日本学術振興会研究拠点形成事業

第 4 回数理論理学とその応用に関するワークショップ

JSPS Core-to-Core Program

Fourth Workshop on Mathematical Logic and its Applications

2020 年 3 月 3 日-5 日
金沢東急ホテル
March 3 – 5, 2020,
Kanazawa Tokyu Hotel

Timetable

Tuesday, 3 March	Wednesday, 4 March	Thursday, 5 March
9:30 -10:00 Registration		
10:00 -10:10 Opening		
10:10 - 10:50 Hidenori Kurokawa	10:10 - 10:50 Makoto Fujiwara	10:10 - 10:50 Kiraku Shintani Nao Hirokawa
10:50 - 11:30 Youan Su Katsuhiko Sano	10:50 - 11:30 Takayuki Kihara	10:50 - 11:30 Makoto Tatsuta Stefano Berardi
Lunch break	Lunch break	Lunch break
14:00 - 14:40 Franz Braußе Konstantin Korovin Margarita Korovina Norbert Müller	14:00 - 14:40 Victor Selivanov	14:00 - 14:40 Daisuke Kimura Koji Nakazawa
14:40 - 15:20 Norbert Müller Margarita Korovina Franz Braußе	14:40 - 15:20 Matthew de Brecht	14:40 - 15:20 Koji Nakazawa Kenji Saotome Daisuke Kimura
Coffee break	Coffee break	Coffee break
15:50 - 16:30 Daichi Hayashi	15:50 - 16:30 Donghyun Lim Martin Ziegler	15:50 - 16:30 Dieter Spreen
16:30 - 17:10 Satoru Niki	16:30 - 17:10 Michal Konečný Florian Steinberg Holger Thies	16:30~ Closing
	18:30 - 21:00 Social dinner	

The social dinner is organized at Amatsubo (4-7 Kakinokibatake).
See https://www.amatsubo.com for how to get there.

Ksmt for solving non-linear constraints*

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We give a detailed overview of the **ksmt** calculus [BKKM19] developed in a conflict driven clause learning framework for checking satisfiability of non-linear constraints over the reals. The **ksmt** calculus successfully integrates strengths of symbolical and numerical methods. The key steps of the decision procedure based on this calculus contain assignment refinements, inferences of linear resolvents driven by linear conflicts, backjumping and constructions of local linearisations of non-linear components initiated by non-linear conflicts. In [BKKM19] we showed that the procedure is sound and makes progress by reducing the search space. This approach is applicable to a large number of constraints involving computable non-linear functions, piecewise polynomial splines, transcendental functions and beyond. In this setting we discuss resent and future research work.

References

[BKKM19] F. Brauße, K. Korovin, M. Korovina, and N. Th. Muller: *A CDCL-style calculus for solving non-linear constraints.* In: *FroCoS 2019, LNCS (LNAI) vol. 11715*, pp 131–148, 2019

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Generalizing Taylor models for multivariate real functions *

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We discuss data structures and algorithms for the approximation of multivariate real functions $f :\subseteq \mathbb{R}^k \to \mathbb{R}$. From the viewpoint of TTE[Wei00], these approximations can be viewed as building blocks for representations. To this end, we apply ideas from the field of Taylor models [MB01], thus generalizing the approach taken in [BKM15]: On domains given as polytopes, functions are approximated by polynomials with (possibly unbounded) interval coefficients. In [DFKT14], there has been a related approach of a 'function interval arithmetic', still lacking the aspects of data reduction in Taylor models.

As an application we aim at the field of SMT solving and present a prototypical implementation: From a (symbolically defined) function f and a value $\boldsymbol{c}$ with $f(\boldsymbol{c}) > 0$ it derives a polytope P and a linear g with $\boldsymbol{c} \in P$, $g(\boldsymbol{c}) > 0$ and $\forall x \in P : f(\boldsymbol{x}) > g(\boldsymbol{x})$ thus separating the graph of f and the point $(\boldsymbol{c}, 0)$. This property is a core requirement for recent CDCL-style SMT solvers [BKKM19,CGI+18].

References

BKKM19. F. Brauße, K. Korovin, M. Korovina, and N. Th. Müller: *A CDCL-style calculus for solving non-linear constraints.* In: *FroCoS 2019, LNCS (LNAI) vol. 11715*, pp 131–148, 2019

BKM15. F. Brauße, M. Korovina, and N. Th. Müller: *Towards using exact real arithmetic for initial value problems.* In: *PSI: 10th Ershov Informatics Conference, LNCS vol. 9609,* pp. 61-75, 2015

CGI+18. A. Cimatti, A. Griggio, A. Irfan, M. Roveri, and R. Sebastiani: *Experimenting on solving nonlinear integer arithmetic with incremental linearization.* In: *Proceedings SAT 2018 - 21st International Conference*, pages 383–398, 2018.

DFKT14. J. Duracz, A. Farjudian, M. Konecný, and W. Taha: *Function interval arithmetic.* In: *Mathematical Software - ICMS 2014 - 4th International Congress, Seoul, South Korea, August 5-9, 2014. Proceedings*, pages 677–684, 2014.

