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第 4 回数理論理学とその応用に関するワークショップ

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March 22 – 24, 2021,
Online meeting

Timetable

Monday, 22 March	Tuesday, 23 March	Wednesday, 24 March
11:00 -11:20 Opening		
11:20 - 12:00 Hidenori Kurokawa	11:20 - 12:00 Youan Su Katsuhiko Sano	
Lunch break		
13:30 - 14:10 Daichi Hayashi	13:30 - 14:10 Donghyun Lim	13:30 - 14:10 Koji Nakazawa Kenji Saotome Daisuke Kimura
14:10 - 14:50 Satoru Niki	14:10 - 14:50 Michal Konečný Florian Steinberg Holger Thies	14:10 - 14:50 Yukihiro Masuoka Makoto Tatsuta
Break	Break	Break
15:20 - 16:00 Matthew de Brecht	15:20 - 16:00 Takayuki Kihara	15:20 - 16:00 Franz Brauße Konstantin Korovin Margarita Korovina Norbert Müller
16:00 - 16:40 Victor Selivanov	16:00 - 16:40 Makoto Fujiwara	16:00 - 16:40 Norbert Müller Margarita Korovina Franz Brauße
Break	Break	
17:00 - 17:40 Dieter Spreen	17:00 - 17:40 Masahiro Doi Nao Hirokawa	

Ksmt for solving non-linear constraints*

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We give a detailed overview of the **ksmt** calculus developed in a conflict driven clause learning framework for checking satisfiability of non-linear constraints over the reals. Non-linear constraint solving naturally arises in the development of formal methods for verification of safety critical systems, program analysis and information management. Implementations of formal methods are widely used to approve in advance that designed systems satisfy all specification requirements, such as reliability, safety and reachability. Historically, there have been two main approaches to deal with non-linear constrains: the symbolic one originated by Tarski's decision procedure for the real closed fields and the numerical one based on interval constraint propagations. It is well known that both approaches have their strength and weakness concerning completeness, efficiency and expressiveness. Nowdays, merging strengths of symbolical and numerical approaches is one of the challenging research aria in theoretical and applied computer science.

The **ksmt** calculus successfully integrates strengths of symbolical and numerical methods. The key steps of the decision procedure based on this calculus contain assignment refinements, inferences of linear resolvents driven by linear conflicts, backjumping and constructions of local linearisations of non-linear components initiated by non-linear conflicts. In [BKKM19] we showed that the procedure is sound and makes progress by reducing the search space. This approach is applicable to a large number of constraints involving computable non-linear functions, piecewise polynomial splines, transcendental functions and solutions of polynomial differential equations.

In this setting we discuss resent and future research work.

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[BKKM19] F. Brauße, K. Korovin, M. Korovina, and N. Th. Muller: *A CDCL-style calculus for solving non-linear constraints.* In: *FroCoS 2019, LNCS (LNAI) vol. 11715*, pp 131–148, 2019

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Generalizing Taylor models for multivariate real functions ⋆

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We discuss data structures and algorithms for the approximation of multivariate real functions $f : \subseteq\mathbb{R}^k \to \mathbb{R}$. From the viewpoint of TTE[Wei00], these approximations can be viewed as building blocks for representations. To this end, we apply ideas from the field of Taylor models [MB01], thus generalizing the approach taken in [BKM15]: On domains given as polytopes, functions are approximated by polynomials with (possibly unbounded) interval coefficients. In [DFKT14], there has been a related approach of a 'function interval arithmetic', still lacking the aspects of data reduction in Taylor models.

As an application we aim at the field of SMT solving and present a prototypical implementation: From a (symbolically defined) function f and a value $\boldsymbol{c}$ with $f(\boldsymbol{c}) > 0$ it derives a polytope P and a linear g with $\boldsymbol{c} \in P$, $g(\boldsymbol{c}) > 0$ and $\forall x \in P : f(\boldsymbol{x}) > g(\boldsymbol{x})$ thus separating the graph of f and the point $(\boldsymbol{c}, 0)$. This property is a core requirement for recent CDCL-style SMT solvers [BKKM19,CGI+18].

