

# Modeling Urgency in Component-Based Real-time Systems

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**Abstract.** A component-based realtime system is a simple model for the server-client relation with time constraints. This paper presents an efficient algorithm, called a *blackbox testing algorithm*, for detecting the emptiness of a component-based realtime system. This algorithm was originally proposed in [5], but with a certain flaw. We improve it and correct the flaw by using urgency [2] of transitions.

**Keywords:** Component Software, Duration Automata, Automatic Verification, Real-time Systems, Model Checking.

## 1 Introduction

The architectural design for embedded systems often relies on specification of the interface of components only, without accessing their internal behaviors. Based on this observation, a simple model for component-based real-time systems based on duration automata was proposed in [5]. A duration automaton does not have clock variables like a time automaton [1], but simply has an upper bound and a lower bound for each transition. A component-based real-time system is defined as a system consisting of a host, which is a general duration automaton, and several components which are duration automata with certain restrictions. A component-based real-time system can be regarded as a timed automaton, thus its emptiness is PSPACE-complete.

This paper presents an efficient algorithm for detecting the emptiness, called a *blackbox testing algorithm*. This algorithm was originally proposed in [5], but with certain flaws. We improve it and correct these flaws by using urgency of transitions, which was firstly introduced by Bornot et. al. [2] as a technique for choosing time deadline condition in complex system specifications.

## 2 Duration Automata

Duration automata was firstly introduced in [3] for modeling simple real-time systems. A duration automaton is a finite automaton in which each transition must occur in an associated time interval. Let  $\mathbb{R}^+$  be the set of non-negative real numbers, and let  $Intv = \{[l, u] \mid l \in \mathbb{R}^+, u \in \mathbb{R}^+ \cup \{\infty\}\}$ .

**Definition 1.** A duration automaton is a tuple  $M = \langle S, \tilde{\Sigma}, q, R, F \rangle$ , where

1.  $S$  is a finite set of states,
2.  $\tilde{\Sigma}$  is alphabet of actions,
3.  $q \in S$  is the initial state,
4.  $R \subseteq S \times \tilde{\Sigma} \times \text{Intv} \times S$  is timed transition relation, and
5.  $F \subseteq S$  is the set of final states.

Each element of  $M$  is referred by  $S(M)$ ,  $\tilde{\Sigma}(M)$ ,  $R(M)$ ,  $q(M)$ , and  $F(M)$ , respectively. An untimed automaton  $\text{untimed}(M)$  is obtained by forgetting time constraints, i.e., replacing  $R$  with  $\text{untimed}(R) = \{(s, a, s') \mid (s, a, [l, u], s') \in R\}$ . As in standard terminology,

- A *configuration* of  $M$  is a pair  $(s, d) \in S \times \mathbb{R}^+$ .
- The *initial configuration* of  $M$  is  $(q, 0)$ .
- An *acceptance configuration* of  $M$  is a configuration  $(s, d)$  where  $s \in F$ .

A duration automaton is equivalent to a timed automata with a single clock such that each transition resets it. A configuration  $(s, d)$  is regarded as a state  $s$  with a clock  $d$ .

- A transition of  $M$  on configurations is either a *time transition*  $(s, d) \xrightarrow{\delta} (s, d + \delta)$  or a *discrete transition*  $(s, d) \xrightarrow{\delta, a} (s', 0)$  where  $a \in \tilde{\Sigma}$ ,  $\delta \geq 0$ ,  $l \leq d + \delta \leq u$ , and  $(s, a, [l, u], s') \in R$ .
- A (possibly empty) sequence  $w = (a_1, t_1) \dots (a_k, t_k) \in (\tilde{\Sigma} \times \mathbb{R}^+)^*$  is a *timed word* of  $M$  if and only if there is a run  $(s_0, 0) \xrightarrow{\delta_1, a_1} (s_1, 0) \xrightarrow{\delta_2, a_2} \dots \xrightarrow{\delta_k, a_k} (s_k, 0)$  such that  $s_0 = q$ ,  $s_k \in F$ ,  $t_1 = \delta_1$  and  $t_{i+1} - t_i = \delta_{i+1}$  for  $1 \leq i \leq k-1$ .

