Precoders for Message-Passing Detection of Partial-Response Channels

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Abstract — **Parallel message-passing detectors for partial-response channels have the property that a bit is estimated using channel symbols in a window of size W centered upon that bit. Distinct input sequences that produce the same output sequence result in undesirable failure of window decoders, but precoding can eliminate this input-to-output mapping ambiguity. For a class of partial-response channels, we show necessary and sufficient conditions on a precoder to unambiguously map input sequences to output sequences.**

I. Introduction

It is well-known that the BCJR algorithm is an instance of the sum-product algorithm [1], and it can be described by applying a serial schedule to a message-passing graph. When a parallel schedule is applied to such a graph, a new algorithm results, and it has been considered for detection on partial-response channels [2]. This algorithm estimates a bit using channel symbols in a window of size W centered upon that bit. It can distinguish some, but not all of the possible transmitted sequences, even in the absence of noise.

We will represent a sequence of symbols $\ldots, a_{i-1}, a_i, a_{i+1}, \ldots$ using delay operator notation $a(D)$; also, we will write a_1^W to mean $a_1, \ldots, a_W$. We consider the system shown in Fig. 1. The precoder input is the binary sequence $x(D)$. The partial-response channel has input $a(D)$ and output $y(D)$. In this paper, we are considering partial-response channel polynomials of the form $h(D) = (1-D)^m(1+D)^n$, for integers $m, n \geq 0$, m and n not both zero. The window detector matched to the channel produces as its estimate $\hat{a}(D)$, and the output of the postcoder with transfer function $f(D)$ is $\hat{x}(D)$.

Definition Let $a(D)$ and $a'(D)$ be two distinct partial-response channel input sequences (or preimages), and let $y(D) = h(D)a(D)$ and $y'(D) = h(D)a'(D)$. If $y_i = {y'}_i$ for $i = 1, \ldots, W$, then y_1^W is an *ambiguous output sequence.*

II. Precoders for Message-Passing Detectors

Ambiguous output sequences will lead to failure of the window algorithm because the detector must choose between the distinct input sequences a_1^W and ${a'}_1^W$ with equal probability; choosing the wrong input sequence generally results in a large number of bit errors. In systems with additive white Gaussian noise operating at high signal-to-noise ratio, ambiguous output sequences are the dominant source of bit errors, particularly when the window size W is small. Thus, we are motivated to eliminate errors due to ambiguous output sequences.

The solution is to introduce a precoder, which maps binary user data $x(D)$ to the channel preimage $a(D)$. Lemma 1 gives necessary and sufficient conditions for a precoder to eliminate the problem of ambiguous output sequences.

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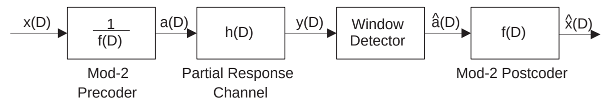

Fig. 1: Block diagram of the system under consideration.

Lemma 1 *Consider a binary input partial-response channel with transfer function $h(D) = (1-D)^m(1+D)^n$ (integers $m, n \geq 0$), preceded by a binary input, binary output precoder with transfer function $1/f(D)$. An ambiguous output sequence of length W greater than or equal to some constant $W^*_{m,n}$ corresponds to a unique precoder input sequence if and only if:*

$$\begin{aligned} (1 \oplus D) \,|\, f(D) \bmod 2 \quad &\text{for} \quad (m = 0 \text{ and } n > 0) \text{ or} \\ & \qquad\; (m > 0 \text{ and } n = 0) \\ (1 \oplus D^2) \,|\, f(D) \bmod 2 \quad &\text{for} \quad (m > 0 \text{ and } n > 0). \end{aligned}$$

When $W \geq W^*_{m,n}$, ambiguous output sequences correspond exactly to null error sequences, which are bi-infinite sequences $\varepsilon_a(D) = a_\infty(D) - {a'}_\infty(D)$ such that $||\varepsilon_a(D)h(D)|| = 0$ [3]. Let $x'(D)$ be the preimage of $a'(D)$. In the proof of Lemma 1, we found restrictions on the coefficients of $f(D)$ such that $x_1^W = {x'}_1^W$, when $a_1^W - {a'}_1^W$ is a substring of a null error sequence.

The following values of $W^*_{m,n}$ were found by computer search: $W^*_{1,0} = 1$ (dicode), $W^*_{1,1} = 1$ (PR4), $W^*_{1,2} = W^*_{1,3} = 4$ (EPR4, E^2PR4).

We note that precoders have been used in the past to avoid the effects of quasi-catastrophic error propagation in partial-response systems [4]. This use of precoders is similar to, but distinct from the use discussed in this paper.

References

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