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On the transferability of results between subcategories of spaces and locales

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Let $\mathbf{\Omega}\colon \mathbf{Top} \to \mathbf{Loc}$ be the usual functor mapping topological spaces to locales. It is well known that $\mathbf{\Omega}$ restricts to an equivalence between the category of sober spaces and the category of spatial locales, but this does not mean that there is an equivalence between topological results on sober spaces and locale theoretical results on spatial locales. For example:

(1) $(\mathbb{Q}, +)$ is a topological subgroup of $(\mathbb{R}, +)$.
(2) Every localic subgroup of a localic group is a closed sublocale [4].

It follows that $(\mathbb{Q}, +)$ is *not* a localic subgroup of $(\mathbb{R}, +)$, even though $\mathbb{Q}$ and $\mathbb{R}$ are included in the sober space $\sim$ spatial locale categorical equivalence. These discrepancies can occur because $\mathbf{\Omega}$ does not preserve products, and the existence of group operations such as $+\colon \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$ depends on the product structure.

A further restriction of $\mathbf{\Omega}$ yields an equivalence between the category **QPol** of quasi-Polish spaces [1] and the category of countably presented locales [3]. Under this restriction, $\mathbf{\Omega}$ now preserves all countable limits, and the categorical equivalence starts to look more like an actual equivalence:

(3) Every quasi-Polish subgroup of a quasi-Polish group is a closed subspace.

Are there any extensions of **QPol** where the equivalence between spaces and locales still behaves so well? At least for countably based spaces we have a partial answer. Based on the results presented in [2], we argue that **QPol** is the largest "reasonable" subcategory of countably based spaces where we can hope for such a natural transfer of results between topology and locale theory.

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Completion and the Infeasibility Problem

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We present a new completion procedure for word problems. While the standard completion procedure [1] solves problems by constructing complete term rewrite systems, our approach employs complete *conditional* term rewriting systems. We anticipate that the new procedure is useful when solving infeasibility problems of conditional term rewrite systems automatically [2].

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Decomposition of König's lemma and its unique variants in constructive reverse mathematics

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König's lemma KL states that any infinite finitely-branching tree has an infinite path. In the context of *classical* reverse mathematics [4], König's lemma KL is equivalent to the arithmetical comprehension axiom ACA over RCA_0. On the other hand, weak König's lemma WKL, which is KL restricted to $\{0, 1\}$-trees, is strictly weaker than KL but still unprovable in RCA_0.

In contrast to the situation in classical reverse mathematics, from the *constructive* viewpoint, KL is derived from WKL. This is because the unique choice principle is (normally) assumed in the axiomatic basis of constructive mathematics (cf. [5, Section 4.1.6]). In this talk, we introduce a weak variant of the unique choice principle which is necessary and sufficient to derive KL from WKL constructively. Combining this with a known decomposition of WKL, KL is decomposed into a logical principle and two choice principles. In addition, we determine a variant of the unique choice principle which captures the difference between WKL!! and KL!!, where WKL!! and KL!! are WKL and KL with a uniqueness hypothesis in the sense of Moschovakis [2] respectively. Furthermore, generalizing the arguments in Schwichtenberg [3], we obtain a similar decomposition result on WKL! and KL!, where WKL! and KL! are WKL and KL with a uniqueness hypothesis in the sense of Berger and Ishihara [1]. In the end, we overview the relation between the variants of the unique choice principle, König's lemma and its unique variants.