**Theorem 1.** Duration automata is closed under union, intersection and complementation. Decision problems for duration automata are decidable.

*Proof.* (Sketch) For a given duration automaton  $M$ , one can reduce  $M$  to a finite automaton  $M'$ . We first list the endpoints of intervals (lower and upper bounds of intervals) of transitions in  $M$  as an increasing sequence, say,  $0 = p_0 < p_1 < p_2 \dots < p_n < \infty$ . This is possible because the number of transitions of  $M$  is finite. Secondly, we define the set of basic intervals  $BI = \{[p_0, p_1], \dots, [p_{n-1}, p_n], [p_n, \infty)\}$ . Since each interval appeared in a transition of  $M$  is the union of certain basic intervals. So, each transition of  $M$  can be divided into several ones. For instance,  $(s, a, [p_0, p_2], s')$  can be divided into  $(s, a, [p_0, p_1], s')$  and  $(s, a, [p_1, p_2], s')$ . We now construct a finite automaton  $M'$  such that  $S(M') = S(M)$ ,  $F(M') = F(M)$ , the input alphabet of  $M'$  is  $\tilde{\Sigma}(M') = \tilde{\Sigma}(M) \times BI$ . Let  $(s, (a, [p_i, p_{i+1}]), s') \in R(M')$  if  $(s, a, [p_i, p_{i+1}], s') \in R(M)$ . Clearly,  $M'$  accepts a word  $(a_1, [l_1, u_1]) \dots (a_n, [l_n, u_n])$  if and only if  $M$  accepts the timed word  $(a_1, t_1) \dots (a_n, t_n)$ , where  $t_0 = 0$  and  $(l_i \leq t_i - t_{i-1} \leq u_i)$  for  $1 \leq i \leq n$ . Thus, the emptiness and the closure properties of duration automata are reduced to that of finite automata, respectively.  $\square$

### 3 Synchronized Composition Systems

Duration interface automata is duration automata in which the input alphabet  $\tilde{\Sigma}$  is decomposed into pairwise disjoint alphabets  $\Sigma, \Delta$  and  $\nabla$ , which correspond to internal, input and output actions, respectively.

**Definition 2.** A host is a duration interface automaton. A component is a duration interface automaton  $X = \langle S, \Sigma \cup \Delta \cup \nabla, q, R, F \rangle$  that satisfies:

- $\Sigma = \emptyset$  (i.e., no “explicit” internal actions).
- $(s, a, [l, u], s') \in R \wedge a \in \Delta$  implies  $l = 0 \wedge u = \infty$  (i.e., an input can occur anytime).
- $(s, a, [l, u], s') \in R \wedge a \in \nabla$  implies  $u = \infty$  (i.e., when an output is ready, it can be sent at any time afterward).

**Definition 3.** A synchronized composition system  $Sys = \langle M, X_1, \dots, X_k \rangle$  consists of a single host  $M$  and components  $X_1, \dots, X_k$  such that  $\tilde{\Sigma}(X_i) \cap \tilde{\Sigma}(X_j) = \emptyset$  for each  $i \neq j$ ,  $\Sigma(M) \cap \tilde{\Sigma}(X_i) = \emptyset$  for each  $i$ ,  $\Delta(M) = \bigcup_{i=1}^k \nabla(X_k)$ ,  $\nabla(M) = \bigcup_{i=1}^k \Delta(X_k)$ , and