MB01. K. Makino and M. Berz. *Higher order verified inclusions of multidimensional systems by taylor models.* In *Nonlinear Analysis: Theory, Methods & Applications*, 47(5):3503 – 3514, 2001. Proc. of the Third World Congress of Nonlinear Analysts.

Wei00. K. Weihrauch. *Computable analysis: an introduction.* Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2000.

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On the transferability of results between subcategories of spaces and locales

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Let $\mathbf{\Omega}\colon \mathbf{Top} \to \mathbf{Loc}$ be the usual functor mapping topological spaces to locales. It is well known that $\mathbf{\Omega}$ restricts to an equivalence between the category of sober spaces and the category of spatial locales, but this does not mean that there is an equivalence between topological results on sober spaces and locale theoretical results on spatial locales. For example:

(1) $(\mathbb{Q}, +)$ is a topological subgroup of $(\mathbb{R}, +)$.
(2) Every localic subgroup of a localic group is a closed sublocale [4].

It follows that $(\mathbb{Q}, +)$ is *not* a localic subgroup of $(\mathbb{R}, +)$, even though $\mathbb{Q}$ and $\mathbb{R}$ are included in the sober space $\sim$ spatial locale categorical equivalence. These discrepancies can occur because $\mathbf{\Omega}$ does not preserve products, and the existence of group operations such as $+\colon \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$ depends on the product structure.

A further restriction of $\mathbf{\Omega}$ yields an equivalence between the category **QPol** of quasi-Polish spaces [1] and the category of countably presented locales [3]. Under this restriction, $\mathbf{\Omega}$ now preserves all countable limits, and the categorical equivalence starts to look more like an actual equivalence:

(3) Every quasi-Polish subgroup of a quasi-Polish group is a closed subspace.

Are there any extensions of **QPol** where the equivalence between spaces and locales still behaves so well? At least for countably based spaces we have a partial answer. Based on the results presented in [2], we argue that **QPol** is the largest "reasonable" subcategory of countably based spaces where we can hope for such a natural transfer of results between topology and locale theory.

References

1. M. de Brecht, *Quasi-Polish spaces*, Annals of Pure and Applied Logic **164** (2013), 356–381.
2. M. de Brecht, *A note on the spatiality of localic products of countably based sober spaces*, presented at the workshop CCC 2019: Computability, Continuity, Constructivity - From Logic to Algorithms (2019), https://www.fmf.uni-lj.si/~simpson/CCC2019_abstracts.pdf.
3. R. Heckmann, *Spatiality of countably presentable locales (proved with the Baire category theorem)*, Math. Struct. in Comp. Science **25** (2015), 1607–1625.
4. J. Isbell, I. Kříž, A. Pultr, and J. Rosický, *Remarks on localic groups*, Categorical Algebra and its Applications, Springer (1988), 154–172.

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König's lemma and the decidable fan theorem in reverse mathematics

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König's lemma KL states that any infinite finitely-branching tree has an infinite path. Weak König's lemma WKL is the restriction of KL to infinite binary trees. It is well-known in classical reverse mathematics [2] that KL is strictly stronger than WKL over a subsystem RCA of second-order arithmetic.

The so-called fan theorem is first studied in Brouwer's intuitionistic mathematics, which is inconsistent with classical logic since a continuity principle is involved in the assertion. In modern constructive mathematics, the decidable fan theorem $\mathrm{FAN_D}(T_{01})$ which is consistent with classical logic plays an important role, and it states that for any decidable predicate A, if $\forall \alpha \in \{0,1\}^{\mathbb{N}} \exists n^{\mathbb{N}} A(\overline{\alpha}n)$, then $\exists m^{\mathbb{N}} \forall \alpha \in \{0,1\}^{\mathbb{N}} \exists n \leq m\, A(\overline{\alpha}n)$, where $\overline{\alpha}n$ denotes the finite sequence consisting of the initial segment of α up to length n. As mentioned in [3, Section 4.7.5], $\mathrm{FAN_D}(T_{01})$ is constructively equivalent to a generalized version $\mathrm{FAN_D}$ which states that for any finitely-branching spread T and any decidable predicate A, if $\forall \alpha \in T \exists n^{\mathbb{N}} A(\overline{\alpha}n)$, then $\exists m^{\mathbb{N}} \forall \alpha \in T \exists n \leq m\, A(\overline{\alpha}n)$. Here a spread is an extended notion of infinite trees, and the complete binary tree $\{0,1\}^{\mathbb{N}}$ is a particular instance of spreads. If one formalizes $\mathrm{FAN_D}$ and $\mathrm{FAN_D}(T_{01})$ as schemata with quantifier-free predicates A over intuitionistic analysis EL [3, Section 3.6], then the former is a sort of contrapositive of KL while the latter is so for WKL (cf. [3, Section 4.7.2]).