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Extending supervaluation-style Frege structure by the limit axiom

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Feferman [4] introduced *explicit mathematics* (EM) as a framework of Bishop's constructive mathematics. Especially, Feferman gave an impredicative theory T_0 of EM which has the *inductive generation* axioms (IG). As a similar framework of set theory, Aczel's *Frege structure* (FS) [1] has been studied as truth theories over applicative theories (cf. e.g. [2]). As Aczel already expected [1], EM and FS as formal theories are closely related, that is, for many weaker variants of T_0 we can find a proof-theoretically equivalent one of FS (cf. e.g. [5]).

Therefore, the purpose of this talk is to find a theory of FS as strong as T_0. Especially, we give a theory VFU, which is an extension of Cantini's theory VF [2, 5] by the *limit axiom* [5]. Here, the theory VF has the axioms stating that the internal logic of the truth predicate follows supervaluation scheme. For example, VF can derive the formula $\mathrm{T}\ulcorner A \vee \neg A \urcorner$, the truth of the law of excluded middle $A \vee \neg A$, without assuring neither $\mathrm{T}\ulcorner A \urcorner$ nor $\mathrm{T}\ulcorner \neg A \urcorner$.

Theorem 1. VFU *and* T_0 *are proof-theoretically equivalent.*

The proof of the theorem consists of two parts. To obtain the lower-bound of VFU, we give a relative interpretation of T_0 into VFU. As for the upper-bound of VFU, we generalize Cantini's *truth*-as-*provability* interpretation [2]. To put it simply, the truth $\mathrm{T}\ulcorner A \urcorner$ of a sentence A in VFU is interpreted as the derivability of A in a certain infinitary derivation system.

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Some Lifschitz-like realizability notions separating non-constructive principles*

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Abstract. There is a way of assigning a realizability notion to each degree of incomputability. In our setting, we make use of Weihrauch degrees (degrees of incomputability of partial multi-valued functions) to obtain Lifschitz-like relative realizability toposes. In particular, we lift some separation results on Weihrauch degrees to those over intuitionistic Zermelo-Fraenkel set theory **IZF**.

Keywords: Realizability topos · Constructive reverse mathematics · Weihrauch degree.

This is a contribution to *constructive reverse mathematics* initiated by Ishihara. Here we do not include the axiom of countable choice $\mathsf{AC}_{\omega,\omega}$ in our base system of constructive reverse mathematics, because including $\mathsf{AC}_{\omega,\omega}$ makes it difficult to compare the results with Friedman/Simpson-style classical reverse mathematics. Our aim is to separate various non-constructive principles which are equivalent under countable choice, and our main tool is (a topological version of) Weihrauch reducibility.

We discuss a hierarchy between LLPO and WKL which collapses under countable choice:

- The lessor limited principle of omniscience LLPO states that for any reals $x, y \in \mathbb{R}$, either $x \leq y$ or $y \leq x$ holds.
- The binary expansion principle BE states that every real has a binary expansion.
- The robust division principle RDIV states that for any reals $0 \leq x \leq y \leq 1$ there is $z \in [0, 1]$ such that $x = yz$.
- The intermediate value theorem IVT states that for any continuous function $f\colon [0, 1] \to [-1, 1]$ if $f(0)$ and $f(1)$ have different signs then there is a real $x \in [0, 1]$ such that $f(x) = 0$.
- Weak König's lemma WKL states that every infinite binary tree has an infinite path.

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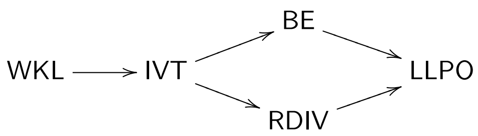
The principle RDIV is known to be related to problems of finding Nash equilibria in bimatrix games and of executing Gaussian elimination. The following implications hold in Troelstra's elementary analysis EL_0:



We use the infinite game version of Weihrauch reducibility. The game closure of a Weihrauch degree **d** always induces a monad on the category **Rep** of represented spaces and relatively continuous functions, and the Kleisli morphisms for this monad yield a realizability notion which obeys the original Weihrauch degree **d** if **d** has the "unique choice" property. Combining Weihrauch separation results with the above idea, we obtain the following:

Theorem 1. *Each of the following items is internally valid in some realizability models:*

1. $\mathsf{LLPO} + \neg\mathsf{RDIV} + \neg\mathsf{BE}$.
2. $\mathsf{RDIV} + \neg\mathsf{BE}$.
3. $\mathsf{BE} + \neg\mathsf{RDIV}$.
4. $\mathsf{RDIV} + \mathsf{BE} + \neg\mathsf{IVT}$.
5. $\mathsf{IVT} + \neg\mathsf{WKL}$.