- The set of configurations is  $\{(s_0, d_0), (s_1, d_1), \dots, (s_k, d_k) \mid s_0 \in S(M), s_1 \in S(X_1), \dots, s_k \in S(X_k), d_i \in \mathbb{R}^+\}$ .
- A transition is  $((s_0, d_0), (s_1, d_1), \dots, (s_k, d_k)) \xrightarrow{\delta, a} ((s'_0, d'_0), (s'_1, d'_1), \dots, (s'_k, d'_k))$  for  $\delta \geq 0$  and  $a \in \bigcup_{i=1}^k \tilde{\Sigma}(X_i)$ , if there exists  $i$  with  $1 \leq i \leq k$  such that
  - $a \in \tilde{\Sigma}(X_i)$ ,
  - $l_0 \leq d_0 + \delta \leq u_0$  and  $l_i \leq d_i + \delta \leq u_i$  (called synchronization condition) for  $(s_i, a, [l_i, u_i], s'_i) \in R(X_i)$  and  $(s_0, a, [l_0, u_0], s'_0) \in R(M)$ ,
  - $d'_0 = d'_i = 0$ , and
  - $(s'_j, d'_j) = (s_j, d_j + \delta)$  for  $j \neq 0, i$ .
- A run is a sequence of transitions that starts from the initial configuration  $((q(M), 0), (q(X_1), 0), \dots, (q(x_k), 0))$ .
- A timed word  $(a_1, t_1) \dots (a_k, t_k)$  with  $t_1 = \delta_1$  and  $t_{i+1} = t_i + \delta_{i+1}$  is accepted if there is a run  $((q(M), 0), (q(X_1), 0), \dots, (q(x_k), 0)) \xrightarrow{\delta_1, a_1} \dots \xrightarrow{\delta_k, a_k} ((s_0, d_0), (s_1, d_1), \dots, (s_k, d_k))$  with  $s_0 \in F(M), s_1 \in F(X_1), \dots, s_k \in F(X_k)$ .

**Theorem 2.** A synchronized composition system  $Sys = \langle M, X_1, \dots, X_k \rangle$  is a timed automaton with  $k+1$  clocks such that each transition with a time constraint  $l_i \leq d_i \leq u_i$  on a clock  $d_i$  will reset  $d_i$  to 0.

*Proof.* (Sketch) Let  $\mathcal{C}$  be the set of time constraints  $[l_j, u_j]$  appearing in a host  $M$  and components  $X_i$ . Note that  $l_j, u_j \in \mathbb{R}^+$ . Assume that we can choose  $\mathcal{C}'$  (a digitization of  $\mathcal{C}$ ) consisting of rational time constraints  $[l'_j, u'_j]$  such that there is a run of  $Sys$  if and only if there is a run of  $Sys'$ , where  $Sys'$  is obtained replacing each  $[l_j, u_j]$  with its digitization  $[l'_j, u'_j]$ . Then, the proof has done.

Let  $rat(\mathcal{C})$  be the set of rational numbers appearing in  $\mathcal{C}$  and let  $m$  be a common multiplier of denominators of positive elements in  $rat(\mathcal{C})$ . Let  $irr(\mathcal{C})$  be the set of irrational numbers appearing in  $\mathcal{C}$  and let  $lin(\mathcal{C})$  be the set of all possible

linear combinations of  $irr(\mathcal{C})$  with natural numbers (i.e.,  $lin(\mathcal{C}) = \{n_1\alpha_1 + \dots + n_l\alpha_l \mid n_j \in \mathbb{N}, \alpha_j \in irr(\mathcal{C})\}$ ). Assume that  $(\alpha, \beta)$  is the pair such that  $\alpha \in irr(\mathcal{C})$ ,  $\beta \in lin(\mathcal{C})$ , and  $\epsilon_{\alpha, \beta} = \frac{\alpha}{\beta} - \lfloor \frac{\alpha}{\beta} \rfloor > 0$ . Since a pair  $(\alpha, \beta)$  with  $\alpha \in irr(\mathcal{C})$ ,  $\beta \in lin(\mathcal{C})$ , and  $\beta < \alpha$  is finitely many,  $(\alpha, \beta)$  with  $\epsilon_{\alpha, \beta}$  to be the least exists. We choose a sufficient large multiplier  $\bar{m}$  of  $m$  such that  $\frac{1}{\bar{m}} < \min(\frac{\epsilon_{\alpha, \beta}}{2}, \frac{1 - \epsilon_{\alpha, \beta}}{2})$ , and set  $l'_j = \frac{\lfloor \bar{m}l_j \rfloor}{\bar{m}}$  and  $u'_j = \frac{\lfloor \bar{m}u_j \rfloor}{\bar{m}}$  for each  $l_j, u_j \in \mathbb{C}$ .  $\square$

*Example 1.* Fig. 3 shows a simple synchronized composition system  $Sys = \langle X_1, X_2 \rangle$  and its corresponding timed automaton  $\mathcal{A}$ .

