Computability Theory
and
Foundations of Mathematics

September 7th – 11th, 2015

Tokyo Institute of Technology, Tokyo, Japan

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Preface

Welcome to CTFM (Computability Theory and Foundations of Mathematics)!

CTFM 2015 is the fifth conference of the CTFM conference series. The aim of this conference is to provide participants with the opportunity to exchange ideas, information and experiences on active and emerging topics in logic, including but not limited to: Computability Theory, Reverse Mathematics, Nonstandard Analysis, Proof Theory, Set Theory, Philosophy of Mathematics, Constructive Mathematics, Theory of Randomness and Computational Complexity Theory.

It is our great pleasure to celebrate Professor Kazuyuki Tanaka's 60th birthday. This CTFM conference series was started by Professor Tanaka and advanced by his group of logicians in Tohoku University (Sendai) and their collaborators. CTFM and its predecessor meetings have taken place in Matsushima (2008, 2009), Inawashiro (2010), Akiu (2011), Harumi in Tokyo (2012), Tokyo Tech (2013, 2014). In honor of his birthday, this year's conference will include a special session covering areas in which Professor Tanaka has worked.

This conference is held jointly with a workshop of the Bilateral Joint Research Project sponsored by the Japan Society for the Promotion of Science and the National University of Singapore. CTFM acknowledges support from Inoue Foundation for Science, National University of Singapore, Tohoku University, JAIST (Japan Advanced Institute of Science and Technology) and Tokyo Tech (Tokyo Institute of Technology).

September 2015

Takeshi Yamazaki
Keita Yokoyama
Organizing Committee

Conference Venue

Ookayama campus of Tokyo Institute of Technology

2-12-1 Ookayama, Meguro-ku, Tokyo, 152-8550, Japan.

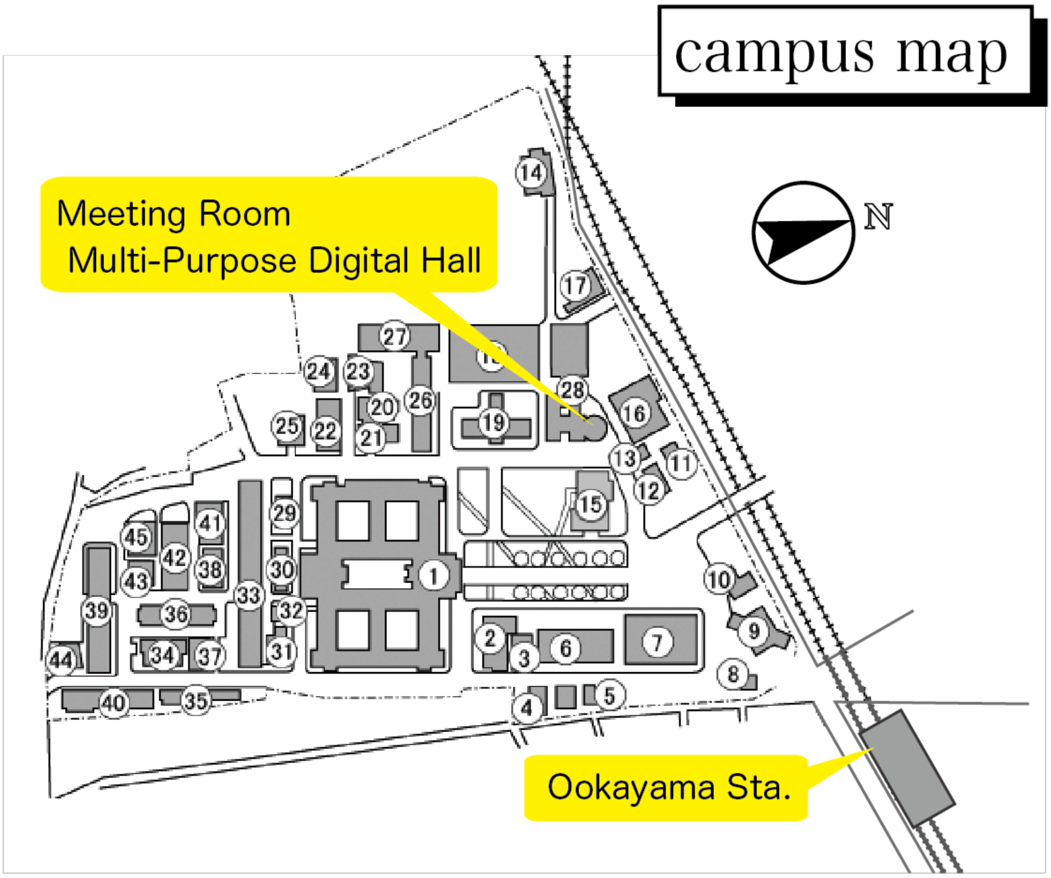
Meeting Room

Multi-Purpose Digital Hall

Banquet

Date: September 9, 2015

Place: Cafe La Bohéme Jiyugaoka



Computability Theory and Foundations of Mathematics 2015

Sep 7 (Mon)	Sep 8 (Tue)	Sep 9 (Wed)	Sep 10 (Thu)	Sep 11 (Fri)
Registration & Opening (9:30-10:30)	T. Yamazaki (9:30-9:55)	JSPS-NUS Discussion Session (9:30-11:30)	K. M. Ng (9:30-10:10)	N. Peng (9:30-9:55)
	Y. Takahashi (9:55-10:20)			K. Miyabe (9:55-10:20)
			L. Yu (10:20-11:00)	T. Sato (10:20-10:45)
G. Wu (10:30-11:10)	D. Raghavan (10:30-11:10)			
				S. Friedman (11:00-11:50)
S. Binns (11:20-12:00)	J. D. Hamkins (11:20-12:00)	Lunch break	L. Kołodziejczyk (11:20-12:10)	
Lunch break	Lunch break		Lunch break	Lunch break
		M. Fujiwara (13:00-13:25)		
		T. Kihara (13:25-13:50)		
A. Marcone (14:00-14:50)	N. Weaver (14:00-14:50)	H. Kurokawa (14:00-14:40)	T. Yamazaki (14:00-14:30)	H. Sakai (14:00-14:40)
			T. Wong (14:40-15:20)	
		W. Dean (14:50-15:30)		S. Fuchino (14:50-15:30)
C. T. Chong (15:00-15:25)	K. Okamoto (15:00-15:40)			
Y. Yang (15:25-15:50)			Coffee break	
	Coffee break			Coffee break
Coffee break			T. Suzuki (15:50-16:30)	
		Mini Concert (16:00-17:00)		T. Usuba (16:00-16:25)
	E. Frittaion (16:10-16:35)			
N.-Y. Suzuki (16:20-16:45)				K. Yokoyama (16:25-16:50)
	F. Pelupessy (16:35-17:00)		K. Yoshii (16:40-17:20)	
T. Kawano (16:45-17:10)				*Closing*
	M. Ilić (17:00-17:25)			

Monday, 7 September

9:30 – 10:20 **Registration**

10:20 – 10:30 **Opening**

10:30 – 12:00 **Session on Computability Theory**

10:30 Guohua Wu (Nanyang Technological University)
Some progress on Kierstead's conjecture

11:10 (Short Break)

11:20 Stephen Binns (Qatar University)
Randomness and Effective Dimension

12:00 – 14:00 (Lunch Break)

14:00 – 14:50 **Plenary Talk**

14:00 Alberto Marcone (The University of Udine)
From well-quasi-orders to Noetherian spaces: the reverse mathematics viewpoint

15:00 – 15:50 **JSPS-NUS Contributed Session** (2 Talks)

15:00 Chi Tat Chong (National University of Singapore)
Partial functions and domination

15:25 Yang Yue (National University of Singapore)
Reverse mathematics and Whitehead groups

15:50 – 16:20 (Coffee Break)

16:20 – 17:10 **CTFM Contributed Session** (2 Talks)

16:20 Nobu-Yuki Suzuki (Shizuoka University)
Axiom schema of Markov's principle preserves disjunction and existence properties