Motivated by this incomprehensible situation, we examine the proper relation between KL, WKL, $\mathrm{FAN_D}$ and $\mathrm{FAN_D}(T_{01})$ over (some fragment of) EL which contains the quantifier-free number-number choice $\mathrm{QF\text{-}AC}^{0,0}$ only. By such a fine-grained investigation in the spirit of constructive reverse mathematics [1], as a corollary, we have that KL and WKL are equivalent to $\mathrm{FAN_D}$ and $\mathrm{FAN_D}(T_{01})$ respectively over the classical counterpart RCA of EL. Then it follows from the above mentioned fact in classical reverse mathematics that some strong choice principle is necessary for deriving $\mathrm{FAN_D}$ from $\mathrm{FAN_D}(T_{01})$ constructively.

References

1. Ishihara, H.: Constructive reverse mathematics: compactness properties. In: From sets and types to topology and analysis, Oxford Logic Guides, vol. 48, pp. 245–267. Oxford Univ. Press, Oxford (2005)
2. Simpson, S.G.: Subsystems of second order arithmetic. Perspectives in Logic, Cambridge University Press, Cambridge, second edn. (2009)
3. Troelstra, A.S., van Dalen, D.: Constructivism in mathematics, An introduction, Vol. I, Studies in Logic and the Foundations of Mathematics, vol. 121. North Holland, Amsterdam (1988)

On two theories of Frege structure equivalent to Feferman's T_0

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Explicit mathematics (EM) has been introduced by Feferman ([4]) as a framework of Bishop's constructive mathematics. Especially, Feferman gives an impredicative theory T_0 of EM which has the *inductive generation* axioms (IG). As a similar framework of set theory, Aczel's *Frege structure* (FS) ([1]) has been studied as truth theories over applicative theories (cf. e.g. [2]). As Aczel already expected ([1]), EM and FS as formal theories are closely related, that is, for many variants of EM we can find a proof-theoretically equivalent one of FS (cf. e.g. [6]).

The purpose of this talk is to explore the further correspondence between proof-theoretically stronger formulations of EM and FS. In this talk, we especially give two theories of FS and show that they are proof-theoretically equivalent to T_0. Firstly, we formulate a truth theory corresponding to the theory LUN of Jäger et al. ([5]), which can be in a sense regarded as an extension of Burgess-Kripke-Feferman truth theory KF_μ ([3]). Secondly, we extend Cantini's supervaluation-style truth theory (cf. [2]) by the *limit axiom* (cf. e.g. [6]).

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References

1. P. Aczel, *Frege Structures and the Notions of Proposition, Truth and Set*, The Kleene Symposium ed by J. Barwise, H. Keisler, K. Kunen, vol. 101, North-Holland, 1980, pp. 31–59.
2. A. Cantini, *Logical Frameworks for Truth and Abstraction*, North-Holland, Amsterdam, 1996.
3. A. Cantini, *About truth and types*, Advances in Proof Theory, Progress in Computer Science and Applied Logic ed by R. Kahle, T. Strahm, T. Studer, Birkhäuser, 2016, pp. 31–64.
4. S. Feferman, *A language and axioms for explicit mathematics*, Algebra and Logic, Lecture Notes in Mathematics, ed by J. Crossley, Springer, Berlin, 1975, pp. 87–139.
5. G. Jäger, R. Kahle, T. Studer, *Universes in explicit mathematics*, Annals of Pure and Applied Logic, vol. 109 (2001), no. 3, pp. 141–162.
6. R. Kahle, *Truth in applicative theories*, Studia Logica, vol. 68 (2001), no. 1, pp. 103–128.

Some Lifschitz-like realizability notions separating non-constructive principles*

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Abstract. There is a way of assigning a realizability notion to each degree of incomputability. In our setting, we make use of Weihrauch degrees (degrees of incomputability of partial multi-valued functions) to obtain Lifschitz-like relative realizability toposes. In particular, we lift some separation results on Weihrauch degrees to those over intuitionistic Zermelo-Fraenkel set theory **IZF**.

Keywords: Realizability topos · Constructive reverse mathematics · Weihrauch degree.

This is a contribution to *constructive reverse mathematics* initiated by Ishihara. Here we do not include the axiom of countable choice $\mathsf{AC}_{\omega,\omega}$ in our base system of constructive reverse mathematics, because including $\mathsf{AC}_{\omega,\omega}$ makes it difficult to compare the results with Friedman/Simpson-style classical reverse mathematics. Our aim is to separate various non-constructive principles which are equivalent under countable choice, and our main tool is (a topological version of) Weihrauch reducibility.

We discuss a hierarchy between LLPO and WKL which collapses under countable choice:

- The lessor limited principle of omniscience LLPO states that for any reals $x, y \in \mathbb{R}$, either $x \leq y$ or $y \leq x$ holds.
- The binary expansion principle BE states that every real has a binary expansion.
- The robust division principle RDIV states that for any reals $0 \leq x \leq y \leq 1$ there is $z \in [0,1]$ such that $x = yz$.
- The intermediate value theorem IVT states that for any continuous function $f\colon [0,1] \to [-1,1]$ if $f(0)$ and $f(1)$ have different signs then there is a real $x \in [0,1]$ such that $f(x) = 0$.
- Weak König's lemma WKL states that every infinite binary tree has an infinite path.