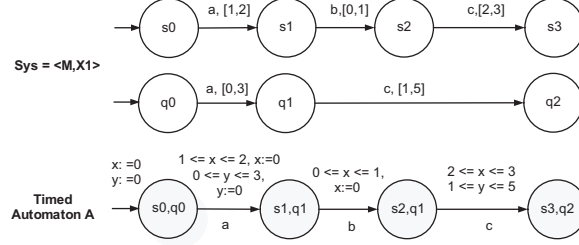


Fig. 1. Synchronized Composition System as a Timed Automaton

From Theorem 2, the emptiness problem of a component-based realtime system is decidable. However, its complexity is expensive, i.e., PSPACE-complete [1] after digitization of time constraints.

## 4 Component-based realtime systems

**Definition 4.** A component  $X$  is input/output deterministic if

- for  $a \in \Delta(X)$ ,  $(s, a, [0, \infty), s')$ ,  $(s, a, [0, \infty), s'') \in R(X)$  implies  $s' = s''$  (input determinism), and
- for  $b \in \nabla(X)$  and  $b' \in \nabla(X) \cup \Delta(X)$ ,  $(s, b, [l, \infty), s')$ ,  $(s, b', [l', u'], s'') \in R(X)$  implies  $s' = s''$ ,  $l' = l$ ,  $u' = \infty$ , and  $b' = b$  (output determinism).

A synchronized composition system  $Sys = \langle M, X_1, \dots, X_k \rangle$  is a component-based realtime system [5] if each component  $X_i$  is input/output deterministic.

**Definition 5.** We borrow notations from Definition 3. In a component-based system  $Sys = \langle M, X_1, \dots, X_k \rangle$ , a transition  $((s_0, d_0), (s_1, d_1), \dots, (s_k, d_k)) \xrightarrow{\delta} \xrightarrow{a} ((s'_0, d'_0), (s'_1, d'_1), \dots, (s'_k, d'_k))$  is urgent if  $\delta$  is the minimum among synchronization conditions of all possible transitions from  $((s_0, d_0), (s_1, d_1), \dots, (s_k, d_k))$ , and delayable otherwise. We also say a corresponding transition  $(s_0, a, [l_0, u_0], s'_0) \in R(M)$  of a host is urgent, and delayable otherwise.

**Definition 6.** Let  $w = (a_1, t_1) \cdots (a_k, t_k)$  and let  $a_i \in A$ . For  $B \subseteq A$ , the projection  $w|_B$  is the subsequence of  $w$  obtained by filtering each element  $(a_j, t_j)$  with  $a_j \in B$ . For  $a_j \in B$ ,  $(a_h, t_h)$  is a local predecessor of  $(a_j, t_j)$  wrt  $B$ , if  $a_h \in B$ ,  $h < j$ , and  $a_i \notin B$  for each  $i$  with  $h < i < j$ .

**Definition 7.** Let  $Sys = \langle M, X_1, \dots, X_k \rangle$  be a component-based real-time system. For a timed word  $w = (a_1, t_1) \dots (a_n, t_n)$ , let  $a_j \in \nabla(X_i)$  and let  $(a_h, t_h)$  be the local predecessor of  $(a_j, t_j)$  wrt  $\tilde{\Sigma}(X_i)$ . For  $(s', a_j, [d_j, \infty), s'') \in R(X_i)$  with  $q(X_i) \xrightarrow{\text{untime}(w|_{\tilde{\Sigma}(X_i)})} s'$  in  $\text{untimed}(X_i)$ ,  $d_j$  is the minimum delay at  $(a_j, t_j)$ .

**Definition 8.** A consecutive sequence of transitions  $(s_{i-1}, a_i, [l_i, u_i], s_i) \in R(M)$  ( $i = 1, \dots, n$ ) is called an accepted sequence of transitions of the host  $M$  if  $s_0 = q(M)$  and  $s_n \in F(M)$ .