Computable analysis and exact real computation in Coq*

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We present some of our recent work on the INCONE library [2], a formalization of ideas from computable analysis in the COQ proof assistant. The library provides a generalized notion of a represented space [1] that can be used to assign computational content to infinite objects such as real numbers and functions.

A representation for real numbers via rational approximations and realizers for arithmetic operations and a limiting procedure can be defined by using the types for real and rational numbers from COQ's standard library. However, this representation is not very useful for doing actual computations as it is extremely inefficient. We develop a framework to easily study and compare more efficient representations for exact real computation. We use this to give a fully formally verified and efficient implementation of exact real computation in COQ based on interval computation and consider several examples. As our algorithms rely on Sierpinski space and the space of Kleeneans, we have developed some of their theory too. To capture the semantics of non-sequential operations on these spaces, such as the "parallel or", we make use of the theory of multivalued functions.

As we do not work in a constructive setting and make use of some of the more complicated parts of COQ's dependent type system, maintaining executability required some effort. In particular, it has lead us to develop a framework of continuous machines that captures the exact information about a continuous function that is considered appropriate in computable analysis and may be of separate theoretical interest.

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Labelled sequent calculi for relevant logics

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Abstract. Relevant logics have been one of the major classes of non-classical logics extensively studied. Although in the earlier stage of development they were studied axiomatically, relevant logics have also been studied by a variety of semantic methods since a certain period in its history. Among them, Routley-Meyer's ternary relational semantics introduced in [4], where a ternary relation as a kind of "accessibility" relation is used, has been one of the best known semantic methods for relevant logics. Proof theory of relevant logics has also been developed, we have practically no approach so far in proof theory of relevant logics in which the following two items are combined: i) labelled sequent calculi in the sense of [1], which use ternary relation symbols for expressing accessibility relations; ii) structural proof theory in the sense of [2] and [5], i.e., G3-style sequent calculi are used in which structural rules are admissible. In this talk, we try to fill this gap in the literature. We first formulate G3-style labelled sequent calculi for systems of relevant logics in [3] by way of the indexed modality and canonical geometric formulas. Secondly, we discuss semantic soundness and completeness of the labelled sequent calculi with respect to Routley-Meyer semantics. Thirdly, we present some lemmas, such as invertibility, admissibility of structural rules, and then 'syntactic' cut-elimination. One interesting feature of our approach is that our labelled sequent calculi enjoy admissibility of structural rules, although relevant logics are known as their substructural features. This may raise a question: 'structural rules are features a logic or a proof system?' (This talk is based on a joint work with Sara Negri.)

Keywords: relevant logic · Routley-Meyer semantics · labelled sequent calculus · structural proof theory.

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Degrees of Second and Higher Order Polynomials*

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The classical theory of computational complexity measures cost in (worst-case) dependence on one integer parameter $n = |x|$ denoting the length of the input $x \in \{0,1\}^*$. Efficient computation according to Cobham means cost bounded by a polynomial in n. Function inputs $f : \{0,1\}^* \to \{0,1\}^*$, presented for example as oracles, are not captured by one natural number only, but instead by number function $\ell : \mathbb{N} \to \mathbb{N}$ with $|f(y)| \le \ell(|y|)$. Here, efficient computation is understood as cost bounded by a so-called second-order polynomial: depending on both $n = |x|$ and ℓ [4]. These characterize the 'basic feasible functionals' [1] and have applications in complexity theory of operators in analysis [2, 5].