16:45 Tomoaki Kawano (Tokyo Institute of Technology)
Sequent calculi of quantum logic with strict implication

16:20 – 17:10 **JSPS-NUS Free Discussion**

Tuesday, 8 September

9:30 – 10:20 **JSPS-NUS / CTFM Contributed Session** (2 Talks)

9:30 Takeshi Yamazaki (Tohoku University)
Reverse mathematics and equilibria of continuous games

9:55 Yasuhiro Takahashi (NTT Communication Science Laboratories)
On the computational power of constant-depth exact quantum circuits

9:30 – 10:20 **CTFM Free Discussion**

10:30 – 12:00 **Session on Set Theory**

10:30 Dilip Raghavan (National University of Singapore)
Cardinal invariants of density

11:10 (Short Break)

11:20 Joel David Hamkins (City University of New York)
Universality and embeddability amongst the models of set theory

12:10 – 14:00 (Lunch Break)

14:00 – 14:50 **Plenary Talk**

14:00 Nik Weaver (Washington University)
A consistent formal system which verifies its own consistency

15:00 – 15:40 **Session on Philosophy**

15:00 Kengo Okamoto (Tokyo Metropolitan University)
Intuitionistic provability, classical validity and situation-dependent propositions – A consideration based on Gödel's modal embedding

15:40 – 16:10 (Coffee Break)

16:10 – 17:25 **CTFM Contributed Session** (3 Talks)

16:10 Emanuele Frittaion (Tohoku University)
Coloring rationals in reverse mathematics

16:35 Florian Pelupessy (Tohoku University)
On the "finitary" infinite Ramsey's theorem and the parametrised Paris-Harrington principle

17:00 Mirjana Ilić (University of Belgrade)
Cut in positive relevant logics without 't'

16:10 – 17:25 **JSPS-NUS Free Discussion**

Wednesday, 9 September

9:30 – 11:30 **JSPS-NUS Discussion Session**

11:30 – 13:00 (Lunch Break)

13:00 – 13:50 **JSPS-NUS Contributed Session** (2 Talks)

13:00 Makoto Fujiwara (JAIST)
Some principles weaker than Markov's principle

13:25 Takayuki Kihara (University of California, Berkeley)
Effective reducibility for smooth and analytic equivalence relations on a cone

14:00 – 15:30 **Session on Philosophy**

14:00 Hidenori Kurokawa (Kobe University)
On the interpretation of HPC in the Kreisel-Goodman theory of constructions

14:40 (Short Break)

14:50 Walter Dean (University of Warwick)
The completeness theorem, WKL_0*, and the origins of reverse mathematics*

16:00 – 17:00 **Mini Concert**

18:30 – 20:30 **Banquet**

Thursday, 10 September

9:30 – 11:00 **Session on Computability Theory**

9:30 Keng Meng Ng (Nanyang Technological University)
Instant structures and categoricity

10:10 (Short Break)

10:20 Liang Yu (Nanjing University)
On the reals which can be random

11:20 – 12:10 **Plenary Talk**

11:20 Leszek Kołodziejczyk (University of Warsaw)
How unprovable is Rabin's decidability theorem?

12:10 – 14:00 (Lunch Break)

14:00 – 17:20 **Special Session on Professor Tanaka's Works**

14:00 Introduction by Takeshi Yamazaki (Tohoku University)

14:30 (Short Break)

14:40 Tin Lok Wong (Kurt Godel Research Center)
Models of weak König's lemma

15:20 (Cofee Break)

15:50 Toshio Suzuki (Tokyo Metropolitan University)
Kazuyuki Tanaka's work on AND-OR trees and subsequent development

16:30 (Short Break)

16:40 Keisuke Yoshii (Okinawa National College of Technology)
A survey of determinacy of infinite games in second order arithmetic, dedicating to 60's birthday of Professor Tanaka

Friday, 11 September

9:30 – 10:45 **CTFM Contributed Session** (3 Talks)

9:30 Ningning Peng (National University of Singapore)
Equilibriums of independent distributions on uniform AND-OR trees

9:55 Kenshi Miyabe (Meiji University)
Reducibilities as refinements of the randomness hierarchy

10:20 Takashi Sato (Tohoku University)
Reverse mathematics, Rees theorem and Artin-Wedderburn theorem

9:30 – 10:45 **JSPS-NUS Free Discussion**

11:00 – 11:50 **Plenary talk**

11:00 Sy-David Friedman (Kurt Gödel Research Center)
The complexity of isomorphism

12:00 – 14:00 (Lunch Break)

14:00 – 15:30 **Session on Set Theory**

14:00 Hiroshi Sakai (Kobe University)
Cofinality of classes of ideals with respect to Katětov and Katětov-Blass orders

14:40 (Short Break)

14:50 Sakaé Fuchino (Kobe University)
A reflection principle as a reverse mathematical fixed point over ZFC

15:30 – 16:00 (Coffee Break)

16:00 – 16:50 **JSPS-NUS Contributed Session** (2 Talks)

16:00 Toshimichi Usuba (Kobe University)
Set-theoretic geology with large cardinals

16:25 Keita Yokoyama (JAIST)
A simple conservation proof for ADS

16:00 – 16:50 **CTFM Free Discussion**

16:50 – 17:00 **Closing**

Abstract Booklet

Contents

Plenary Talks

The complexity of isomorphism

Sy-David Friedman

Kurt Gödel Research Center, University of Vienna

I'll discuss the complexity of the equivalence relation of isomorphism when restricted to the computable, hyperarithmetic, ω_1-computable and finite structures. In some cases this relation is complete for Σ_1^1 equivalence relations, in some cases it is not, and in the case of finite structures this is an open problem of computational complexity theory.

How unprovable is Rabin's decidability theorem?

Leszek Kołodziejczyk

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Rabin's decidability theorem states that the monadic second order (MSO) theory of the infinite binary tree $\{0,1\}^*$ with the left- and right-successor relations S_0, S_1 is decidable. We study the strength of set existence axioms needed to prove this theorem.

We first consider a result known as the complementation theorem for tree automata, which is a crucial ingredient of typical proofs of Rabin's theorem. We show that the complementation theorem is equivalent over ACA_0 to a determinacy principle implied by the positional determinacy of all parity games and implying the determinacy of all $\mathrm{Bool}(\boldsymbol{\Sigma}^0_2)$ Gale-Stewart games. It follows that the complementation theorem is provable in Π^1_3-CA_0 but not Δ^1_3-CA_0.

We then use results due to MedSalem-Tanaka, Möllerfeld and Heinatsch-Möllerfeld to prove that over Π^1_2-CA_0, the decidability of the Π^1_3 fragment of the MSO theory of $(\{0,1\}^*, S_0, S_1)$ (or the Π^1_n fragment for any fixed $n \geq 3$) is equivalent to the complementation theorem for tree automata. This means in particular that Rabin's decidability theorem is not provable in Δ^1_3-CA_0.

The talk is based on joint work with Henryk Michalewski.

From Well-Quasi-Orders to Noetherian Spaces: the Reverse Mathematics Viewpoint

Alberto Marcone*

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If $(Q, \leq_Q)$ is a quasi-order we can equip Q with several topologies. We are interested in the Alexandroff topology $\mathcal{A}(Q)$ (the closed sets are exactly the downward closed subsets of Q) and the upper topology $\mathcal{U}(Q)$ (the downward closures of finite subsets of Q are a basis for the closed sets). $\mathcal{A}(Q)$ and $\mathcal{U}(Q)$ are (except in trivial situations) not T_1, yet they reflect several features of the quasi-order.

Recall that a topological space is Noetherian if all open sets are compact or, equivalently, there is no strictly descending chain of closed sets. Noetherian spaces are important in algebraic geometry.

It is fairly easy to show that $(Q, \leq_Q)$ is a well-quasi-order (wqo: well-founded and with no infinite antichains) if and only if $\mathcal{A}(Q)$ is Noetherian. Moreover, if $(Q, \leq_Q)$ is wqo then $\mathcal{U}(Q)$ is Noetherian.

