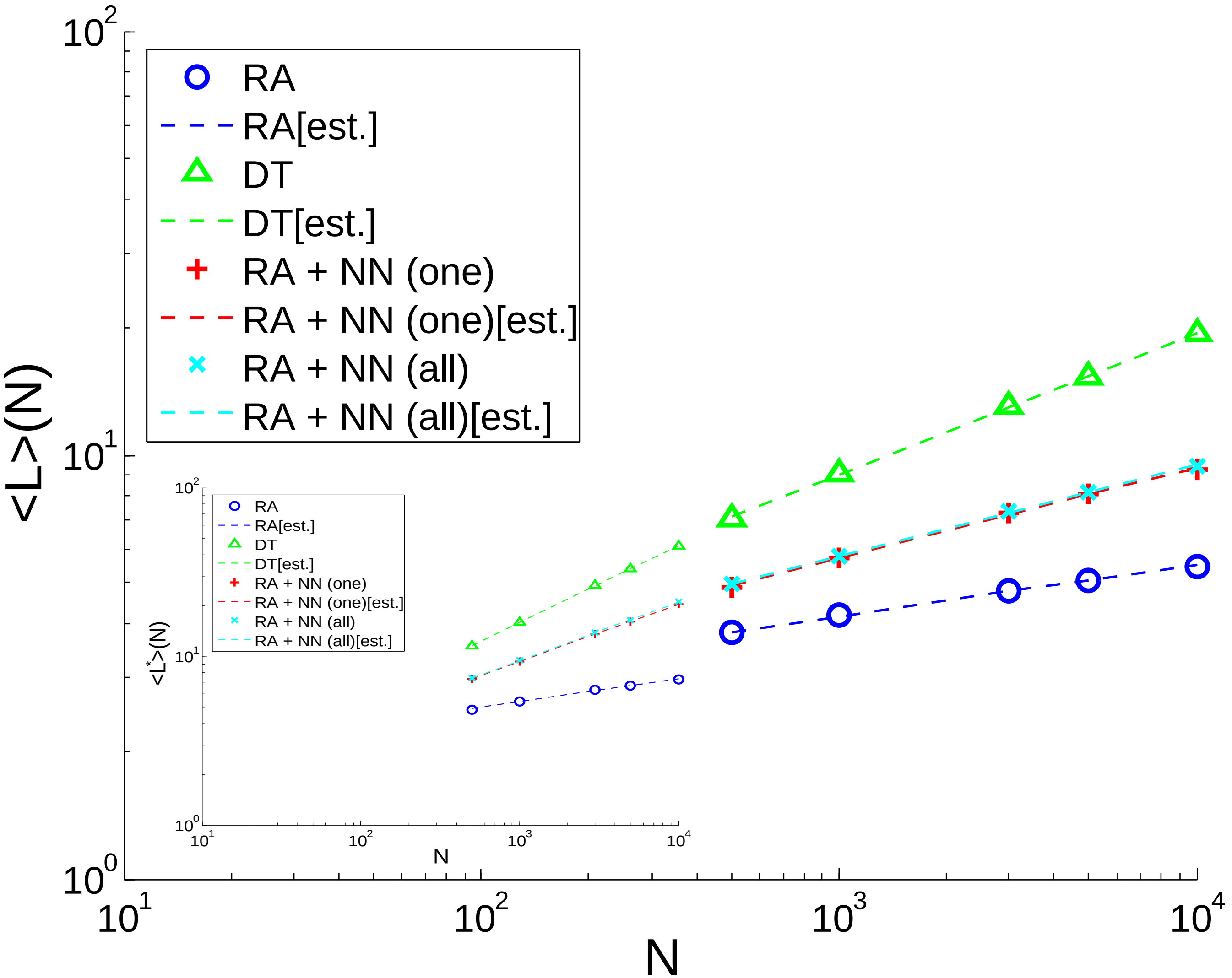
$\langle L' \rangle$**:** average number of hops on the shortest paths

6-4. Distance for Movement



$$\langle D \rangle \sim (\ln N)^{\beta_d}, \text{ Inset: } \langle D' \rangle \sim (\ln N)^{\beta_{d'}}$$