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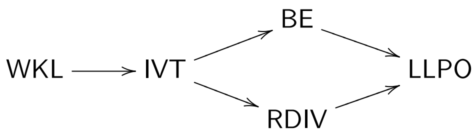
The principle RDIV is known to be related to problems of finding Nash equilibria in bimatrix games and of executing Gaussian elimination. The following implications hold in Troelstra's elementary analysis EL_0:



We use the infinite game version of Weihrauch reducibility. The game closure of a Weihrauch degree **d** always induces a monad on the category **Rep** of represented spaces and relatively continuous functions, and the Kleisli morphisms for this monad yield a realizability notion which obeys the original Weihrauch degree **d** if **d** has the "unique choice" property. Combining Weihrauch separation results with the above idea, we obtain the following:

Theorem 1. *Each of the following items is internally valid in some realizability models:*

1. LLPO + ¬RDIV + ¬BE.
2. RDIV + ¬BE.
3. BE + ¬RDIV.
4. RDIV + BE + ¬IVT.
5. IVT + ¬WKL.

On Cut-Elimination for Cyclic Proof System of Bunched Implication

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One of the nice properties of the sequent-calculus style proof system is to give us a way for mechanical proof-search procedures that find a proof of a given sequent. A naive approach to extend the proof-search procedure with inductive defined predicates is to apply unfolding inductively-defined predicates repeatedly when the procedure encounters them. However this procedure would produce infinite trees, which may not be represented in a finite way. This is an obstacle when we try to develop an automated proof search.

A cyclic-proof system [1] is a sequent-calculus style proof system for proving properties involving inductively defined predicates. A proof structure of a cyclic-proof system is a finite tree with back edges. A back edge connects a bud, which is an open assumption at a leaf position, and its companion, which is a node with the same sequent to the bud. A condition called "global trace condition" is also required for ensuring soundness, since a proof figure with this cyclic structure is not sound in general. This cyclic structure gives a solution for the issue described above, since it naturally gives a finite representation of a class of infinite trees. Several automated proof-search tools based on cyclic proofs for several logics, such as classical logic and separation logic, have been proposed.

In this context, the cut-elimination property for cyclic-proof systems is not only of theoretical interest, because needing arbitrary cuts would mean that there is a limit to what one would be able to prove by a naive mechanical proof search. Unfortunately some negative results that cyclic-proof systems for several logics, such as classical logic, linear logic, and separation logic, do not enjoy the cut-elimination property, have been reported.

The logic of bunched implication (BI), which is an extension of classical first-order logic, offers a reasonable formalism for expressing properties of memory-states and is used as an assertion language for verifying pointer-manipulating programs. Although a cyclic-proof system for BI is already proposed [2], so far it is not known whether this system enjoys the cut-elimination property. In this talk we discuss this point.

References

1. J. Brotherston, A Simpson, *Sequent calculi for induction and infinite descent*, Journal of Logic and Computation, vol 21 (6) (2011), pp. 1177–1216.
2. J. Brotherston, *Formalised Inductive Reasoning in the Logic of Bunched Implications*, Proceedings of SAS 2007, *LNCS* 4634, Springer, 2007, pp.87–103.

Computable analysis and exact real computation in Coq*

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We present some of our recent work on the INCONE library [2], a formalization of ideas from computable analysis in the COQ proof assistant. The library provides a generalized notion of a represented space [1] that can be used to assign computational content to infinite objects such as real numbers and functions.

A representation for real numbers via rational approximations and realizers for arithmetic operations and a limiting procedure can be defined by using the types for real and rational numbers from COQ's standard library. However, this representation is not very useful for doing actual computations as it is extremely inefficient. We develop a framework to easily study and compare more efficient representations for exact real computation. We use this to give a fully formally verified and efficient implementation of exact real computation in COQ based on interval computation and consider several examples. As our algorithms rely on Sierpinski space and the space of Kleeneans, we have developed some of their theory too. To capture the semantics of non-sequential operations on these spaces, such as the "parallel or", we make use of the theory of multivalued functions.

As we do not work in a constructive setting and make use of some of the more complicated parts of COQ's dependent type system, maintaining executability required some effort. In particular, it has lead us to develop a framework of continuous machines that captures the exact information about a continuous function that is considered appropriate in computable analysis and may be of separate theoretical interest.