Given the quasi-order $(Q, \leq_Q)$, consider the following quasi-orders on the powerset $\mathcal{P}(Q)$:

$$A \leq^\flat B \iff \forall a \in A\, \exists b \in B\, a \leq_Q b;$$
$$A \leq^\sharp B \iff \forall b \in B\, \exists a \in A\, a \leq_Q b.$$

We write $\mathcal{P}^\flat(Q)$ and $\mathcal{P}^\sharp(Q)$ for the resulting quasi-orders, and $\mathcal{P}^\flat_f(Q)$ and $\mathcal{P}^\sharp_f(Q)$ for their restrictions to the collection of finite subsets of Q.

When $(Q, \leq_Q)$ is wqo, $\mathcal{P}^\flat_f(Q)$ is also wqo, but $\mathcal{P}^\flat(Q)$ and $\mathcal{P}^\sharp_f(Q)$ are not always wqos. However Goubault-Larrecq proved that if $(Q, \leq_Q)$ is wqo then $\mathcal{U}(\mathcal{P}^\flat(Q))$, $\mathcal{U}(\mathcal{P}^\sharp(Q))$, and $\mathcal{U}(\mathcal{P}^\sharp_f(Q))$ are Noetherian. These results support the view that Noetherian spaces can be viewed as topological versions, or generalizations, of well-quasi-orders. Moreover Goubault-Larrecq provided applications of his theorems to verification problems.

We study Goubault-Larrecq's theorems and some of their consequences from the viewpoint of reverse mathematics. To this goal we first need to formalize statements about topological spaces which are far from being metrizable. When dealing with $\mathcal{P}^\sharp_f(Q)$ and $\mathcal{P}^\flat_f(Q)$, which are countable second countable spaces, we can use ideas originally introduced by Dorais. In the more general case of the uncountable spaces $\mathcal{P}^\sharp(Q)$ and $\mathcal{P}^\flat(Q)$ we need to code the upper topology using a metatheoretic framework that includes, besides our spaces, the well-known

* Joint work with Emanuele Frittaion, Matthew Hendtlass, Paul Shafer, and Jeroen Van der Meeren

coding of separable complete metric spaces, as well as the MF-spaces studied by Mummert.

We can thus state and prove our main theorem:

Theorem 1. *The following are equivalent over the base theory* RCA_0*:*

(i) ACA_0*;*
(ii) If Q *is wqo, then* $\mathcal{A}(\mathcal{P}^\flat_f(Q))$ *is Noetherian;*
(iii) If Q *is wqo, then* $\mathcal{U}(\mathcal{P}^\flat_f(Q))$ *is Noetherian;*
(iv) If Q *is wqo, then* $\mathcal{U}(\mathcal{P}^\flat(Q))$ *is Noetherian;*
(v) If Q *is wqo, then* $\mathcal{U}(\mathcal{P}^\sharp_f(Q))$ *is Noetherian;*
(vi) If Q *is wqo, then* $\mathcal{U}(\mathcal{P}^\sharp(Q))$ *is Noetherian.*

A consistent formal system which verifies its own consistency

Nik Weaver

The notion of a *proof* — in the semantic sense of "perfect rational justification", not the syntactic sense of a legal derivation within some formal system — is central to constructive mathematics. However, attempts to axiomatize the proof relation have not been successful.

In contrast, we find that the concept of *provability* (again, in the informal semantic sense) can be given a simple, intuitive axiomatization. And there are good reasons why this notion should be better behaved.

To avoid confusion with formal provability, we can the philosophical term *assertibility.* Assertibility predicates resemble classical truth predicates, but there are essential differences. Assertibility can be consistently used in a self-referential manner. Indeed, when Peano arithmetic is augmented with an assertibility predicate, it remains consistent while gaining the ability to formally derive a sentence which intuitively expresss its own consistency.

Invited Talks

Randomness and Effective Dimension

Stephen Binns

Department of Mathematics, Statistics, and Physics
Qatar University, Doha, Qatar

The idea of effective dimension of a real (infinite binary sequence), defined in terms of the Kolmogorov complexity of its initial segments, has been much studied. If X is such an an infinite binary sequence, then we define its effective Hausdorff dimension as

$$\dim_{\mathcal{H}} X = \liminf_n \frac{C(X \upharpoonright n)}{n}.$$

A dual notion of effective packing dimension is defined as

$$\dim_p X = \limsup_n \frac{C(X \upharpoonright n)}{n}.$$

If these two quantities are equal we refer to X as being *regular* and define the effective dimension of X as $\dim X = \lim \frac{C(X \upharpoonright n)}{n}$. These notions can be extended to introduce interesting geometric ideas into the study of effective dimension. We define a relativised version of these definitions: if X and Y are two reals then we define

$$d(Y \to X) = \limsup_n \frac{C(X \upharpoonright n \mid Y \upharpoonright n)}{n}.$$

$d(Y \to X)$ obeys the triangle inequality in the direction of the arrow, and can be used to constuct a pre-metric on the set of all reals by defining

$$d(X, Y) = \max\{d(X \to Y), d(Y \to X)\}.$$

Two reals of distance 0 from each other are deemed d-equivalent. Along with this metric, we can define a notion of scalar multiplication, so that if $\alpha \in [0, 1]$, and X is a regular real, then αX can be defined so that

$$\dim(\alpha X) = \alpha \dim(X).$$

This scalar multiplication *dilutes* the information in X and reduces its dimension by a factor of α.

Furthermore, using these ideas, it is possible to introduce natural notions of angle and projection into the geometry and to define a notion of flatness using them.

The main result presented will be to show that any regular real is d-equivalent to a dilution of a 1-random, thereby giving a sense in which all regular complexity is a diluted form of randomness.

The completeness theorem, WKL_0, and the origins of reverse mathematics

Walter Dean

University of Warwick

One of the founding goals of the Reverse Mathematics program is the study of which set existence axioms of second-order arithmetic are needed to prove theorems of classical mathematics whose statements are not overtly set theoretic in nature. One reason this program has been of interest to philosophers of mathematics is the hope articulated by Simpson (1988, 2009) that the major subsystems of second-order arithmetic formally characterize foundational standpoints such as finitism or predicativism. The principle known as Weak König's Lemma [WKL] appears to be an outlier in both respects: not only does it fail to have the form of an unqualified assertion of set existence (as exemplified by a comprehension scheme), it is also not initially clear how it demarcates a difference between the philosophical schools and figures mentioned by Simpson. The first goal of this talk will be to put these observations into historical context by considering how WKL came to be isolated from König's Infinity Lemma as a combinatorial principle in its own right. (This story has much to do with what we now call the Arithmetized Completeness Theorem and the origins of computable model theory.) The second goal will be to attempt to compare the role of this principle as a minimally non-constructive posit to other means of expressing ontological commitment which have been discussed by philosophers of mathematics.

A reflection principle as a reverse mathematical fixed point over ZFC

Sakaé Fuchino

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Many "mathematical" assertions are known to be equivalent to the Continuum Hypothesis (CH) over the Zermelo-Fraenkel axiom system of set theory with Axiom of Choice (ZFC). In spite of some arguments which claim the irrelevance of the CH (in the "real" mathematics) the abundance of these assertions support the relevance and import of CH over ZFC.

The significance of the set-theoretic principle called Fodor-type Reflection Principle (FRP) can be argued similarly: this principle is known to be equivalent to many "mathematical" reflection assertions previously known to hold under strong axioms like $\mathrm{MA}^+(\sigma\text{-closed})$.

One of such mathematical reflection assertion goes as follows: For any locally compact topological space X, if all subspaces of X of cardinality $\leq \aleph_1$ are metrizable, then X itself is also metrizable.

Unlike ZFC + CH, which is equiconsistent with ZFC, ZFC + FRP implies a fairly large large cardinal property. At the moment the exact equiconsistency of FRP is not yet known but Tadatoshi Miyamoto proved that a local version of FRP has the consistency strength strictly less than that of the corresponding local versions of some other known set-theoretic reflection principles.

