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Theoretical Analysis and Model Construction for Optimal R&D Investment Control

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Abstract

A hypothetical view is postulated on the basis of the observation of structural change in Japan's techno-economic behavior. In order to construct a virtuous cycle trajectory between R&D investment, technology stock and economic growth, R&D investment decision making has become a crucial issue. An optimal R&D investment model based on the optimal control theory postulated by Pontryagin is constructed.

1. Introduction

R&D is a key determinant of long-run productivity and consumer welfare. According to the observation of structural change in Japan's techno-economic behavior, there is a fear to vicious cycle between R&D investment, technology stock and economic growth. Therefore, R&D investment decision making has become a crucial issue. Furthermore, this decision is difficult because of the complex interrelationships among governing factors of industry R&D. A number of studies have analyzed R&D contribution to growth[1]. However, there are hardly satisfactory in identifying optimal R&D investment trajectory.

In this paper, section 2 empirically examines the hypothetical view of the fear to vicious cycle between R&D and growth. Section 3 constructs an optimal R&D investment control model and provides the analytic solution of the model. Section 4 briefly summarizes the conclusions.

2. Examination of the Fear to Vicious Cycle between R&D and Growth

Currently, the stagnation of technology development has become a crucial structural problem common to many advanced economies [2]. Similarly, Japan has been suffering from a collapse of its long lasting "virtuous cycle" between technology development and economic growth [6]. The structural stagnation of Japanese industry's R&D activities can be demonstrated by trends in change rate of R&D intensity in major sectors of its manufacturing industry (MA: manufacturing average; FD: food; PM: primary metals; CH: chemicals; and EM: electrical machinery) over the period 1975-1996 as shown in **Fig. 1**.

Another noteworthy trends in the Japanese manufacturing industry's techno-economic behavior under increasing technology spillover [7] can be observed in the rise and fall of marginal productivity of technology, stagnation of technology substitution for scarce resources and stagnation of assimilation capacity (*AC:* the ability to utilize spillover technology).

Fig. 2 illustrates trends in marginal productivity of technology (*MPT*) in three of Japan's leading manufacturing industries (EM, CH and PM) over the period 1960-1997 [8]. We note that *MPT* is sensitive to economic circumstances in respective period.

Fig. 3 illustrates trends in the elasticity of technology substitution for labor (*TSL*) in Japan's manufacturing industry (MA) and the same three leading sectors (EM, CH and PM) over the period 1981-1997 [8]. Fig. 3 demonstrates that *TSL* started to decrease in the 1980s and continued to decrease in the 1990s.

Fig. 4 illustrates trends in assimilation capacity for leading sectors (EM, CH and PM) over the period 1981-1995 [7]. We note that *AC* in EM and PM increased before the bubble economy in 1987. However, this changed to a dramatic decrease starting from the period of the bubble economy. While assimilation capacity of CH continues to decline from 1983.

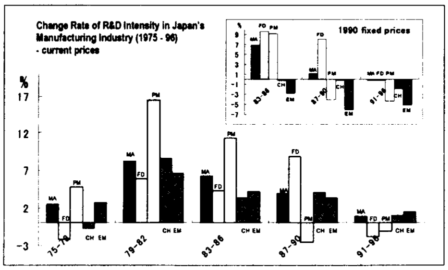


Fig. 1 Trends in Change Rate of R&D Intensity in Japan's Manufacturing Industry (1975-1996) - %

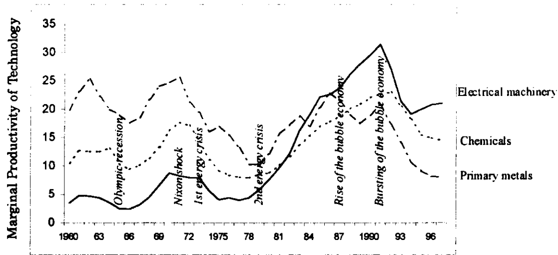


Fig. 2 Trends in *MPT* in Japan's Major Manufacturing Industries (1960-1997) - %

[1] See details of relevant existing works in [9].