References

1. Kreitz, C., Weihrauch, K.: Theory of representations. Theoretical computer science **38**, 35–53 (1985)
2. Steinberg, F., Thery, L., Thies, H.: Quantitative continuity and computable analysis in coq. arXiv preprint arXiv:1904.13203 (2019)

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Labelled sequent calculi for relevant logics

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Abstract. Relevant logics have been one of the major classes of non-classical logics extensively studied. Although in the earlier stage of development they were studied axiomatically, relevant logics have also been studied by a variety of semantic methods since a certain period in its history. Among them, Routley-Meyer's ternary relational semantics introduced in [4], where a ternary relation as a kind of "accessibility" relation is used, has been one of the best known semantic methods for relevant logics. Proof theory of relevant logics has also been developed, we have practically no approach so far in proof theory of relevant logics in which the following two items are combined: i) labelled sequent calculi in the sense of [1], which use ternary relation symbols for expressing accessibility relations; ii) structural proof theory in the sense of [2] and [5], i.e., G3-style sequent calculi are used in which structural rules are admissible. In this talk, we try to fill this gap in the literature. We first formulate G3-style labelled sequent calculi for systems of relevant logics in [3] by way of the indexed modality and canonical geometric formulas. Secondly, we discuss semantic soundness and completeness of the labelled sequent calculi with respect to Routley-Meyer semantics. Thirdly, we present some lemmas, such as invertibility, admissibility of structural rules, and then 'syntactic' cut-elimination. One interesting feature of our approach is that our labelled sequent calculi enjoy admissibility of structural rules, although relevant logics are known as their substructural features. This may raise a question: 'structural rules are features a logic or a proof system?' (This talk is based on a joint work with Sara Negri.)

Keywords: relevant logic · Routley-Meyer semantics · labelled sequent calculus · structural proof theory.

References

1. Negri, S. Proof Analysis in Modal Logic, *Journal of Philosophical Logic*, Vol.34, No.5-6, 507-544, 2005.
2. Negri, S. & von Plato, J., *Structural Proof Theory*, Cambridge U. P., 2001.
3. Routley, R., Plumwood, V., Meyer, R. K. & Brady, R. T. *Relevant Logics and Their Rivals*, Ridgeview, 1982.
4. Routley, R. & Meyer, R. K. The Semantics of Entailment. In *Truth, Syntax, and Modality: Proceedings Of The Temple University Conference On Alternative Semantlcs*, (ed.) H. Leblanc, North-Holland Publishing Company. pp. 199-243, 1973.
5. Troelstra, A.S. & H. Schwichtenberg, *Basic Proof Theory* (2nd ed.), Cambridge University Press, 2000

Towards quantitative versions of the "Main Theorem" of Computable Analysis

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The Kreitz-Weihrauch (aka "Main") Theorem of Computable Analysis [Wei00, Theorem 3.2.11] characterizes continuity of functions in terms of continuous realizers. Towards a computational complexity theory of spaces of continuum cardinality beyond real numbers/functions [KSZ16,AK18,Lim19], we explore quantitative versions of the qualitative Main Theorem. Multifunctions are unavoidable in real computation [Luc77]. Classical mathematical notions like *hemi*continuity for multifunctions do not make said Main Theorem generalize. Inspired by [BH94], a new notion of uniform continuity was developed in [PZ13, §4.1]:

Definition 1. *A non-decreasing subadditive continuous real function* $\mu : [0; \infty) \to [0; \infty)$ *with* $\mu(0) = 0$ *is called a* modulus of continuity*. For metric spaces* (X, d) *and* (Y, e)*, a total multifunction* $F : X \rightrightarrows Y$ *is called* μ-continuous *if it satisfies:*

$$\begin{aligned} &\forall n \in \mathbb{N} \quad \forall x_0 \in X \;\; \exists y_0 \in F(x_0) \\ &\qquad \forall x_1 \in X \;\; \exists y_1 \in F(x_1) : \;\; e(y_0, y_1) \leq \mu\big(d(x_0, x_1)\big) \quad \wedge \\ &\qquad \forall x_2 \in X \;\; \exists y_2 \in F(x_2) : \;\; e(y_1, y_2) \leq \mu\big(d(x_1, x_2)\big) \qquad \wedge \; \ldots \; \wedge \\ &\qquad \forall x_n \in X \;\; \exists y_n \in F(x_n) : \;\; e(y_{n-1}, y_n) \leq \mu\big(d(x_{n-1}, x_n)\big) \;\; . \end{aligned} \tag{1}$$

Theorem 2. *Suppose* Y *satisfies the Strong Triangle Inequality. Let* $G :\subseteq \mathcal{C} \rightrightarrows Y$ *be pointwise compact and* μ*-continuous, for some arbitrary modulus of continuity* μ*, with compact domain* $\mathrm{dom}(G) \subseteq \mathcal{C}$*. Then* G *admits a* μ*-continuous single-valued selection* $g : \mathrm{dom}(G) \to Y$*.*

Recall that the *Strong* Triangle Inequality means $e(x, z) \leq \max\big\{e(x, y), e(y, z)\big\}$ and the metric $d : (\bar{x}, \bar{x}') \mapsto 2^{-\min\{n : x_n \neq x'_n\}}$ on Cantor space actually satisfies the Strong Triangle *E*quality. Note that a Lipschitz-continuous multifunction with domain $[0; 1]$ in general does *not* admit a continuous selection [PZ13, Fig.5]. Theorem 2 yields an elegant quantitative counterpart to the qualitative *Main Theorem*, omitted here due to limited space.