In this talk, I shall give a survey of the results around the principle FRP. Most of the results I mention here are obtained in joint work with Istvan Juhász, Hiroshi Sakai, Lajos Soukup, Zoltan Szentmiklóssy and Toshimichi Usuba.

Universality and embeddability amongst the models of set theory

Joel David Hamkins[1,2,3,⋆]

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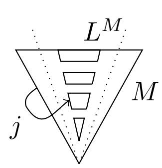
Abstract. Recent results on the embeddability phenomenon and universality amongst the models of set theory are an appealing blend of ideas from set theory, model theory and computability theory. Central questions remain open.

Keywords: embeddings, universality, hypnagogic digraph

A surprisingly vigorous embeddability phenomenon has recently been uncovered amongst the countable models of set theory. It turns out, for instance, that among these models embeddability is linear: for any two countable models of set theory, one of them embeds into the other ([4]). Indeed, one countable model of set theory M embeds into another N just in case the ordinals of M order-embed into the ordinals of N. This leads to many surprising instances of embeddability: every forcing extension of a countable model of set theory, for example, embeds into its ground model, and every countable model of set theory, including every well-founded model, embeds into its own constructible universe.



$$x \in y \longleftrightarrow j(x) \in j(y)$$

Although the embedding concept here is the usual model-theoretic embedding concept for relational structures, namely, a map $j : M \to N$ for which $x \in^M y$ if and only if $j(x) \in^N j(y)$, it is a weaker embedding concept than is usually

⋆ The author's research has been supported in part by NSF grant DMS-0800762, PSC-CUNY grant 64732-00-42 and Simons Foundation grant 209252. Commentary can be made at http://jdh.hamkins.org/universality-and-embeddability-ctfm-2015-japan.

considered in set theory, where embeddings are often elementary and typically at least Δ_0-elementary. Indeed, the embeddability result is surprising precisely because we can easily prove that in many of these instances, there can be no Δ_0-elementary embedding.

The proof of the embedding theorem makes use of universality ideas in digraph combinatorics, including an acyclic version of the countable random digraph, the countable random $\mathbb{Q}$-graded digraph, and higher analogues arising as uncountable Fraïssé limits, leading to the hypnagogic digraph, a universal homogeneous graded acyclic class digraph, closely connected with the surreal numbers. Thus, the methods are a blend of ideas from set theory, model theory and computability theory.

Results from [2] show that the embedding phenomenon does not generally extend to uncountable models. Current work [1] is concerned with questions on the extent to which the embeddings arising in the embedding theorem can exist as classes inside the models in question. Since the embeddings of the theorem are constructed externally to the model, by means of a back-and-forth-style construction, there is little reason to expect, for example, that the resulting embedding $j : M \to L^M$ should be a class in M. Yet, it has not yet known how to refute in ZFC the existence of a class embedding $j : V \to L$ when $V \neq L$. However, many partial results are known. For example, if the GCH fails at an uncountable cardinal, if $0^\sharp$ exists, or if the universe is a nontrivial forcing extension of some ground model, then there is no embedding $j : V \to L$. Meanwhile, it is consistent that there are non-constructible reals, yet $\langle P(\omega), \in\rangle$ embeds into $\langle P(\omega)^L, \in\rangle$.

References

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On the interpretation of HPC in the Kreisel-Goodman Theory of Constructions

Hidenori Kurokawa

Kobe University

In this talk, we discuss the Kreisel-Goodman Theory of Constructions (ToC). This theory was originally introduced by Kreisel as an untyped theory which can handle the notion of mathematical constructions used in the BHK interpretation of intuitionistic logical constants. One of the theoretical goals of ToC is to provide a formal theory of mathematical constructions "in terms of which the formal rules of Heyting's predicate calculus [HPC] can be interpreted (Kreisel, 1962)." However, the version of ToC which satisfies all the desiderata considered by Kreisel turns out to be inconsistent (the Kreisel-Goodman paradox). We first present our own analysis of the paradox and propose a consistent sub-theory of the inconsistent version of ToC. We then discuss an outline of Goodman's proof of the soundness of the interpretation of HPC into this weaker version of ToC. This is a joint work with Walter Dean.

Instant structures and categoricity

Keng Meng Ng

Nanyang Technological University

In this talk we will study various aspects of instant structures. An instant structure is a countable structure with domain omega, and where the relations and functions are primitive recursive. Various related notions have been studied in the literature, notably the early work of Cenzer and Remmel. We give some intuition and explain why these structures are interesting. We also compare with classical computable structures. We discuss notions of instant categoricity, and show the surprising result that instant categoricity is different from classical computable categoricity.

Intuitionistic Provability, Classical Validity and Situation-Dependent Propositions

A Consideration based on Gödel's Modal Embedding

Kengo Okamoto

Tokyo Metropolitan University

It is well-known that there exists a sound and faithful embedding of the sequents deducible in intuitionistic logic (IL) into those deducible in the modal logic S4: for any sequent S of IL, IL proves S if and only if S4 proves the modal translation (i.e. the so-called Gödel translation) of S. Moreover, we can show that there exists a more exact model theoretic correspondence between IL propositions (i.e. IL formulae) and their S4 translations: for any Kripke model M of S4, we can construct i) a mapping f on the set W of possible worlds in M and ii) a Kripke model L of IL that is defined in terms of the mapping f, such that for any S4 proposition P that is the modal translation of some IL proposition Q (i.e. P is the Gödel translation of Q), (1) f not only strictly preserves the truth (i.e. P is true in M iff Q is true in L), (2) but also maps the truth set of P in M (i.e. the set of the words in W in which P is true) exactly into that of Q in L (i.e. the set of possible states of L in which Q is true). The converse mapping from IL Kripke models to S4 Kripke models can be also (rather trivially) constructed.

One might say these facts justify the claim that intuitionistic propositions be literally construed in accordance with their modal translations: for example, for any IL primitive proposition q, we could construed it as stating that the corresponding S4 primitive proposition (usually this latter proposition is itself symbolized by "q", but we are free to assign to it any S4 primitive proposition whatever) is "necessary", since the Gödel translation of q is the necessitation of the corresponding S4 primitive proposition. Note that the notion of necessity in question here is nothing other than that of the intuitionistic provability.

Now, similar observations could be made with respect to classical logic (CL) and the modal logic S5: (1) the modal translation embeds sequents deducible in CL into those deducible in S5 in a sound and faithful way and (2) there exists an exact model theoretic correspondence between CL propositions and their S5 translations. As might easily be seen, in this case the relevant notion of necessity is nothing other than that of classical validity: for example, the CL primitive proposition q could be construed as stating that the corresponding S5 primitive proposition is classically valid.

These considerations induce the following question: what are the (primitive) propositions of S4 and S5? We attempt to show that they are in general to be identified with situation-dependent propositions (some authors say they are the propositions seen from the "internal" or "local" point of view) and that this construal also throw some light on the relationships between intuitionistic and classical formal arithmetical theories.

Cardinal invariants of density

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I will talk about some recent work on cardinal invariants associated with the ideal $\mathcal{Z}_0$ of sets of asymptotic density 0. In particular I will discuss some upper bounds for $\mathrm{cov}^*(\mathcal{Z}_0)$, which is the minimal number of density 0 sets needed to intersect every infinite subset of ω on an infinite set. This dualizes to give a lower bound for $\mathrm{non}^*(\mathcal{Z}_0)$. Some of the results I will present are joint work with Shelah.

Cofinality of Classes of Ideals with Respect to Katětov and Katětov-Blass Orders

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The Katětov order $\leq_{\mathrm{K}}$ and the Katětov-Blass order $\leq_{\mathrm{KB}}$ are orders on ideals over ω. For ideals $\mathcal{I}$ and $\mathcal{J}$ over ω, $\mathcal{I} \leq_{\mathrm{K}} \mathcal{J}$ if there is a function $f : \omega \to \omega$ such that $f^{-1}[A] \in \mathcal{J}$ for any $A \in \mathcal{I}$. Moreover $\mathcal{I} \leq_{\mathrm{KB}} \mathcal{J}$ if there is such a finite to one function f.