We consider syntax and semantics of second-order polynomials. The syntax is given by the rule $P, Q ::= 1 \mid n \mid P + Q \mid P * Q \mid \ell(P)$. The semantics $[\![P]\!] : \mathbb{N}^{\mathbb{N}} \times \mathbb{N} \to \mathbb{N}$ is the canonical interpretation as a second-order function. We prove that the syntax and the semantics coincide: $[\![P]\!] = [\![Q]\!]$ if and only if P and Q are related under a suitable notion of syntactic equivalence. Work in progress generalizes this to higher-order polynomials in terms of typed lambda calculus.

We consider a notion of degree of a second-order polynomial to be a (first-order) polynomial; we show it to be well-defined, and such that the degree of the degree coincides with the nesting depth of ℓ [3].

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Cut-elimination in cyclic proof system for first-order logic

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A *cyclic proof system*, whose proof figures are finite trees with cycles, is an alternative proof system to the proof system with explicit induction. Brotherston proposed the cyclic proof system $\mathtt{CLKID}^{\omega}$ for first-order logic with inductive definitions [1]. He also conjectured the cut-elimination property of $\mathtt{CLKID}^{\omega}$ does not hold [2]. This talk shows the conjecture is correct by giving a counterexample. The counterexample uses two different inductive predicates, each of which defines the addition in natural numbers. Our goal of this talk is to show a sequent with these predicates is not provable without the cut rule but is provable in the system with the cut rule.

Acknowledgments

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Decidability of variables in constructive logics

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Heyting's intuitionistic logic differs from classical logic in its rejection of *the law of excluded middle* (LEM). Consequently, to assure that a classical theorem is always derivable intuitionistically, some instances of LEM have to be assumed. For instance, it is well-known that $A \vee \neg A$ is derivable in intuitionistic propositional logic once we assume each propositional variable p in A is *decidable*, i.e. $p \vee \neg p$ is assumed. It is from this direction that Ishihara [1] questioned:

> *What set V of propositional variables suffices for $\Pi_V, \Gamma \vdash_i A$ whenever $\Gamma \vdash_c A$?* (where $\Pi_V = \{p \vee \neg p : p \in V\}$)

The answer to this problem in [1] is that $V = (\mathcal{V}^-(\Gamma) \cup \mathcal{V}^+(A)) \cap (\mathcal{V}^+_{ns}(\Gamma) \cup \mathcal{V}^-(A))$ suffices. ($\mathcal{V}^+(A), \mathcal{V}^-(A)$ and $\mathcal{V}^+_{ns}(A)$ are sets of *positive*, *negative* and *non-strictly positive* propositional variables in A)

Later, Ishii [2] presented different classes (that are incomparable with that of [1]). These are any $V \in V^*(A)$ where $V^*(A)$ is defined inductively, by: ($\mathcal{V}(A)$ denotes the set of all propositional variables in A.)

$$\begin{aligned}
V^*(p) &= \{\{p\}\},\\
V^*(\bot) &= \{\emptyset\},\\
V^*(A \wedge B) &= \{V_1 \cup V_2 : V_1 \in V^*(A), V_2 \in V^*(B)\},\\
V^*(A \vee B) &= \{V_1 \cup \mathcal{V}(B) : V_1 \in V^*(A)\} \cup \{\mathcal{V}(A) \cup V_2 : V_2 \in V^*(B)\},\\
V^*(A \to B) &= V^*(B).
\end{aligned}$$

In this talk, we shall discuss some refinements on the result in [2]. We shall observe that a full LEM in the assumption can often be replaced with weaker axioms $\neg\neg p \vee \neg p$ (WLEM) or $\neg\neg p \to p$ (DNE) for the preservation of classical theorem. This replacement in turn allows us to extend Ishii's result to *Glivenko's logic*, a logic obtained by weakening the *ex falso quodlibet* axiom $\bot \to A$ (EFQ) to its double negation $\neg\neg(\bot \to A)$ [3]. The talk will also discuss what classes of atomic EFQ in addition to classes for LEM would suffice for the preservation.