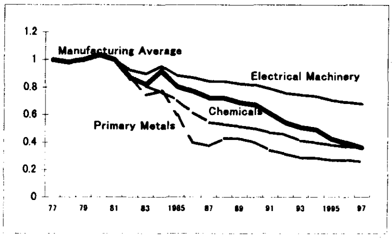


Fig. 3 Trends in *TSL* in Japan's Major Manufacturing Industries (1981-1997): Index: 1981 = 1

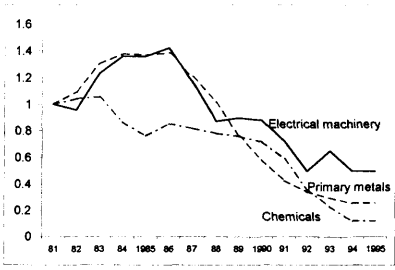


Fig. 4 Trends in Assimilation Capacity of Japan's Major Manufacturing Industries (1981-1995) - Index: 1981 = 1

By combining with some empirical analyses, these observations suggest the following hypothetical view:

(i) R&D intensity (*r/y* or *r/V*), the marginal productivity of technology (*MPT*), technology substitution for labor (*TSL*), and assimilation capacity (*AC*) correlate with each other constructing a comprehensive subtle system as illustrated in **Fig. 5**.

(ii) This system has both possibilities leading to virtuous or vicious spin cycle between R&D and growth.

(iii) R&D intensity (*r/y*) plays a trigger role deciding this trajectory.

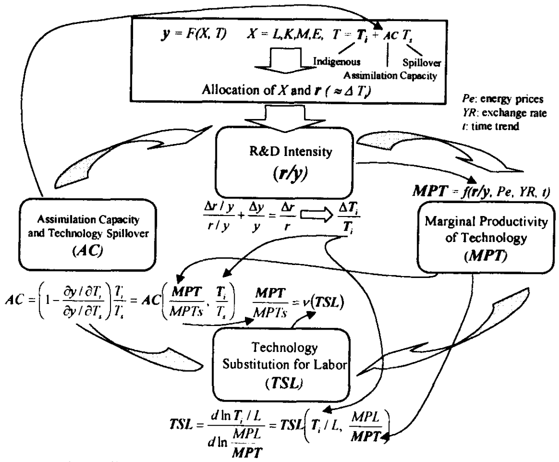
(iv) Due to its stagnation, empirical analyses demonstrate a strong fear of vicious spin cycle between R&D and growth.

(v) Therefore, in order to avoid this fear, optimal R&D intensity (*r/y*) control has become critical.

3. Construction and Solution of the Optimal R&D Investment Control

3.1 The System Model

To construct the dynamic model of manufacturing and R&D investment, the following variables are used: $y = y(t)$: manufacturing production; t: time trend; $\dot{y}/y$: change rate of production where $\dot{y} = dy/dt$; $T = T(t)$: technology knowledge stock (accumulated R&D investment r); $\dot{T} \approx r = r(t)$: change in technology knowledge stock (approximated by R&D investment); r/y: R&D intensity; $X (=L, K, M, E)$: production factors



Relationship between *r/V* and *MPT* in Japan's Manufacturing Industry (1975-1996)	adj. R^2	DW
$\ln MPT = 83.30 + 2.90 \ln r/V + 0.21 \ln Pe + 0.33 \ln YR - 0.04t$ (3.51) (14.77) (4.04) (2.15) (-3.27)	0.962	2.06

Governing Factors of *AC* in Japan's Leading Manufacturing Industries (1981-1995)		adj. R^2	DW
Electrical machinery	$\ln ACem = -3.71 + 0.97 \ln MPTem + 4.25 \ln TSLem$ (7.90) (14.24)	0.940	2.38
Chemicals	$\ln ACch = -12.10 + 2.73 \ln MPTch + 3.62 \ln TSLch$ (12.23) (20.25)	0.971	1.57
Primary metals	$\ln ACpm = -6.33 + 1.69 \ln MPTpm + 0.52 \ln TSLpm$ (9.55) (4.49)	0.948	2.55

Relationship between *r/V* and *V* in Japan's Manufacturing Industry (1975-1996)	adj. R^2	DW
$\ln r/V = -9.40 + 0.53 \ln V_{-1} + 0.18 \ln MPT_{-1}$ (-25.27) (13.56) (5.02)	0.978	1.20

Fig. 5 Schematic Representation of the Relation between R&D Intensity, *MPT, TSL* and *AC*

(labor, capital, materials and energy), which involve both factors for manufacturing and R&D; and X_T $(=L_T, K_T, M_T, E_T)$: factors input directing to R&D.