The Katětov order was introduced by Katětov [2] to study convergence in topological spaces. After that it has turned out that many combinatorial properties of ideals over ω can be characterized using $\leq_{\mathrm{K}}$ and $\leq_{\mathrm{KB}}$. For example, Solecki [3] introduced a certain F_σ-ideal $\mathcal{S}$ and proved that an ideal $\mathcal{I}$ over ω has the Fubini property if and only if $\mathcal{S} \not\leq_{\mathrm{K}} \mathcal{I}|X$ for any $X \in \mathcal{I}^+$. Moreover, in these characterizations of properties of ideals, it is often the case that there is some critical Borel ideal such as $\mathcal{S}$ in Solecki's characterization of the Fubini property. Hrušák [1] studies $\leq_{\mathrm{K}}$ and $\leq_{\mathrm{KB}}$ on Borel ideals systematically.

In this talk we discuss the cofinal types of classes of ideals with respect to $\leq_{\mathrm{K}}$ and $\leq_{\mathrm{KB}}$. Among other things, we study the classes of all F_σ-ideals, Analytic P-ideals and Borel ideals. This is a joint work with Hiroaki Minami.

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Kazuyuki Tanaka's work on AND-OR trees and subsequent development

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Searching a game tree is an important subject of artificial intelligence. In the case where the evaluation function is bi-valued, the subject is interesting for logicians, because a game tree in this case is a Boolean function. Among such trees, the most basic one is a binary uniform NAND tree. By moving negations, we may identify such a tree with an AND-OR tree.

Kazuyuki Tanaka has a wide range of research interests which include complexity issues on AND-OR trees. In the joint paper with C.-G. Liu (2007) [3], he studies distributional complexity of AND-OR trees. We overview this work and subsequent development.

A truth assignment to the leaves is given and hidden. The goal of a tree searching algorithm is to find the value of the root. For this purpose, an algorithm successively makes queries to leaves. The cost is measured by the number of leaves probed during the searching.

The *alpha-beta pruning algorithm* is a famous efficient searching algorithm (Knuth and Moore [2]). In the case of an AND-OR tree, an alpha-beta pruning algorithm is characterized by the following two requirements. (1) It is depth-first, that is, whenever it probes a leaf that is a descendant of an internal node, say v, it never probes descendants of siblings of v until it knows the value of v. (2) Whenever the algorithm knows a child of an AND-node has value 0 (false) it knows that the AND-node has the same value without probing the other siblings. A similar rule applies to OR-nodes, too.

In the case of independent and identical distributions (IID), the optimality of the alpha-beta pruning algorithms is studied by Baudet [1] and Pearl [4], and the optimality is shown by Pearl [5] and Tarsi [9].

Yao [10] observed a variation of von Neumann's minimax theorem. The *distributional complexity* P of a given tree T is defined as follows.

$$P = \max_{d} \min_{A_D} C(A_D, d)$$

Here, d runs over the distributions on the truth assignments, and A_D runs over the deterministic algorithms. $C(A_D, d)$ denotes (expected value of) the cost.

The *randomized complexity* R of a given tree T is defined as follows.

$$R = \min_{A_R} \max_{x} C(A_R, x)$$

Here, A_R runs over the randomized algorithms, and x runs over the truth assignments. Yao's principle is an assertion that $P = R$. Saks and Wigderson [6] show basic results on the equilibriums.

Liu and Tanaka [3] extend the work of Saks and Wigderson. In [3], a probability distribution on the truth assignment that achieves the distributional complexity is called an *eigen-distribution*.

(1) The case of ID (independent distribution on the truth assignments): They study which distribution is an eigen-distribution, and investigate asymptotic behavior of eigen-distributions with respect to the height of the tree.

(2) The case of CD (correlated distributions): By extending the concepts of the *reluctant inputs* of Saks and Wigderson, they introduce the concepts of E_1*-distribution*, and show that E_1-distribution is the unique eigen-distribution.

The result (2) is extended to the case where distributions and algorithms run over the elements of given classes (S. and Nakamura, 2012 [7]).

In the course of showing (1), Liu and Tanaka state, without a proof, that if a distribution achieves the distributional complexity among IDs then it is an IID (Theorem 4 of [3]). S. and Niida [8] show fundamental relationships between probability and expected cost of uniform binary OR-AND trees, and by means of the relationships, give a rigorous proof for Theorem 4 of [3]. More recently, NingNing Peng et al. extends the result in [8] to uniform level-by-level k-branching AND-OR trees.

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Models of Weak König's Lemma

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Weak König's Lemma (WKL) states that every infinite binary tree contains an infinite branch. Its formalization in second-order arithmetic occupies a prominent position in the foundations of mathematics.

Tanaka and his collaborators made significant contributions to the understanding of nonstandard models of WKL. On the one hand, he introduced self-embeddings to second-order arithmetic, and explained why WKL is relevant in such constructions [2]. On the other hand, research about Tanaka's conjecture on the conservativity of WKL led to the discovery of a novel technique, due jointly to Simpson, Tanaka and Yamazaki [1], for producing very similar yet very different models of WKL.

In my talk, I will survey these results, and report on some ongoing work in collaboration with Ali Enayat (Gothenburg) in refining them.

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Some progress on Kierstead's conjecture

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A well-known theorem of Dushnik-Miller says that any countably infinite linear ordering admits a nontrivial self-embedding. The effective version of Dushnik-Miller theorem is not true (Hay and Rosenstein). Downey and Lempp proved that the proof strength of the Dushnik-Miller theorem is the same as ACA_0.

We are interested in the complexity of automorphisms associated. One of Kierstead's work initiates the study of η-like linear orderings, where Kierstead shows the existence of a computable linear ordering of order type 2η with no nontrivial Π^0_1-automorphism. Kierstead also conjectured that every computable copy of a linear order L has a strongly nontrivial Π^0_1-automorphism if and only if it contains an interval of order type η. Kierstead proved it for 2η and Downey and M. Moses proved it for discrete linear orderings.

Harris, Lee and Cooper extended these results by proving that Kierstead ' s conjecture is true for a quite general subclass of η-like computable linear orderings. In this talk, we will present our recent work towards further progress on Kierstead's conjecture. This is joint work with Zubkov.

A survey of determinacy of infinite games in second order arithmetic, dedicating to 60's birthday of Professor Tanaka.

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What set existence axioms are needed to prove the theorems of ordinary mathematics? In 1970s, Harvey Friedman introduced this theme of reverse mathematics, and many mathematician are now working on this field all over the world. Tanaka started his research on determinacy of infinite games in second order arithmetic in 1980s, the early age of reverse mathematics.

It is now known that most of classical mathematics theorems are equivalent to only five systems of second order arithmetic such as $\mathsf{RCA}_0, \mathsf{WKL}_0, \mathsf{ACA}_0, \mathsf{ATR}_0$ and Π^1_1-CA_0. Determinacy of infinite games are treated in the strong systems of second order arithmetic such as ATR_0, Π^1_1-CA_0, and more. After John Steel showed in [2] that determinacy of open games (Σ^0_1-Det) is equivalent to ATR_0, Tanaka formalized Σ^1_1 inductice definition (Σ^1_1-ID_0) in second order arithmetic ([3]) and introduced it as a new axiom system. The importance of Σ^1_1 inductive definitions had been already established in descriptive set theory. Tanaka showed that Σ^1_1-ID is derived from Σ^0_2-Det over ACA_0 and, what is more, showed that the reversal is not derived from over ACA_0, considering the light face versions of them. It showed that the theme of reverse mathematics does not hold in that case and emphasized the importance to establish proof-theoretic relations among lightface statements. Other important aspect of determinacy in second order arithmetic could be that equivalences between comprehension axioms and determinacy of Σ^0_2 and more complex classes can not be proved. Indeed, Δ^1_2 comprehension axiom is not derived even from Borel determinacy. It makes the situation harder to investigate the proof-theoretic strengths of determinacy, but he and MedSalem succeeded to pin down Δ^0_3 by introducing transfinite combinations of Σ^1_1 inductive definitions [1].