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Failure of cut-elimination in the cyclic proof system of bunched logic with inductive propositions

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Cyclic proof systems [1–3] are sequent-calculus style proof systems that allow circular structures representing induction, and they are considered suitable for automated inductive reasoning. However, Kimura et al. [4] have shown that the cyclic proof system for the symbolic heap separation logic does not satisfy the cut-elimination property, one of the most fundamental properties of proof systems. They guessed in [4] that the cut-elimination would not hold for the bunched logic [5, 1] either, but it is hard to adapt the existing proof technique, where chasing contradictory paths in cyclic proofs, since the bunched logic contains the structural rules, the weakening and the contraction rules.

This study proposes a new proof technique called proof unrolling for proving that the cyclic proof system for the bunched logic with only nullary inductive predicates does not satisfy the cut-elimination property. The proof unrolling can be adapted to the symbolic heap separation logic, and it also‘ shows that the cut-elimination fails even if we restrict the inductive predicates to nullary ones in the symbolic heap separation logic.

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Effective Wadge hierarchy in computable quasi-Polish spaces

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Classical descriptive set theory (DST) was extended by M. de Brecht from the usual context of Polish spaces to the much larger class of quasi-Polish spaces which contains many important non-Hausdorff spaces. Computable analysis especially needs an effective DST for reasonable effective versions of topological spaces. Effective versions of classical Borel, Hausdorff and Luzin hierarchies are naturally defined for every effective space but, as also in the classical case, they behave well only for spaces of special kinds. Recently, a convincing version of a computable quasi-Polish space (CQP-space for short) was suggested independently in [1, 2].

Here we continue to develop effective DST in CQP-spaces where effective analogues of some important properties of the classical hierarchies hold. Namely, we develop an effective Wadge hierarchy (including the hierarchy of k-partitions) in such spaces which subsumes the effective Borel and Hausdorff hierarchies (as well as many others) and is in a sense the finest possible hierarchy of effective Borel sets. In particular, we show that levels of such hierarchies are preserved by the computable effectively open surjections, that if the effective Hausdorff-Kuratowski theorem holds in the Baire space then it also holds in every CQP-space, and we extend the effective Hausdorff theorem for CQP-spaces [3] to k-partitions. We hope that these results (together with those already known) show that effective DST reached the state of maturity.

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The Compact Hyper-space Monad, a Constructive Approach*

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As is well known, the collection $\mathcal{K}(X)$ of all non-empty compact subsets of a compact Hausdorff space X is a compact Hausdorff space again with respect to the Vietoris topology. The functor $\mathcal{K}$ defines a monad $(\mathcal{K}, \eta, U)$, where η_X maps points $x \in X$ to $\{x\}$, and U_X compact sets $\mathbb{K} \in \mathcal{K}(\mathcal{K}(X))$ to their union $\bigcup \mathbb{K}$.

In this talk a constructive version of the result will be presented. We work in intuitionistic logic extended by inductive and co-inductive definitions (cf. Berger/Tsuiki [3]). As in Berger/Spreen [2], only compact metric spaces X are considered that come equipped with a distinguished finite set D of contracting functions $d\colon X \to X$ the ranges of which cover the underlying space.

Let co-inductively $\mathbb{C}_X$ be the largest subset of X so that

$$x \in \mathbb{C}_X \to (\exists d \in D)(\exists y \in \mathbb{C}_X)\, x = d(y).$$

Then it follows classically that $X = \mathbb{C}_X$. We only work with the subspaces $\mathbb{C}_X$.