The classical production function is used to construct dynamics as follows:

$$y = F(t, (L - L_T), (K - K_T), (M - M_T), (E - E_T), T) \quad (1)$$

Assume that the functional dependence between the L_T, K_T, M_T, E_T and the accumulated R&D investment T is given by function of substitution type:

$$T = T(L_T, K_T, M_T, E_T) = \min\{h_1(L_T), h_2(K_T), h_3(M_T), h_4(E_T)\} \quad (2)$$

and the inverse relations exist

$$L_T = L_T(T) = h_1^{-1}(T), \qquad K_T = K_T(T) = h_2^{-1}(T)$$
$$M_T = M_T(T) = h_3^{-1}(T), \qquad E_T = E_T(T) = h_4^{-1}(T) \quad (3)^2$$

Differentiating (1) by time t and taking into account (3),

$$\frac{\dot{y}}{y} \approx \frac{\partial F}{\partial t}\frac{1}{y} + \sum \frac{\partial F}{\partial X}\frac{X}{y}\frac{\dot{X}}{X} - \sum \frac{\partial F}{\partial X}\frac{\partial X_T}{\partial T}\frac{\dot{T}}{y} + \frac{\partial F}{\partial T}\frac{\dot{T}}{y} \quad (4)$$

Here we can approximately treat $\dot{T} \approx r$. In line with the previous approach [5], technology knowledge stock in time t, T_t can be measured as follow:

$$T_t = r_{t-m} + (1-\rho)\, T_{t-1} \quad (5)$$
$$T_0 = r_{1-m} / (\theta + \rho) \quad (6)$$

[2] The rationality of the existence of this reverse relation has been checked by using the empirical analysis in Japan's manufacturing industry [9].

where r_{t-m}: R&D expenditure in time $t-m$; m: time lag between R&D and commercialization; ρ: rate of obsolescence of technology; and θ: increase rate of r.

Using $\dot{T} \approx r$ and rewrite (4) in the form of:

$$\frac{\dot{y}}{y} = f - p\frac{r}{y} + q\frac{r}{y} \quad (7)$$

where terms related to the production factors X (=L, K, M, E), effects of institutional change (e.g. learning and scale effects) are combined into function f

$$f = \frac{\partial F}{\partial t}\frac{1}{y} + \sum \frac{\partial F}{\partial X}\frac{X}{y}\frac{\dot{X}}{X} \quad (8)$$

decrease in manufacturing due to R&D spending X_T (=L_T, K_T, M_T, E_T) is collected into function p

$$p = p(t) = \sum \frac{\partial F}{\partial X}\frac{\partial X_T}{\partial T} \quad (9)$$

increase in manufacturing by technology knowledge stock is described by the marginal productivity of technology q

$$q = q(t) = \frac{\partial F}{\partial T} \quad (10)$$

the control parameter r stands for change in technology knowledge stock $\dot{T}$.

Collecting the terms $(r/y)p$, $(r/y)q$ which depend on the control parameter r into the net contribution by R&D intensity $(r/y)g$, the equation for the dynamic control process can be obtained as follow:

$$\frac{\dot{y}}{y} = f - g\frac{r}{y} \quad (11)$$

where $g = g(t) = p(t) - q(t) > 0$ (12)

3.2 Utility of the System Trajectory

In order to formalize the goal for designing the control parameter $r = r(t)$ and indicate the profit of R&D investment in the long-run, the utility function represented by the present value of the consumption of the invented products[3] with the discount coefficient η is considered.