In this talk, dedicating to 60's birthday of Professor Tanaka, we overview his works on determinacy in second order arithmetic and some of related recent works.

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On the reals which can be random

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We investigate which reals can be L-random respect to some continuous measure. This is a joint work with Yizheng Zhu.

Contributed Talks

Partial functions and domination

Chi Tat Chong

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A partial function f dominates a partial function g if for all but finitely many inputs x, whenever $x \in Dom(f)$ then $f(x) \leq g(y)$ for some $y \leq x$ in the domain of g. A set A is *pdominant* if there is an e such that Φ_e^A dominates every partial recursive function. We discuss some recursion-theoretic properties of pdominant sets. The talk is based on joint work with Gordon Hoi, Frank Stephan and Dan Turetsky.

Coloring rationals in reverse mathematics

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(Joint work with Ludovic Patey)

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I will present some new results about the reverse mathematics of a theorem due to Erdös and Rado about colorings of rationals ([1])

Theorem 1 (Erdős, Rado 1952). *The partition relation $\eta \to (\aleph_0, \eta)^2$ holds, that is, for every coloring $c\colon [\mathbb{Q}]^2 \to 2$ there exists either an infinite 0-homogeneous set or a dense 1-homogeneous set.*

This Erdős-Rado theorem is known to lie between ACA_0 and RT^2_2, but its reverse mathematics status remains open. However, we show that it does not computably reduce to RT^2_2, even though we are not able to generalize this "one-step" separation to a separation over ω-models, as in the case of the tree theorem for pairs (see [2]). Moreover, we show that one of its consequences, that can be regarded as the "pigeonhole principle" over the rationals, is provable in Σ^0_2 induction and properly stronger than Σ^0_2 bounding, a feature shared by the tree theorem for singletons (see [3]).

Keywords: Reverse Mathematics, computable reducibility

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Some principles weaker than Markov's principle

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It is known that most of Bishop's constructive mathematics can be formalized within the system EL of intuitionistic second-order arithmetic (which is known as a system of elementary analysis). On the other hand, Russian constructive recursive mathematics has been accepted Markov's principle, which is formalized over EL but is known not to be provable in EL. With respect to constructive reverse mathematics (e.g. [5, 2]), several principles which are strictly weaker than Markov's principle but are not provable in EL, has been introduced in the previous studies ([4, 6, 1]). In this talk, we discuss the interrelations between these principles as well as some other related principles: weak variants of the law of excluded middle, de Morgan's Law, and Markov's principle. In particular, we show over EL that Δ^0_1-LEM in the sense of [1] is strictly weaker than disjunctive Markov's principle $\mathrm{MP}^\vee$, and is not derived from weak Markov's principle WMP ([3]).

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Cut in positive relevant logics without $'t'$

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The first sequent calculi for positive relevant logics were formulated by Dunn and Minc in [5] and [8]. In those calculi, the cut rule has the following form:

$$\frac{\Pi \vdash \varphi \qquad \Gamma[\varphi] \vdash \gamma}{\Gamma[\Pi] \vdash \gamma}\ \text{(cut)}$$

where $\Gamma[\Pi]$ is the result of replacing arbitrarily many occurrences of φ in $\Gamma[\varphi]$ by Π if Π is non–empty, and otherwise by $'t'$. The constant truth $'t'$ is needed to disable the inference of the modal fallacy $\vdash \alpha \to (\beta \to \beta)$. Really, without $'t'$, we would have:

$$\dfrac{\dfrac{\vdash \beta \to \beta \qquad \dfrac{\beta \to \beta \vdash \beta \to \beta}{\alpha, \beta \to \beta \vdash \beta \to \beta}\ \text{(extensional thinning)}}{\alpha \vdash \beta \to \beta}\ \text{(cut)}}{\vdash \alpha \to (\beta \to \beta)}\ (\to \text{r})$$

However, with $'t'$, the admissibility of *modus ponens*, essential for the proof of the equivalence between sequent calculi and their Hilbert–style formulations, cannot be proved. Really, with $'t'$ added as above, it cannot be proved that whenever $\vdash \alpha$ and $\vdash \alpha \to \beta$ are both derivable in a sequent system, so is $\vdash \beta$. This is the reason why some authors (e.g. Dunn [5], [6], Giambrone [4]), first add $'t'$, essential for the admissibility of cut, but once cut–elimination is established, they develop the techniques to get rid of $'t'$.

We propose another formulation of the cut rule, for positive relevant logics, where the constant $'t'$ is not needed. Our cut rule is of the following forms:

$$\frac{\Pi \vdash \varphi \qquad \Gamma[\varphi] \vdash \gamma}{\Gamma[\Pi] \vdash \gamma}\ \text{(cut–i)}$$

$$\frac{\vdash \varphi \qquad \Gamma[\varphi; \Pi] \vdash \gamma}{\Gamma[\Pi] \vdash \gamma}\ \text{(cut–ii)}$$

$$\frac{\vdash \varphi \qquad \varphi \vdash \gamma}{\vdash \gamma}\ \text{(cut–iii)}$$

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where Π is non–empty. In (cut-i), $\Gamma[\Pi]$ is the result of replacing exactly one occurrence of φ in $\Gamma[\varphi]$ by Π, in (cut-ii) the single occurrence of φ in $\Gamma[\varphi; \Pi]$ is replaced by an empty multiset and similarly in (cut-iii).

The various versions of our cut rule, ensure that the modal fallacy remains unprovable. Furthermore, they are enough for the proof of the equivalence between Hilbert–style formulation and the corresponding sequent calculus (e.g., this form of cut is used in the the sequent calculus formulation for the positive contraction–less relevant logic RW_+°, in [7]). However, it should be mentioned that the use of $'t'$ remains crucial in sequent calculus for TW_+ and in sequent calculi for other weaker, permutation–less, relevance logics such as B_+, E_+ and even $T_\to$ (there is a sequent calculus for $T_\to$ without $'t'$ in [2], however the one with $'t'$ is much easier to use), where $'t'$ precludes intensional structures from becoming scrambled, see e.g. [3].

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Sequent calculi of quantum logic with strict implication

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Quantum logic is the logic witch describe a propositional space of quantum physics. It corresponds to a complete orthomodular lattice. Orthomodular lattice has been studied from the point of view of physics and also from the point of view of logic. One feature of quantum logic is that the distributive law is not satisfied in a orthomodular lattice like the law of excluded middle is not satisfied in a intuitionistic logic.

Many syntax of quantum logic are studied. There are Hilbert style calculi, natural deduction, sequent calculi, and so on. In general, they only include negation, conjunction and disjunction. When we try to add the notion of implication in quantum logic, there is some problems. In quantum logic, if we treat a implication as $\neg A \vee B$ as in classical logic, modus ponens is failed. There is some main conditions which implication has to satisfy like modus ponens. These conditions are index for making a new implication.

When we think about Kripke semantics, most plausible and descriptive implication is strict implication as its definition is similar to the implication in intuitionistic logic. I will present about a new sequent calculi for quantum logic which include this special implication and prove completeness and cut-elimination.

Effective Reducibility for Smooth and Analytic Equivalence Relations on a Cone

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The *oracle relativization* of a computability-theoretic concept sometimes has applications in other areas of mathematics which does *not* involve any notion concerning computability. For example, authors [2–4] discovered unexpected applications of Turing degree spectra *on a cone* (or *relative to an oracle*) in various areas of mathematics such as descriptive set theory, infinite dimensional topology, and Banach space theory. As for another example, Becker [1] and Knight-Montalbán (see [5]) independently showed that if there is no first-order axiomatizable class of countable structures whose isomorphism relation is intermediate w.r.t. *computable reducibility on a cone*, then the Vaught conjecture, one of the most notable conjectures in model theory, turns out to be true; that is, the number of countable models of a first-order theory is at most countable or $2^{\aleph_0}$.