It is well-known that $\mathcal{K}(X)$ equipped with the Hausdorff distance function is a metric space again. For $d_1, \ldots, d_r \in D$ define $[d_1, \ldots, d_r]\colon \mathcal{K}(X)^r \to \mathcal{K}(X)$ by

$$[d_1, \ldots, d_r](K_1, \ldots, K_r) := \bigcup_{\kappa=1}^{r} d_\kappa(K_\kappa),$$

and set

$$\mathcal{K}(D) := \{\, [d_1, \ldots, d_r] \mid d_1, \ldots, d_r \in D \text{ pairwise distinct with } r > 0 \,\}.$$

Now, similarly to above, let co-inductively $\mathbb{C}_{\mathcal{K}(X)}$ be the largest subset of $\mathcal{K}(X)$ such that

$$K \in \mathbb{C}_{\mathcal{K}(X)} \to (\exists [d_1, \ldots, d_r] \in \mathcal{K}(D))\\ (\exists K_1, \ldots, K_r \in \mathbb{C}_{\mathcal{K}(X)}) K = [d_1, \ldots, d_r](K_1, \ldots, K_r).$$

Then $\mathcal{K}(X) = \mathbb{C}_{\mathcal{K}(X)}$, classically. Again, we only work with $\mathbb{C}_{\mathcal{K}(X)}$.

Unfortunately, the construction cannot be extended to higher powers $\mathcal{K}^n(X)$, with $n > 1$. An extension of the framework that allows doing so is presented in the talk.

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In [1] a co-inductive inductive characterisation of the uniformly continuous maps $f\colon [0,1]^n \to [0,1]$ has been given. As shown in [4], it can be extended to the more general case of spaces considered here and is used to define the hom-sets of the category.

By constructively reasoning on the basis of co-inductive and/or inductive definitions computational content is derived. Realisability facilitates the extraction of algorithms from the corresponding proof. The framework presented here in particular allows to deal with compact-valued maps and their selection functions. Maps of this kind abundantly occur in applied mathematics. They have applications in areas such as optimal control and mathematical economics, to mention a few. In addition, they are used to model non-determinism.

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First-Order Expansion of Intuitionistic Epistemic Logic

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Artemov and Protopopescu (2016) gave an intuitionistic epistemic logic based on a verification reading of the intuitionistic knowledge in terms of Brouwer-Heyting-Kolmogorov interpretation. They proposed that a proof of a formula KA (read "it is known that A") is the conclusive verification of the existence of a proof of A. Then $A \supset KA$ expresses that, when a proof of A is given, the conclusive verification of the existence of the proof of A can be constructed. Since a proof of A itself is the conclusive verification of the existence of a proof of A, they claimed that $A \supset KA$ is valid. But $KA \supset A$ (usually called *factivity* or *reflection*) is not valid, since the verification does not always give a proof. They provided a Hilbert system of intuitionistic epistemic logic **IEL** as the intuitionistic propositional logic plus the axioms schemes $K(A \supset B) \supset KA \supset KB$, $A \supset KA$ and $\neg K\bot$. They also gave the Kripke semantics for **IEL** and proved the Hilbert system is sound and complete for the semantics.

We study the first-order expansion **QIEL** of **IEL**, where Kripke semantics for **QIEL** is naturally defined. We propose the sequent calculus for **QIEL**. The sequent calculus for propositional **IEL** has been given by Krupski and Yatmanov (2016), though one inference rule in their system for **IEL** does not satisfy the subformula property. This talk gives a new analytic sequent calculus $\mathcal{G}$(**QIEL**) of the first-order intuitionistic epistemic logic.

As corollaries of the syntactic cut-elimination theorem, $\mathcal{G}$(**QIEL**) enjoys the disjunction property and the existence property. Furthermore, Craig interpolation theorem of $\mathcal{G}$(**QIEL**) holds. Finally, with the method of Hermant (2005), we also establish the cut-free completeness of $\mathcal{G}$(**QIEL**), which implies a semantic proof of cut-elimination theorem of $\mathcal{G}$(**QIEL**).

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