References

[AK18] Svetlana Selivanova Martin Ziegler Akitoshi Kawamura, Donghyun Lim. Representation theory of compact metric spaces and computational complexity of continuous data. *arXiv*, 1809.08695, 2018.

[BH94] Vasco Brattka and Peter Hertling. Continuity and computability of relations, 1994.

[KSZ16] Akitoshi Kawamura, Florian Steinberg, and Martin Ziegler. Complexity theory of (functions on) compact metric spaces. In *Proceedings of the 31st Annual ACM/IEEE Symposium on Logic in Computer Science, LICS '16, New York, NY, USA, July 5-8, 2016*, pages 837–846, 2016.

[Lim19] Donghyun Lim. Representations of totally bounded metric spaces and their computational complexity. Master's thesis, KAIST, School of Computing, 2019.

[Luc77] Horst Luckhardt. A fundamental effect in computations on real numbers. *Theoretical Computer Science*, 5(3):321–324, 1977.

[PZ13] Arno Pauly and Martin Ziegler. Relative computability and uniform continuity of relations. *Journal of Logic and Analysis*, 5(7):1–39, 2013.

[Wei00] Klaus Weihrauch. *Computable Analysis.* Springer, Berlin, 2000.

Restriction on cut in cyclic proof system for symbolic heaps

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It has been shown that some variants of cyclic proof systems for symbolic heap entailments in separation logic do not enjoy cut elimination property. In bottom-up proof search, we have to consider the cut rule, which requires some heuristics to find cut formulas. Hence, it is expected to achieve some restricted variant of cut rule which does not change provability and does not interfere automatic proof search without heuristics.

In this work, we give a strict limit on this challenge. We propose a restricted cut rule, called the presumable cut, in which cut formula is restricted to those which may occur below the cut. We show that there is an entailment which is provable with full cuts in cyclic proof system for symbolic heaps, but not with only presumable cuts.

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Decidability of variables in constructive logics

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Heyting's intuitionistic logic differs from classical logic in its rejection of *the law of excluded middle* (LEM). Consequently, to assure that a classical theorem is always derivable intuitionistically, some instances of LEM have to be assumed. For instance, it is well-known that $A \vee \neg A$ is derivable in intuitionistic propositional logic once we assume each propositional variable p in A is *decidable*, i.e. $p \vee \neg p$ is assumed. It is from this direction that Ishihara [1] questioned:

> *What set V of propositional variables suffices for $\Pi_V, \Gamma \vdash_i A$ whenever $\Gamma \vdash_c A$?* (where $\Pi_V = \{p \vee \neg p : p \in V\}$)

The answer to this problem in [1] is that $V = (\mathcal{V}^-(\Gamma) \cup \mathcal{V}^+(A)) \cap (\mathcal{V}^+_{ns}(\Gamma) \cup \mathcal{V}^-(A))$ suffices. ($\mathcal{V}^+(A), \mathcal{V}^-(A)$ and $\mathcal{V}^+_{ns}(A)$ are sets of *positive*, *negative* and *non-strictly positive* propositional variables in A)

Later, Ishii [2] presented different classes (that are incomparable with that of [1]). These are any $V \in V^*(A)$ where $V^*(A)$ is defined inductively, by: ($\mathcal{V}(A)$ denotes the set of all propositional variables in A.)

$$\begin{aligned}
V^*(p) &= \{\{p\}\},\\
V^*(\bot) &= \{\emptyset\},\\
V^*(A \wedge B) &= \{V_1 \cup V_2 : V_1 \in V^*(A), V_2 \in V^*(B)\},\\
V^*(A \vee B) &= \{V_1 \cup \mathcal{V}(B) : V_1 \in V^*(A)\} \cup \{\mathcal{V}(A) \cup V_2 : V_2 \in V^*(B)\},\\
V^*(A \to B) &= V^*(B).
\end{aligned}$$

In this talk, we shall discuss some refinements on the result in [2]. We shall observe that a full LEM in the assumption can often be replaced with weaker axioms $\neg\neg p \vee \neg p$ (WLEM) or $\neg\neg p \to p$ (DNE) for the preservation of classical theorem. This replacement in turn allows us to extend Ishii's result to *Glivenko's logic*, a logic obtained by weakening the *ex falso quodlibet* axiom $\bot \to A$ (EFQ) to its double negation $\neg\neg(\bot \to A)$ [3]. The talk will also discuss what classes of atomic EFQ in addition to classes for LEM would suffice for the preservation.

References

1. Hajime Ishihara, *Classical propositional logic and decidability of variables in intuitionistic logic*, Logical Methods in Computer Science, vol. 10 (2014), no. 3:1, pp. 1–7.
2. Katsumasa Ishii, *A note on decidability of variables in intuitionistic propositional logic*, Mathematical Logic Quarterly, vol. 64 (2018), no. 3, pp. 183–184.
3. Krister Segerberg, *Propositional logics related to Heyting's and Johansson's*, Theoria, vol. 34 (1968), no. 1, pp. 26–61.

