A Parametric Representation of Linguistic Hedges in Zadeh's Fuzzy Logic

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Abstract

This paper proposes a model for the parametric representation of linguistic hedges in Zadeh's fuzzy logic. In this model each linguistic truth value, which is generated from a primary term of the linguistic truth variable, is identified by a real number r depending on the primary term. It is shown that the model yields a method of efficiently computing linguistic truth expressions accompanied with a rich algebraic structure of the linguistic truth domain, namely De Morgan algebra. Also, a fuzzy logic based on the parametric representation of linguistic truth values is introduced.

Key words: Linguistic hedges, linguistic variable, distributive lattice, De Morgan algebra, fuzzy logic, approximate reasoning

1 Introduction

In 1970s, L. Zadeh introduced and developed the theory of approximate reasoning based on the notions of linguistic variable and fuzzy logic [18–22]. Informally, by a linguistic variable we mean a variable whose values are words in a natural or artificial language. For example, *Age* is a linguistic variable whose values are linguistic such as *young*, *old*, *very young*, *very old*, *quite young*, *more or less young*, *not very young and not very old*, etc. As is well-known, the values of a linguistic variable are generated from primary terms (e.g., *young* and *old* in the case of linguistic variable *Age*) by various linguistic hedges (e.g., *very*, *more or less*, etc.) and connectives (e.g., *and, or, not*).

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In Zadeh's view of fuzzy logic, the truth-values are linguistic, e.g., of the form "*true*", "*very true*", "*more or less true*", "*false*", "*possible false*", etc., which are expressible as values of the linguistic variable *Truth*, and the rules of inference are approximate rather than exact. In this sense, approximate reasoning (also called fuzzy reasoning) is, for the most part, qualitative rather than quantitative in nature, and almost all of it falls outside of the domain of applicability of classical logic (see Zadeh [2,21,22]). The primary aim of the theory of approximate reasoning is to mimic human linguistic reasoning particularly in describing the behaviour of human-centered systems.

Throughout this paper, by a fuzzy logic we mean a fuzzy logic in the sense of Zadeh, that is, its truth-values are linguistic values of the linguistic truth variable, which are represented by fuzzy sets in the interval $[0, 1]$.

According to Zadeh's rule for truth qualification [22], a proposition such as "Lucia is very young" is considered as being *semantically equivalent* with the proposition "Lucia is young is very true". This semantic equivalence relation plays an important role in approximate reasoning. In fuzzy set based approaches to fuzzy reasoning [7,21,22,2], the primary linguistic truth-values such as *true* and $false$ are correspondingly assigned to fuzzy sets defined over the interval $[0, 1]$, which are designed to interpret the meaning of these primary terms. The composite linguistic truth-values are then computed by using the following procedure:

- Linguistic hedges [1], for example *very* and *more or less*, are defined as unary operators on fuzzy sets, for example CON, DIL, respectively;
- The logic connectives such as *and*, *or*, *not* and $if \ldots then$ are defined generally as operators such as *t-norm*, *t-conorm*, *negation*, and *implication* respectively.

As is well known, one of inherent problems in a model of fuzzy reasoning is that of linguistic approximation, i.e., how to name by a linguistic term a resulted fuzzy set of the deduction process. This depends on the shape of the resulted fuzzy set in relation with the primary fuzzy sets and the operators.

Based on two characteristics of linguistic variables introduced by Zadeh (namely, the context-independent meaning of linguistic hedges and connectives, and the universality of linguistic domains), and the meaning of linguistic hedges in natural language, Nguyen and Wechler [14,15] proposed an algebraic approach to the structure of linguistic domains (term-sets) of linguistic variables. It is shown in [12,13] that the obtained structute is rich enough for the investigation of some kinds of fuzzy logic. Furthermore, the approach also provides a possibility for introducing methods of linguistic reasoning that allow us to handle linguistic terms directly, and hence, to avoid the problem of linguistic

[1] also called linguistic modifiers [6].

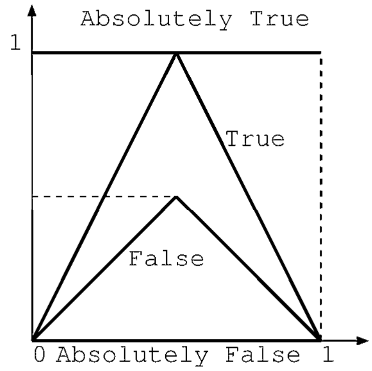

Fig. 1. Membership functions of truth-values by Lascio's model

approximation [10,11].

It is of interest that in [6], Lascio et al. have proposed a model for representation of linguistic terms satisfying the hypotheses imposed on linguistic hedges introduced by Nguyen and Wechler [14]. In their model, each linguistic term of a linguistic variable is characterised by three parameters and can be identified by only a positive real number. It is shown that the set of liguistic terms of the linguistic truth variable in Lascio's model exhibited interesting semantic properties justified by intuitive meaning of linguistic hedges, which were axiomatically formulated in the terms of hedge algebras [14]. However, going back to the membership function representation, Lascio's model does not give a good interpretation at the intuitive level on logical basis behind the shape of membership functions of linguistic truth values (see Fig. 1).

It is important to note that in the conventional approach to fuzzy reasoning, fuzzy logic, which a method of fuzzy reasoning bases on, can be viewed as a fuzzy extension of a underlying multi-valued logic (i.e., *base logic*), in which the truth-values are fuzzy sets of the truth-value set of the base logic (see, e.g., [2,21,22]). Although membership functions of primary terms such as $true$ or $false$ are defined subjectively, it will be natural to hope that a fuzzy logic should meet the base logic at the limited cases. For example, for membership function of the **unitary** truth-value *u-true* [22], that is $\mu_{u\text{-}true}(v) = v$ for $v \in [0,1]$, and the linguistic hedge *very* defined by CON operation, we have $very^n true$ tends to $Absolutely\ true$ as n tends to infinity, where $Absolutely\ true$ is identified with 1 as a nonfuzzy truth-value, see Fig. 2. Unfortunately, this is not the case for Lascio's model, again see Fig. 1.

In this paper, we introduce a new representation model for linguistic terms of the linguistic truth variable in fuzzy logic. In this model, each linguistic truth value generated from a primary term of the linguistic truth variable is identified by a real number r depending on the primary term. It will be shown that the model not only satisfies the interesting semantic properties justified by intuitively meaning of linguistic hedges as Lascio's model, but also meets