Here we say, for equivalence relations E and F on spaces $\mathcal{X}$ and $\mathcal{Y}$ respectively, that E is computable reducible to F on a cone (written as $E \leq^{\mathrm{cone}}_{\mathrm{eff}} F$) if there is an oracle r such that for any oracle $z \geq_T r$, there is a partial computable function $f :\subseteq \omega \to \omega$ such that for any indices $d, e \in \omega$, whenever the i- and j-th Turing computations Φ^z_i and Φ^z_j with oracle z determine points in $\mathcal{X}$, $\Phi^z_{f(i)}$ and $\Phi^z_{f(j)}$ also determine points in $\mathcal{Y}$ and $\Phi^z_i E \Phi^z_j$ if and only if $\Phi^z_{f(i)} F \Phi^z_{f(j)}$. An equivalence relation is intermediate w.r.t. $\leq^{\mathrm{cone}}_{\mathrm{eff}}$ if it is neither Borel nor $\leq^{\mathrm{cone}}_{\mathrm{eff}}$-complete among analytic equivalence relations.

In this talk, we study how cone-computable reducibility behaves differently from continuous and Borel reducibility. In particular, we give a few results on smooth equivalence relations and intermediate analytic equivalence relations w.r.t. $\leq^{\mathrm{cone}}_{\mathrm{eff}}$. This is ongoing work with Antonio Montalbán.

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Reducibilities as refinements of the randomness hierarchy

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The theory of algorithmic randomness has studied many randomness notions, most of which are linearly ordered in the sense that one randomness notion implies another randomness notion. For instance, 2-randomness implies ML-random, which in turn implies Schnorr randomness, which also implies Kurtz randomness. Here, we say that 2-randomness is stronger than ML-randomness and so on, and we call this order randomness hierarchy. Notice that this fact can be used as a measure of how random a set is.

Another way of measuring randomness is reducibility. Levin-Schnorr's theorem says that a set A is ML-random if and only if $K(A \upharpoonright n) > n - O(1)$ where K is the prefix-free Kolmogorov complexity. With this in mind, we say that A is K-reducible to B, denoted by $A \leq_K B$, if $K(A \upharpoonright n) < K(B \upharpoonright n) + O(1)$, whose intuitive meaning is that B is more random than A. This K-reducibility has been studied the most, while similar reducibilities also has been studied.

We expect that, if B is more random than A and A is random, then B should be random. However, this does not hold for K-reducibility and Schnorr randomness in the sense that, even if $A \leq_K B$ and A is Schnorr random, B may not be Schnorr random. Thus, the two measures of randomness are not completely consistent.

Definition 1. *We say that a reduciblity $\leq_r$ of randomness is* consistent *with a randomness notion R if the following holds: if $A \leq_r B$ and A is R-random, then B is R-random.*

Now, we ask which pairs of reducibilities and randomness notions are consistent. The answers are immediate from known results for most pairs. For instance, $\leq_K$ and $\leq_C$ are consistent with ML-randomness and n-randomness for all $n \geq 2$, but not with Schnorr randomness and Kurtz randomness.

In contrast, the consistency is not obvious for some pairs. The decidable prefix-free machine reducibility $\leq_{dm}$ is consistent with Kurtz randomness, Schnorr randomness, ML-randomness and n-randomness for all $n \geq 2$. Thus, $\leq_{dm}$ can be seen as a refinment of this part of the randomness hierarchy. This is somewhat understandable when considering that dedicable prefix-free machines characterize many randomness notions. Furthermore, the total-machine reducibility $\leq_{tm}$ is also consistent with Kurtz randomness, Schnorr randomness, ML-randomness, and n-randomness for all $n \geq 2$.

We need to do some work for Schnorr reducibility $\leq_{Sch}$.

Theorem 1. *The Schnorr reducibility is consistent with 2-randomness.*

For the proof of this, we extended the counting theorem for a machine that may not be universal.

In contrast, it is open whether the Schnorr reducibility is consistent with ML-randomness. The counterexample should be deep in the sense of Bennett.

In this study we also obtain the following.

Theorem 2. *The following are equivalent for a set:*

(i) *X is 2-Z-random.*
(ii) $C(X \upharpoonright (Z \upharpoonright n)) > Z \upharpoonright n - O(1)$.

Corollary 1. *A set X is 3-random if and only if* $C(X \upharpoonright (\Omega \upharpoonright n)) > \Omega \upharpoonright n - O(1)$.

This is a characterization of 3-randomness via complexity. The characterization uses Ω (or any set in the degree $0'$), but not as an oracle. It seems that how random a 2-random set is can be measured by when complexities of the initial segment is maximal up to a constant.

On the "finitary" infinite Ramsey's theorem and the parametrised Paris–Harrington principle

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We examine two different "finitary" Ramsey principles and compare them to the infinite Ramsey's theorem. One is based on Gaspar and Kohlenbach's "finitary" infinite pigeonhole principle in [1], the other is based on the Weiermann-style parametrisation of the Paris–Harrington principle from [2].

Definition 1 (RT^k_d)**.**
For every $C\colon [\mathbb{N}]^d \to k$ there exists an infinite C-homogeneous set.

Definition 2 (AS)**.** *A function $F\colon \{$(codes of) finite subsets of $\mathbb{N}\} \to \mathbb{N}$ is asymptotically stable if for every sequence $X_0 \subseteq X_1 \subseteq X_2 \ldots$ of finite sets there exists i such that $F(X_j) = F(X_i)$ for all $j \geq i$.*

Definition 3 (FRT^k_d)**.** *For every $F \in \mathrm{AS}$ there exists R such that for all $C\colon [0,R]^d \to k$ there exists C-homogeneous H of size $> F(H)$.*

Definition 4 (PPH^k_d)**.** *For all $f\colon \mathbb{N} \to \mathbb{N}, a$ there exists R such that for all $C\colon [a,R]^d \to k$ there exists C-homogeneous H of size $f(\min H)$.*

The latter statement is a direct subcase of the first one. We have the following implications for these statements:

Theorem 1.

(a) $\mathrm{RCA}_0 \vdash \mathrm{FRT}^k_d \to \mathrm{RT}^k_d$,
(b) $\mathrm{WKL}_0 \vdash \mathrm{RT}^k_d \to \mathrm{FRT}^k_d$,
(c) $\mathrm{RCA}_0 \vdash \mathrm{WO}(\varepsilon_0) \leftrightarrow \forall d, k.\mathrm{PPH}^k_d$,
(d) $\mathrm{RCA}_0 \vdash \forall k.\mathrm{PPH}^k_d \to \mathrm{WO}(\omega_d)$.

Where $\mathrm{WO}(\alpha)$ is the statement "α is well-founded" , $\omega_1 = \omega$ and $\omega_{d+1} = \omega^{\omega_d}$.

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Equilibriums of Independent Distributions on Uniform AND-OR Trees

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In 2007, Liu and Tanaka [2] showed that for any uniform binary AND-OR tree on the assignments that are independently distributed (ID), the distributional complexity is achieved only if the assignments are also identically distributed (IID).

We generalize Liu-Tanaka's result to level-by-level uniform multi-branching AND-OR tree. The proof technique is different from available ones. One ingredient of our proof is a generalization of Suzuki-Niida's [3] "fundamental relationships between costs and probabilities". Another ingredient of our proof is a careful analysis of the algorithms involved.

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Reverse Mathematics, Rees Theorem and Artin-Wedderburn Theorem

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Abstract. Artin-Wedderburn Theorem is the structural theorem of rings. Affected by this theorem, Rees proved the structural theorem of semigroups. I will talk about the ongoing research of reverse mathematics of Rees Theorem and Artin-Wedderburn Theorem. This research is along the way of [1] which analyze the fundamental theorem of finitely generated abelian groups-the structural theorem of groups.