Effective Wadge hierarchy in computable quasi-Polish spaces

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Classical descriptive set theory (DST) was extended by M. de Brecht from the usual context of Polish spaces to the much larger class of quasi-Polish spaces which contains many important non-Hausdorff spaces. Computable analysis especially needs an effective DST for reasonable effective versions of topological spaces. Effective versions of classical Borel, Hausdorff and Luzin hierarchies are naturally defined for every effective space but, as also in the classical case, they behave well only for spaces of special kinds. Recently, a convincing version of a computable quasi-Polish space (CQP-space for short) was suggested independently in [1, 2].

Here we continue to develop effective DST in CQP-spaces where effective analogues of some important properties of the classical hierarchies hold. Namely, we develop an effective Wadge hierarchy (including the hierarchy of k-partitions) in such spaces which subsumes the effective Borel and Hausdorff hierarchies (as well as many others) and is in a sense the finest possible hierarchy of effective Borel sets. In particular, we show that levels of such hierarchies are preserved by the computable effectively open surjections, that if the effective Hausdorff-Kuratowski theorem holds in the Baire space then it also holds in every CQP-space, and we extend the effective Hausdorff theorem for CQP-spaces [3] to k-partitions. We hope that these results (together with those already known) show that effective DST reached the state of maturity.

References

1. M. de Brecht, A. Pauly, M. Schröder, *Overt choice*, arXiv 1902.05926v1 [math. LO], 2019.
2. M. Hoyrup, C. Rojas, V. Selivanov, D. Stull, *Computability on quasi-Polish spaces*, Lecture Notes in Computer Science (Proc. of DCFS), (Michal Hospodár and Galina Jirásková and Stavros Konstantinidis, editors), vol. 11612, Springer, 2019, pp. 171–183.
3. V.L. Selivanov, . Proc. CiE 2015, LNCS volume , Berlin, Springer, 2015, P. , *Towards the effective descriptive set theory*, Lecture Notes in Computer Science (Proc. of CiE) (Arnold Beckmann and Victor Mitrana and Mariya Ivanova Soskova, editors), vol. 9136, Springer, 2015, pp. 324–333.

A simple proof of the parallel closedness theorem

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Confluence is a property that guarantees uniqueness of computation results and plays a crucial role to control nondeterminism of rewriting-based computation. Huet's parallel closedness [1] is one of fundamental theorems for proving confluence of term rewrite systems. In order to show its correctness Huet employed a special induction measure. Although most part of his proof is straightforward, arguing decreasingness of the measure is notoriously difficult [2–4]. In this talk we propose an alternative measure. With the new measure decreasingness follows trivially.

References

1. G. Huet, *Confluent reductions: Abstract properties and applications to term rewriting systems*, Journal of ACM, vol. 27 (4), pp. 797–821.
2. F. Baader and T. Nipkow. *Term rewriting and all that*, Cambridge University Press, 1998.
3. J. Nagele and A. Middeldorp. *Certification of classical confluence results for left-linear term rewrite systems*, Proceedings of the 7th International Conference on Interactive Theorem Proving, Lecture Notes in Computer Science 9807, Springer, pp. 290–306, 2016.
4. J. Nagele. *Mechanizing confluence: Automated and certified analysis of first- and higher-order rewrite systems*, PhD thesis, University of Innsbruck, 2017.

The Compact Hyper-space Monad, a Constructive Approach

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As is well known, the collection $\mathcal{K}(X)$ of all non-empty compact subsets of a compact Hausdorff space X is a compact Hausdorff space again with respect to the Vietoris topology. The functor $\mathcal{K}$ defines a monad $(\mathcal{K}, \eta, U)$, where η_X maps points $x \in X$ to $\{x\}$, and U_X compact sets $\mathbb{K} \in \mathcal{K}(\mathcal{K}(X))$ to their union $\bigcup \mathbb{K}$.

In this talk a constructive version of the result will be presented. We work in intuitionistic logic extended by inductive and co-inductive definitions (cf. Berger [1, 2]). As in Berger/Spreen [3], only compact Hausdorff spaces X are considered that come equipped with a distinguished finite set D of continuous endo-functions the ranges of which cover the underlying space. By allowing the endo-functions $d \in D$ to be of any finite arity $\mathrm{ar}(d)$ a unified framework can be set up covering both, the point set case as well as that of compact hyper-spaces such that all essential properties are inherited from X to $\mathcal{K}(X)$.

Let co-inductively $\mathbb{C}_X$ be the largest subset of X so that

$$x \in \mathbb{C}_X \to (\exists d \in D)(\exists y_1, \ldots, y_{\mathrm{ar(d)}} \in \mathbb{C}_X)\, x = d(y_1, \ldots, y_{\mathrm{ar(d)}}).$$

Then it follows that $X = \mathbb{C}_X$. We only work with the subspaces $\mathbb{C}_X$.

In [2] a co-inductive inductive characterisation of the uniformly continuous maps $f\colon [0,1]^n \to [0,1]$ has been given. It can be extended to the more general case of spaces considered here and is used to define the hom-sets of the category.