$$U_t = \int_t^{\infty} e^{-\eta(s-t)} \ln D(s) ds \quad (13)$$

$$D = D(s) = \left(\int_0^n x^{\alpha}(j) dj \right)^{1/\alpha}, \quad n = n(s) \quad (14)$$

$$y = n \cdot x(j), \; y = y(s), \; n = n(s) \quad (15)$$

$$n = n(s) = br^{\beta_1}T^{\beta_2}, \; r = r(s), \; T = T(s) \quad (16)$$

where $D(s)$: demand function; s: running time; t: the initial time; j: current index of innovative goods; $x(j)$: consumption of brand j innovative goods; $n(s)$: number of available varieties at time s; α: parameter of elasticity of substitution between any two innovative goods (ε, $\varepsilon = 1/(1-\alpha)$); and β_1, β_2: elasticities of r and T to n.

[3] Consumer behaves to taste for diversity in consumption represented by number of available varieties to which technology contributes to increase [1].

Combining (14), (15) and (16), the following demand function can be obtained:

$$D(s) = \left[\int_0^n \left(\frac{y}{n}\right)^{\alpha} dj \right]^{1/\alpha} = \frac{y}{n}(n)^{1/\alpha} = y \cdot n^{\frac{1-\alpha}{\alpha}} \quad (17)$$

From equation (5) we can get following formula:

$$T_t - T_{t-1} = -\rho T_{t-1} + r_{t-m} \quad (18)$$

When t is long enough to satisfy $t > t-1 >> m-1$,

$$\Delta T = -\rho T + r \qquad (\Delta T: dT/dt) \quad (19)$$

Solve the differential equation (19),

$$T(t) = \frac{r_t}{\theta + \rho} + e^{-\rho(t-t0)} T_0 (1 - e^{\theta(t0-1+m)}) \quad (20)$$

where θ: the average increase rate of $r(\tau)$.

Under the condition $t_0 - 1 + m \approx 0$

$$T(t) \approx \frac{r_t}{\theta + \rho} \quad (21)$$

$$\therefore n = n(s) = br^{\beta_1}T^{\beta_2} = br^{\beta_1}\left(\frac{r}{\theta+\rho}\right)^{\beta_2} \quad (22)$$

$$\ln n = (\ln b - \beta_2 \ln(\theta+\rho)) + (\beta_1+\beta_2) \ln r \quad (23)$$

Combining (13), (17) and (23), the following expression for the utility function can be obtained:

$$U_t = \int_t^{\infty} e^{-\eta(s-t)} (\ln y + \frac{1-\alpha}{\alpha}((\ln b - \beta_2 \ln(\theta+\rho)) + (\beta_1+\beta_2)\ln r)) ds \quad (24)$$

3.3 The Analytic Solution of the Model

The Pontryagin's maximum principle [3] is used to solve the classical optimal control problem constructed by dynamics (12) and utility function (24). The main elements in the analysis are Hamiltonian H and the adjoint variable ψ.

The Hamiltonian has the form

$$H(y,r,\psi) = \ln y + \frac{1-\alpha}{\alpha}((\ln b - \beta_2 \ln(\theta+\rho)) + (\beta_1+\beta_2)\ln r) + \psi(fy - gr) \quad (25)$$

and represents the utility flow.

Its maximum by parameter r is determined by

$$\frac{\partial H}{\partial r} = \frac{1-\alpha}{\alpha}(\beta_1+\beta_2)\frac{1}{r} - g\psi = 0 \quad (26)$$

So its maximum value is attained at the optimal R&D investment r^0

$$r^0 = \frac{1-\alpha}{\alpha}\frac{\beta_1+\beta_2}{g\psi} \quad (27)$$

ψ is marginal price of production y expressed as $\psi = \partial W / \partial y$ where W is optimal value.