6-5. Min. Hops for Transfer



$\langle L \rangle \sim N^{\alpha_l}$, Inset: $\langle L' \rangle \sim N^{\alpha_{l'}}$, stronger than the SW

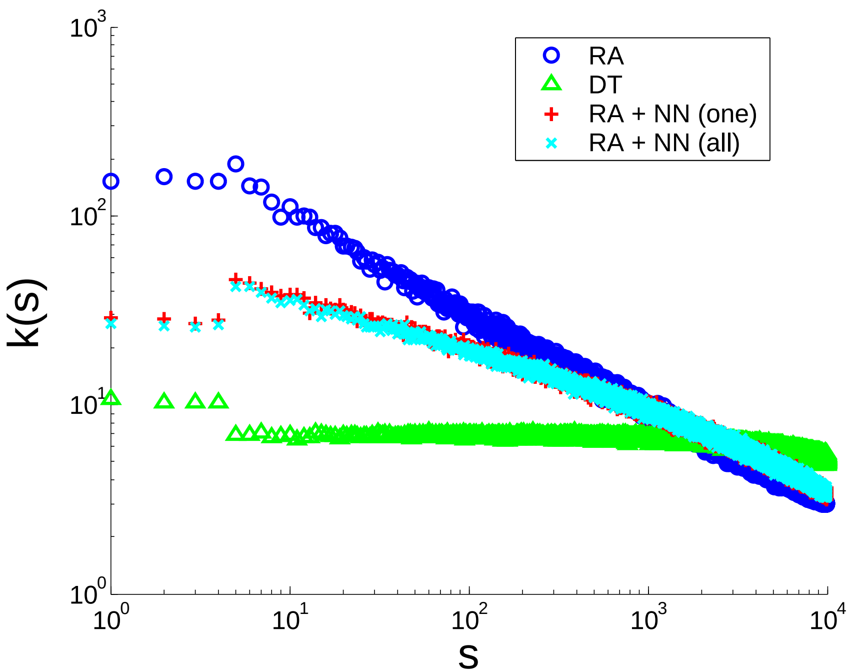
7. Summary

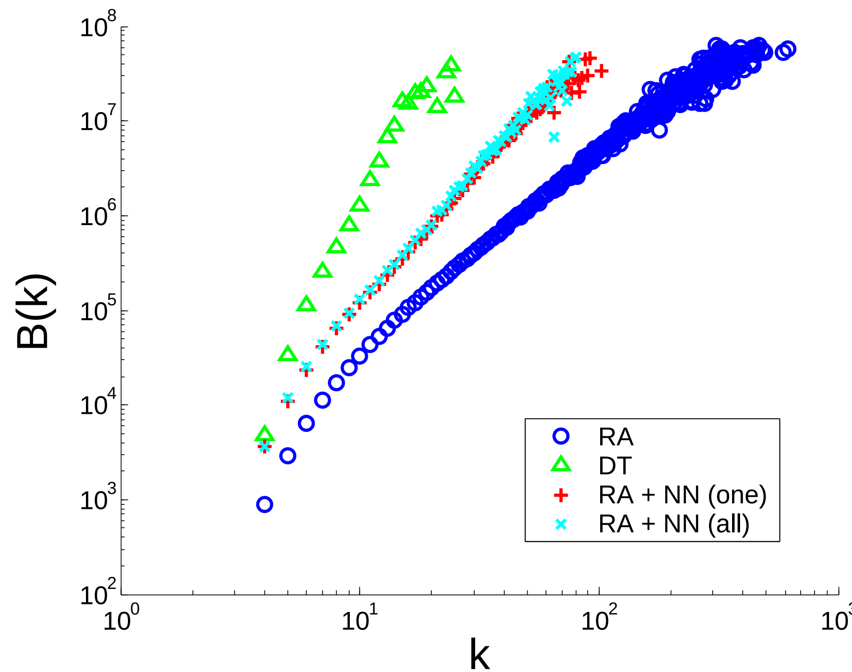
- We've briefly reviewed recent studies of geographical SF net models, and proposed a modified one to reduce long-range links.
- The Delaunay-like SF net without crossing links is dynamically constructed by the triangulation and diagonal flips based on local rules.
- Simulation results have shown that our proposed model has short path lengths and small num. of hops (DT is not opt. in minimizing them), which are suitable topological properties for efficient communications.