By constructively reasoning on the basis of co-inductive and/or inductive definitions computational content is derived. Realisability facilitates the extraction of algorithms from the corresponding proof. The framework presented here in particular allows to deal with compact-valued maps and their selection functions. Maps of this kind abundantly occur in applied mathematics. They have applications in areas such as optimal control and mathematical economics, to mention a few. In addition, they are used to model non-determinism.

References

1. Ulrich Berger, *Realisability for induction and coinduction with applications to constructive analysis*, Journal of Universal Computer Science, vol. 16 (2010), no. 18, pp. 2535–2555.
2. Ulrich Berger, *From coinductive proofs to exact real arithmetic: theory and applications*, Logical Methods in Computer Science, vol. 7 (2011), no. 1, pp. 1–24, doi: 10.2168/LMCS7(1:8)2011.
3. Ulrich Berger, Dieter Spreen, *A coinductive approach to computing with compact sets*, Journal of Logic and Analysis, vol. 8 (2016), no. 3, pp. 1–35, doi: 10.4115/jla.2016.8.3.

First-Order Expansion of Intuitionistic Epistemic Logic

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Artemov and Protopopescu (2016) gave an intuitionistic epistemic logic based on a verification reading of the intuitionistic knowledge in terms of Brouwer-Heyting-Kolmogorov interpretation. They proposed that a proof of a formula KA (read "it is known that A") is the conclusive verification of the existence of a proof of A. Then $A \supset KA$ expresses that, when a proof of A is given, the conclusive verification of the existence of the proof of A can be constructed. Since a proof of A itself is the conclusive verification of the existence of a proof of A, they claimed that $A \supset KA$ is valid. But $KA \supset A$ (usually called *factivity* or *reflection*) is not valid, since the verification does not always give a proof. They provided a Hilbert system of intuitionistic epistemic logic **IEL** as the intuitionistic propositional logic plus the axioms schemes $K(A \supset B) \supset KA \supset KB$, $A \supset KA$ and $\neg K\bot$. They also gave the Kripke semantics for **IEL** and proved the Hilbert system is sound and complete for the semantics.

We study the first-order expansion **QIEL** of **IEL**, where Kripke semantics for **QIEL** is naturally defined. We propose the sequent calculus for **QIEL**. The sequent calculus for propositional **IEL** has been given by Krupski and Yatmanov (2016), though one inference rule in their system for **IEL** does not satisfy the subformula property. This talk gives a new analytic sequent calculus $\mathcal{G}$(**QIEL**) of the first-order intuitionistic epistemic logic.

As corollaries of the syntactic cut-elimination theorem, $\mathcal{G}$(**QIEL**) enjoys the disjunction property and the existence property. Furthermore, Craig interpolation theorem of $\mathcal{G}$(**QIEL**) holds. Finally, with the method of Hermant (2005), we also establish the cut-free completeness of $\mathcal{G}$(**QIEL**), which implies a semantic proof of cut-elimination theorem of $\mathcal{G}$(**QIEL**).

References

1. Sergei Artemov and Tudor Protopopescu *Intuitionistic Epistemic logic*, The Review of Symbolic Logic, vol.9, 2016, pp.266-298.
2. Vladimir N Krupski and Alexey Yatmanov *Sequent calculus for intuitionistic epistemic logic* **IEL**. In International Symposium on Logical Foundations of Computer Science, Springer, 2016, pp. 187–201
3. Olivier Hermant *Semantic cut elimination in the intuitionistic sequent calculus*, In Typed Lambda-Calculi and Applications,editor Pawel Urzyczyn, vol. 3461 of Lecture Notes in Computer Science, Nara, Japan, Springer-Verlag 2005, pp. 221–233.

Counterexample to Brotherston's Conjecture

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A cyclic proof system, called CLKID-omega, gives us another way of representing inductive definitions and efficient proof search. The 2006 paper by Brotherston [1] showed that the provability of CLKID-omega includes the provability of LKID, first order classical logic with inductive definitions in Martin-Löf's style, and conjectured the equivalence. The equivalence had been left as an open question until 2011 [2]. This talk gives a counterexample to this conjecture and shows that CLKID-omega and LKID are indeed not equivalent. This talk considers a statement called 2-Hydra in these two systems with the first-order language formed by 0, the successor, the natural number predicate, and a binary predicate symbol used to express 2-Hydra. Then this talk shows that the 2-Hydra statement is provable in CLKID-omega, but the statement is not provable in LKID, by constructing some Henkin model where the statement is false.

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References

1. S. Berardi and M. Tatsuta, Classical System of Martin-Lof's Inductive Definitions is not Equivalent to Cyclic Proof System, In: Proceeding of 20th International Conference on Foundations of Software Science and Computation Structures (FoSSaCS 2017), *Lecture Notes in Computer Science* 10203 (2017) 301–317.
2. J. Brotherston, *Sequent calculus proof systems for inductive definitions,* PhD thesis, Laboratory for Foundations of Computer Science School of Informatics, University of Edinburgh, 2006.