Note that such a minimum delay is well-defined, since each component in  $Sys$  is input/output deterministic. Let  $r$  be the number of states of  $M$ , and let  $m$  is the maximal number of states of components  $X_j, j \leq k$ . Let  $P$  be the length of the longest path (number of transitions) from the initial state to a final state of  $M$  in which any cycle is not repeated more than  $r * m^k$  times. The next theorem reduces the emptiness of a whole component-base realtime system to that of its host under certain conditions.

**Theorem 3.** Let  $Sys = \langle M, X_1, \dots, X_k \rangle$  be a component-based realtime system. There is an accepted timed word of  $Sys$  if and only if there are an accepted sequence of transitions of the host  $M$   $\sigma = (s_0, a_1, [l_1, u_1], s_1)(s_1, a_2, [l_2, u_2], s_3) \dots (s_{n-1}, a_n, [l_n, u_n], s_n)$  with the length  $n \leq P$ , and a real number sequence  $0 = t_0 \leq t_1 \leq \dots \leq t_n$  satisfying following conditions:

- $w_i = a_1 a_2 \dots a_n |_{\tilde{\Sigma}(X_i)}$  is accepted by  $\text{untimed}(X_i)$  for each  $i$  with  $1 \leq i \leq k$ ,
- $l_i \leq t_i - t_{i-1} \leq u_i$  for all  $i$  with  $1 \leq i \leq n$ ,
- When  $a_j \in \nabla(X_i)$ , let  $(a_h, t_h)$  be the local predecessor of  $(a_j, t_j)$  wrt  $\tilde{\Sigma}(X_i)$  and let  $d_j$  be the minimum delay at  $(a_j, t_j)$ . Then,
  - $t_j - t_h \geq d_j$ , and
  - if a transition  $(s_j, 0) \xrightarrow{\delta} \xrightarrow{a_j} (s_{j+1}, 0)$  is urgent,  $t_j = \min \{t \mid t - t_h \geq d_j \wedge l_j \leq t - t_{j-1} \leq u_j\}$ .

*Proof.* (Sketch) We only have to prove the bound  $P$  in “only if” part. Assume that a timed word  $w = (a_1, t_1) \cdots (a_n, t_n)$  is accepted by  $Sys$ . Timed word  $w$  is inductively computed by constructing an accepted sequence of transitions  $\phi = (s_0, a_1, [l_1, u_1], s_1)(s_1, a_2, [l_2, u_2], s_3) \cdots (s_{n-1}, a_n, [l_n, u_n], s_n)$  of the host  $M$ . If  $n \leq P$ , the proof is done. If  $n > P$ , then  $\phi$  must include at least a cycle  $c$  with more than  $r * m^k$  repetitions. By the pumping lemma like argument, we can find a shorter accepted sequence of transitions of  $M$  that satisfies all the conditions in the Theorem.  $\square$

In the next section, the blackbox testing algorithm will be presented by searching an accepted sequence of the host  $M$  satisfying the conditions in Theorem 3 up to the length  $P$ .

## 5 Checking Emptiness of Component-based Realtime Systems

The emptiness problem for a system plays a key role in checking the safety. An algorithm for checking the emptiness of a component-based system using black box testing was originally proposed in [5]. However, there is a flaw such that a component-based realtime system is empty, whereas the algorithm in [5] reports that the system is not empty. For instance, consider the following simple example.

*Example 2.* Let  $Sys = \langle M, X \rangle$  where  $M$  is a host and  $X$  is a component.

- $M = \langle \{s_0, s_1, s'_1\}, \{a\}, s_0, \{(s_0, a, [2, 4], s_1), (s_0, a, [5, 10], s'_1)\}, \{s'_1\} \rangle$ .
- $X = \langle \{q_0, q_1\}, \{a\}, q_0, \{(q_0, a, [3, \infty), q_1)\}, \{q_1\} \rangle$ .

In [5], the state  $(s'_1, q_1)$  is regarded as a successor of  $(s_0, q_0)$ . But,  $(s'_1, q_1)$  is not reachable from  $(s_0, q_0)$ . This is due to the fact that  $Sys$  has already changed from  $(s_0, q_0)$  to  $(s_1, q_1)$  at some point in the time interval  $[3, 4]$ .