We'll further investigate the traffic properties and the fault-tolerance.

Appendix 1.

Average degree $k(s)$ of the node inserted at time s, and the betweenness $B(k)$ of the nodes with degree k





$\Rightarrow$ Old nodes of RA and RA+NNs tend to be hubs, and the traffic load of RA+NNs is the intermediate

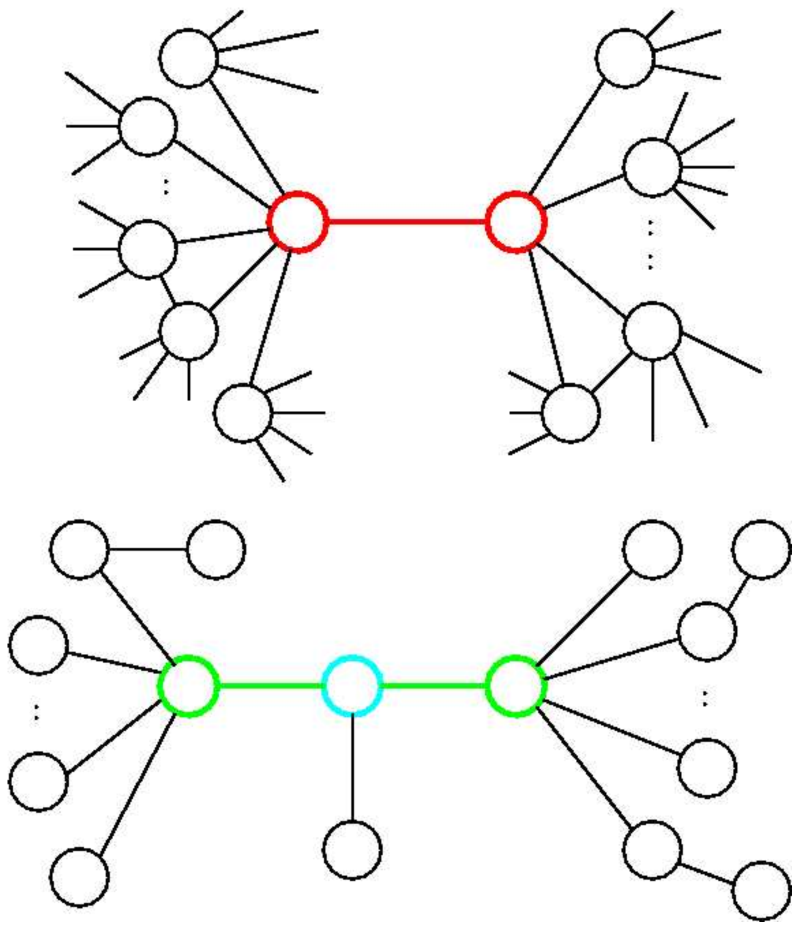
Appendix 2.

Estimated parameters for the degree distribution in each model by a NMSE method

model	estimated function	parameters
RA	$P(k) \sim k^{-\gamma_{RA}}$	$\gamma_{RA} \approx 3$
DT	$P(k) \sim \exp\left(-\frac{(\ln k - \mu)^2}{2\sigma^2}\right)$	$\mu = 1.7755, \sigma = 0.2383$
RA+NN(one)	$P(k) \sim k^{-\gamma} \exp(-ak)$	$\gamma = 2.26,$
		$a = 0.0647$
RA+NN(all)	$P(k) \sim k^{-\gamma} \exp(-ak)$	$\gamma = 1.7248,$
		$a = 0.0979$

Appendix 3.

Assortative and Disassortative correlations observed in social and technological/biological networks



Ass: tend to have connections between similar peers

Dis: between hub and peripheral nodes with low degrees