Halmiton-Jacobi equation depicts that a trajectory of the optimal position can be expressed

$$\frac{\partial W}{\partial t} + H(y,r,\frac{\partial W}{\partial y}) = \frac{\partial W}{\partial t} + H(y,r,\psi) = 0 \quad (28)$$

Optimal trajectory with respect to y is

$$\frac{\partial}{\partial y}\left(\frac{\partial W}{\partial t}\right)+\frac{\partial H}{\partial y}=\frac{\partial}{\partial t}\left(\frac{\partial W}{\partial y}\right)+\frac{\partial H}{\partial y}=\frac{\partial \psi}{\partial t}+\frac{\partial H}{\partial y}=0 \quad (29)$$

Utility function (24) requires the following Hamiltonian in addition to the Hamiltonian (25):

$$H^*(y,r,\psi^*)=e^{-\eta(s-t)}(\ln y+\frac{1-\alpha}{\alpha}((\ln b-\beta_2\ln(\theta+\rho)) + (\beta_1+\beta_2)\ln r))+\psi^*(fy-gr) \quad (30)$$

Under the optimal trajectory condition $\frac{\partial H}{\partial r}=\frac{\partial H^*}{\partial r}=0$,

$$\frac{\partial H}{\partial y}=\frac{1}{y}+f\psi=0 \quad (31)$$

$$\frac{\partial H^*}{\partial y}=e^{-\eta(s-t)}\frac{1}{y}+f\psi^*=0 \quad (32)$$

$$\therefore \psi^*=e^{-\eta(s-t)}\psi \quad (33)$$

$$\frac{\partial H^*}{\partial y}=e^{-\eta(s-t)}\left(\frac{1}{y}+f\psi\right)=e^{-\eta(s-t)}\frac{\partial H}{\partial y} \quad (34)$$

Assume y in an optimal trajectory in (29),

$$\frac{\partial H^*}{\partial y}=-\frac{\partial \psi^*}{\partial t}=-\left(-\eta e^{-\eta(s-t)}\psi+e^{-\eta(s-t)}\dot{\psi}\right) \quad (35)$$

From equations (34) and (35),

$$\frac{\partial H}{\partial y}=\eta\psi-\dot{\psi} \quad (36)$$

Therefore, for dynamics of the conjugate variable ψ one can compose the adjoint equation:

$$\dot{\psi}=\eta\psi-\frac{\partial H}{\partial y}=\eta\psi-\frac{1}{y}-f\psi \quad (37)$$

Combining equations (12) and (26), and changing (37), the following closed system of differential equations are obtained:

$$\frac{\dot{y}}{y}=f-\frac{1-\alpha}{\alpha}(\beta_1+\beta_2)\frac{1}{y\psi} \quad (38)$$

$$\frac{\dot{\psi}}{\psi}=\eta-\frac{1}{y\psi}-f \quad (39)$$

Introducing notation $z=y\psi$ for the production cost and summarizing equations (38) and (39) the following differential equation is obtained:

$$\dot{z}=\eta z-\left[\frac{1-\alpha}{\alpha}(\beta_1+\beta_2)+1\right] \quad (40)$$

By solving this differential equation, the following equation can be obtained:

$$z=z(t)=\frac{1}{\eta}\cdot\left[\frac{1-\alpha}{\alpha}(\beta_1+\beta_2)+1\right] \quad (41)$$

Substituting solution (41) into optimal control (27), the relation between the optimal investment r and the optimal production y is obtained:

$$r=\frac{1}{\varepsilon-1+(\beta_1+\beta_2)}\cdot\frac{\eta}{g}y \quad (42)$$

In case the number of available varieties $n(s)$ in equation (16) is under constant returns to scale with respect to r and T, $\beta_1+\beta_2=1$[4].

Under these conditions:

$$\frac{r}{y}=\frac{\eta}{\varepsilon g} \quad (43)$$

Equation (43) suggests that the optimal R&D intensity depends on the elasticity of substitution ε, the discount rate η and the discounted marginal productivity of technology g, and its level increases as ε and g decrease and η increases.

4. Concluding Remarks

(i) Increasing significance of optimal R&D control is identified by demonstrating the stagnation of R&D intensity, marginal productivity of technology, technology substitution for scarce resources and a decrease in assimilation capacity leading to a vicious cycle between R&D and growth.

(ii) On the basis of a concept of constructing a virtuous cycle trajectory between R&D investment, technology stock and economic growth, a R&D investment model based on the optimal control theory postulated by Pontryagin is constructed to satisfy customer's tastes for diversity in consumption.

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