To deal with this problem, we introduce urgency for transitions to specify time deadline condition of configurations. For the emptiness problem, we first use the **BlackboxTest** algorithm proposed in [5] for solving membership for a component. Secondly, we construct Algorithm 1 to compute time deadline condition of a given configuration. Lastly, with the aid of Algorithm 1 and Theorem 3, we construct Algorithm 2 to check the emptiness of a component-based system using black box testing.

For a sequence of transitions  $\phi$ , let  $label(\phi)$  denote the sequence of the labels corresponding to  $\phi$ . For a given prefix of a generated sequence of transitions  $\sigma = e_1 e_2 \dots e_n$ , where  $e_i = (s_{i-1}, a_i, [l_i, u_i], s_i) \in R(M)$  ( $i = 1..n$ ). Suppose that  $t_0, t_1, \dots, t_n$  are inductively computed in advance. Time deadline of  $s_n$  along  $\sigma$  is denoted by  $deadline_\sigma(s_n)$ . It can be computed by the following algorithm:

ALGORITHM 1. **Deadline** $_\sigma(s_n)$ : (Check the conditions (2) and (3) of Theorem 3)

**Input:** A prefix-generated sequence  $\sigma = e_1 e_2 \dots e_n$

**Output:**  $deadline_\sigma(s_n)$ .

**Method:**

1. Compute the set  $R(s_n) := \{e \mid e = (s_n, a, [l, u], s) \in R(M)\}$ .
2.  $deadline := \infty$ . For  $j \leq k$  let  $m_j$  be the largest index of  $\sigma$  such that  $a_{m_j} \in \tilde{\Sigma}(X_j)$  if it exists, otherwise, set  $m_j = 0$ . For each  $e = (s_n, a, [l, u], s) \in R(s_n)$ .
  - (a) If  $a \in \Delta(X_j) \cup \Sigma(M)$ , if **BlackboxTest** $(X_j, label(\sigma)|_{\tilde{\Sigma}(X_j)}) = \text{“yes”}$  and  $u < deadline$  then  $deadline := u$ .
  - (b) If  $a \in \nabla(X_j)$ . If **BlackboxTest** $(X_j, label(\sigma)|_{\tilde{\Sigma}(X_j)}) = \text{“yes”}$ , let  $d$  be the value of  $d_{X_j}$ .

- **Case 1:  $e$  is delayable.** If  $t_n - t_{m_j} + u \geq d$  and  $u < \text{deadline}$  then  $\text{deadline} := u$ .
  - **Case 2:  $e$  is urgent.**
    - If  $t_n - t_{m_j} + l \leq d \leq t_n - t_{m_j} + u$  and  $d - (t_n - t_{m_j}) < \text{deadline}$  then  $\text{deadline} := d - (t_n - t_{m_j})$ .
    - If  $t_n - t_{m_j} + l \geq d$  and  $l < \text{deadline}$  then  $\text{deadline} := l$ .
3. **return** Delainey;

With the aid of the Algorithm 1, the emptiness of a component-based real-time system can be solved by the following testing procedure.

**ALGORITHM 2. Non-Emptiness(Sys):** (Check all conditions of Theorem 3)

**Input:** Component-based real-time system  $Sys = \langle M, X_1, \dots, X_k \rangle$

**Output:** “Yes” if the set of timed words of  $Sys$  is not empty, “No” otherwise.

**Method:**

1. Compute  $P$ . Generate all accepted sequences of transitions of  $M$  with length less than  $P$ .
2. Check on-the-fly whether any prefix of a generated sequence satisfies the conditions of Theorem 3. This can be done by:

