The proof of correctness of template 3

Yuki Chiba    Takahito Aoto    Yoshihito Toyama

Let \( \langle \mathcal{P}, \mathcal{P}', \mathcal{H} \rangle \) be a template where

\[
\mathcal{P} = \left\{ \begin{array}{l}
    f(a) \rightarrow b \\
    f(c(u, v)) \rightarrow g(e(u, v), f(v))
\end{array} \right.
\]

\[
\mathcal{P}' = \left\{ \begin{array}{l}
    f(u) \rightarrow f_1(u, b) \\
    f_1(a, u) \rightarrow u \\
    f_1(c(u, v), w) \rightarrow f_1(v, g(w, e(u, v)))
\end{array} \right.
\]

\[
\mathcal{H} = \left\{ \begin{array}{l}
    g(b, u) \rightarrow u \\
    g(u, b) \rightarrow u \\
    g(g(u, v), w) \rightarrow g(u, g(v, w))
\end{array} \right.
\]

We now show that \( \langle \mathcal{P}, \mathcal{P}', \mathcal{H} \rangle \) is a correct template.

1. Let \( \mathcal{P}_0 = \mathcal{P} \).

2. Let \( \mathcal{P}_1 = \mathcal{P}_0 \cup \{ f_1(u, v) \rightarrow g(v, f(u)) \} \). Here, \( f_1 \) is a fresh pattern variable.
   Thus, \( \mathcal{P}_0 \Rightarrow \mathcal{P}_1 \) by the introduction rule.

3. Let \( \mathcal{P}_2 = \mathcal{P}_1 \cup \{ f(u) \rightarrow f_1(u, b) \} \). Here, we have
   \[
   f(u) \leftrightarrow_{\mathcal{H}} g(b, f(u)) \leftrightarrow_{\mathcal{P}_1} f_1(u, b).
   \]
   Thus, we have \( \mathcal{P}_1 \Rightarrow \mathcal{P}_2 \) by the Addition rule.

4. Let \( \mathcal{P}_3 = \mathcal{P}_2 \cup \{ f_1(a, v) \rightarrow v \} \). Here, we have
   \[
   f_1(a, v) \leftrightarrow_{\mathcal{P}_2} g(v, f(a)) \leftrightarrow_{\mathcal{H}} v.
   \]
   Thus, we have \( \mathcal{P}_2 \Rightarrow \mathcal{P}_3 \) by the Addition rule.

5. Let \( \mathcal{P}_4 = \mathcal{P}_3 \cup \{ f_1(c(u, v), w) \rightarrow f_1(v, g(w, e(u, v))) \} \). Here, we have
   \[
   f_1(c(u, v), w) \leftrightarrow_{\mathcal{P}_3} g(w, f(c(u, v))) \leftrightarrow_{\mathcal{P}_3} g(w, g(e(u, v), f(v))) \leftrightarrow_{\mathcal{H}} g(g(w, e(u, v)), f(v)) \leftrightarrow_{\mathcal{P}_3} f_1(v, g(w, e(u, v))).
   \]
   Thus, we have \( \mathcal{P}_3 \Rightarrow \mathcal{P}_4 \) by the Addition rule.

6. Finally, applying the Elimination rules three times to \( \mathcal{P}_4 \), we obtain \( \mathcal{P}' \).
   Thus, \( \langle \mathcal{P}, \mathcal{P}', \mathcal{H} \rangle \) is a correct template.