Keywords: Reverse Mathematics, Second Order Arithmetic Rees Theorem, Artin-Wedderburn Theorem

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Axiom schema of Markov's principle preserves disjunction and existence properties

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It is well-known that Heyting arithmetic HA is closed under Markov's rule:

$$\text{if } \vdash \forall x(A(x) \vee \neg A(x)) \text{ and } \vdash \neg\neg\exists x A(x), \text{ then } \vdash \exists x A(x) \ ,$$

which is the formulation of Markov's principle as a rule, and that HA enjoys disjunction property (DP) and numerical existence property (n-EP)[1]:

(DP): if $\mathbf{L} \vdash A \vee B$, then $\mathbf{L} \vdash A$ or $\mathbf{L} \vdash B$.
(n-EP): if $\mathbf{L} \vdash \exists x A(x)$, then there exists a numeral $\overline{m}$ such that $\mathbf{L} \vdash A(\overline{m})$
(cf. Kleene [1], Troelstra[6]).

The properties (DP) and (n-EP) are regarded as distinguishing features and characteristics of constructivity of HA (and other intuitionistic theories). In this talk, we consider the axiom schematic counterpart MP to Markov's principle:

$$MP: \quad \forall x(A(x) \vee \neg A(x)) \wedge \neg\neg\exists x A(x) \supset \exists x A(x) \ .$$

and show that the intermediate predicate logic obtained from intuitionistic predicate logic by adding MP as an additional axiom schema enjoys DP as well as the existence property (EP):

(EP): if $\mathbf{L} \vdash \exists x A(x)$, then there exists a v such that $\mathbf{L} \vdash A(v)$.

Since the language of intermediate predicate logics has neither individual constants nor function symbols, the above v is chosen from individual variables.

We discuss finitely axiomatizable extensions of intuitionistic predicate logic in the setting of intermediate predicate logics which enjoy DP and EP, and obtain the above result as a corollary of our main theorem stated in terms of Kripke semantics for intermediate predicate logics. That is, we show that if an axiom schema satisfies a semantical condition described in terms of Kripke semantics, then the intermediate predicate logic axiomatized by this axiom schema relative to intuitionistic predicate logic enjoys DP and EP, and MP is shown to satisfy this semantical sufficient condition.

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[1] Also known as *explicit definability property*.

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On the computational power of constant-depth exact quantum circuits

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Abstract. We study the computational power of constant-depth polynomial-size exact quantum circuits with unbounded fan-out gates, which are called $\mathsf{QNC}^0_\mathsf{f}$ circuits. Our main result is that there exists a $\mathsf{QNC}^0_\mathsf{f}$ circuit for the OR function. This is an affirmative answer to the question of Høyer and Špalek [1]. In sharp contrast to the strict hierarchy of the classical complexity classes: $\mathsf{NC}^0 \subsetneq \mathsf{AC}^0 \subsetneq \mathsf{TC}^0$, our main result with Høyer and Špalek's one implies the collapse of the hierarchy of the corresponding quantum classes: $\mathsf{QNC}^0_\mathsf{f} = \mathsf{QAC}^0_\mathsf{f} = \mathsf{QTC}^0_\mathsf{f}$. As an application of our main result, we show that, under a plausible assumption, there exists a classically hard problem that is solvable by a $\mathsf{QNC}^0_\mathsf{f}$ circuit with gates for the quantum Fourier transform. This talk is based on a joint work with Seiichiro Tani [2].

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Set-theoretic geology with large cardinals

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Laver [2] and Woodin showed that a ground model is definable in its forcing extension, and Fuchs-Hamkins-Reitz [1] proved the ground models can be defined uniformly.

Theorem 1 ([1]). *There is a formula $\varphi(r,x)$ such that:*

1. *For every r, the class $W_r = \{x : \varphi(r,x)\}$ is a* ground *of the universe V, that is, W_r is a transitive model of ZFC such that there are a poset $\mathbb{P} \in W_r$ and a $(W_r, \mathbb{P})$-generic filter G with $V = W_r[G]$.*
2. *For every ground M of V, there is r such that $W_r = M$.*

Now the study of the structure of $\{W_r : r \in V\}$ is called *set-theoretic geology.*

Theorem 2 ([1], Reitz [3]).

1. *There is a class forcing which forces "the universe V has no proper grounds".*
2. *Conversely, there is a class forcing which forces "there are class many proper grounds".*

A class forcing in the item 1. preserves almost every large cardinals, and a class forcing in 2. preserves supercompact cardinals. However, it does not preserves large cardinals stronger than supercompact cardinals.

In this talk, we show that the statement that "there are class many proper grounds" is actually inconsistent with some large cardinals.

Definition 1. *Let $n \in \omega$. We say that δ is n-*supercompact *if for every $\lambda \geq \delta$, there is an elementary embedding $j : V \to M$ into some inner model M such that the critical point of j is δ, $\lambda < j(\delta)$, and ${}^{j^n(\lambda)}M \subseteq M$. Where $j^k(\alpha)$ is defined as follows: $j^0(\alpha) = \alpha$, and $j^{k+1}(\alpha) = j(j^k(\alpha))$.*

Clearly 0-supercompactness is the same to the usual supercompactness, and a 1-supercompact cardinal is superhuge and extendible.

Theorem 3. *Suppose there is a 1-supercompact cardinal δ. Then the following hold:*

1. *For every ground M of the universe V, there are a poset $\mathbb{P} \in M \cap V_\delta$ and an $(M, \mathbb{P})$-generic G such that $V = M[G]$. That is, V must be a small forcing extension of M.*
2. *In particular, V has at most δ many proper grounds.*
3. *Moreover if $V = \mathrm{HOD}$ holds, then V has a minimum ground.*

As an immediate corollary of this theorem, we have a following extreme destructibility of 1-supercompact cardinals:

Theorem 4. *Let δ be an infinite cardinal. If a poset $\mathbb{P}$ is not forcing equivalent to a poset of size $< \delta$, then $\mathbb{P}$ forces that "δ is not a 1-supercompact cardinal".*

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Reverse Mathematics and Equilibria of Continuous Games

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Nash equilibrium is one of the most important notions in game theory. In particular, we are interested in complexity of equilibrium. In this talk, we mainly discuss the proof-theoretic strength of the existence of Nash equilibria for continuous games as mixed strategies. First, we think how to formalize a mixed strategy in a weak subsystem of second order arithmetic. Next, we show that Glicksberg's theorem can be proved in ACA_0, modifying some well-known proof of the theorem. Then, we will introduce some reverse mathematical results which appear in the proof.

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Reverse Mathematics and Whitehead Groups

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I will speak about joint works with Frank Stephan (NUS, Singapore), Yang Sen (Inner Mongolia University, China) and Yu Liang (Nanjing University, China). During the IMS-JSPS workshop at National University of Singapore in September 2014, Yang Sen has reported that over the base theory of WKL_0, Stein Theorem is equivalent to ACA_0, where Stein Theorem says every countable Whitehead group is free. This talk is a continuation of Yang Sen's talk and in particular we will look at what happens if we move the base theory to RCA_0.

A simple conservation proof for ADS

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Deciding the proof-theoretic strength of Ramsey's theorem for pairs is a long-term open problem in the study of reverse mathematics. In [1], Chong, Slaman and Yang showed that two combinatorial principles called CAC and ADS, which are both important consequences of Ramsey's theorem for pairs, are both Π^1_1-conservative over $\mathrm{B}\Sigma^0_2$. Thus their proof-theoretic strength are equivalent to $\mathrm{I}\Sigma^0_1$. The proof is rather complicated since they use some recursion theoretic method in nonstandard models. Using these ideas, Kreuzer[2] give a more proof-theoretic proof for this result, and showed that CAC is Π^0_2-conservative over $\mathrm{B}\Sigma^0_2$ even in the higher-order setting. In this talk, I will introduce a simpler proof for a weaker statement that ADS is Π^0_2-conservative over $\mathrm{B}\Sigma^0_2$. The proof is based on the classical Paris argument [3], and just directly calculate the increasing speed of the indicator for ADS.

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