For each prefix of a generated sequence of transitions  $\sigma = e_1 \dots e_{n-1}$ , where  $e_i = (s_{i-1}, a_i, [l_i, u_i], s_i)$  for each  $i$  with  $1 \leq i \leq n-1$ . Suppose that  $t_0, t_1, \dots, t_n$  are inductively computed in advance. For  $j \leq k$  let  $m_j$  be the largest index of  $\sigma$  such that  $a_{m_j} \in \tilde{\Sigma}(X_j)$  if it exists, otherwise, let  $m_j = 0$ . For each transition  $e_n = (s_{n-1}, a_n, [l, u], s)$  of the host  $M$  starting from  $s_{n-1}$ . Compute  $\text{deadline}_\sigma(s_{n-1})$  using Algorithm 1. If  $l \leq \text{deadline}_\sigma(s_{n-1})$  then:

- (a) If  $a_n \in \Delta(X_j)$ , then if: **BlackboxTest** $(X_j, \text{label}(\sigma)|_{\tilde{\Sigma}(X_j)}) = \text{“no”}$ ,  $\sigma e_n$  does not satisfy the conditions of Theorem 3. Otherwise,  $\sigma := \sigma e_n$ ,  $m_j := n$ . If  $e_n$  is delayable then  $t_n := t_{n-1} + u$ . If  $e_n$  is urgent then  $t_n := t_{n-1} + l$ .
- (b) If  $a_n \in \nabla(X_j)$ .

If **BlackboxTest** $(X_j, \text{label}(\sigma)|_{\tilde{\Sigma}(X_j)}) = \text{“yes”}$ , let  $d$  be the value of  $d_{X_j}$ .

**Case 1: If  $e_n$  is delayable**

- i. If  $t_{n-1} - t_{m_j} + u < d$ : then  $\sigma e_n$  does not satisfy the conditions of Theorem 3.
- ii. If  $t_{n-1} - t_{m_j} + u \geq d$ : then  $\sigma := \sigma e_n$ ,  $m_j := n$ ,  $t_n := t_{n-1} + u$ .

**Case 2: If  $e_n$  is urgent**

- i. If  $t_{n-1} - t_{m_j} + u < d$  then  $\sigma e_n$  does not satisfy the conditions of Theorem 3.
- ii. If  $(t_{n-1} - t_{m_j} + l) < d \leq (t_{n-1} - t_{m_j} + u)$  then the conditions of Theorem 3 are satisfied; update  $\sigma := \sigma e_n$ ,  $t_n := t_{m_j} + d$ ,  $m_j := n$ .
- iii. If  $t_{n-1} - t_{m_j} + l \geq d$  then update  $\sigma := \sigma e_n$ ,  $t_n := t_{n-1} + l$ ,  $m_j := n$ .

If **BlackboxTest** $(X_j, \text{label}(\sigma)|_{\tilde{\Sigma}(X_j)}) = \text{“no”}$ , the conditions of Theorem 3 are not satisfied.

- (c) If  $a_n \in \Sigma(M)$  then  $\sigma := \sigma e_n$ ,  $t_n := t_{n-1} + u$ .

3. If a generated sequence satisfying the conditions of Theorem 3 is found, return “Yes”. Otherwise, return “No”.

The complexity for the worst cases of this algorithm is  $O(P^2 * K^{P+1})$ , where  $K = |\tilde{\Sigma}(M)|$  is the size of the alphabet of the system  $Sys$ . Unlike the complexity of checking the emptiness for timed automata, this complexity does not depend on the size of the constants occurring in the time intervals for the transitions.

## 6 Conclusion

This paper presented an efficient algorithm for detecting the emptiness, called a *blackbox testing algorithm*. This algorithm was originally proposed in [5], but with a certain flaw. We improved and corrected it by using urgency of transitions, which was firstly introduced by Bornot et. al. [2] as a technique for choosing time deadline condition in complex system specifications. The urgency enables us to compute the deadline of an accepted behavior of a system using Algorithm 2.

Currently, the algorithm covers checking emptiness only. With the urgency, we can describe a property in Timed Computation Tree Logic (TCTL), such as  $\phi \implies F_{\leq t}\psi$ . The next step is to give an efficient checking algorithm for such TLCL properties of a component-based realtime system.

## Acknowledgments

This research is supported by the 21st Century COE “Verifiable and Evolvable e-Society” funded by Japanese Ministry of Education, Culture, Sports, Science and Technology.

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