Minimal Surface Area of Polyhedral Unfoldings

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An *unfolding* (development) of a convex polyhedron is a connected plane figure obtained by cutting the surface of the polyhedron and unfolding it. Then length and configuration of a minimum perimeter unfolding for each of the regular polyhedra were determined by Akiyama et al. [1]. In this talk, for a 4 dimensional convex polytope we study unfoldings (that is, connected 3-dimensional figures obtained by cutting the surface of the polytope and unfolding it in the 3-dimensional space) with minimal surface area, instead of minimal perimeter.

One of efficient differences between unfoldings in the plane and ones in the 3 dimensional space is that in the plane, minimal perimeter unfoldings of a convex polyhedron are always polygons, on the other hand, in the 3-dimensional space, minimal area surfaces of unfoldings of a 4-dimensional convex polytope are curved in general. So, we restrict to polyhedral unfoldings, and determine minimal surface area among such unfoldings.

For a special case of a 4 dimensional polytope there is a doubly covered polyhedron (that is, a degenerated 4 dimensional polytope consisting of two congruent copies of a polyhedron whose corresponding faces are identified). For a doubly covered regular tetrahedron V, the minimal area surface of unfoldings of V is derived from the soap-films on the frame of edges of V and it consists of six isosceles triangles with the center of gravity.

First, we discuss a doubly covered convex polyhedron such that the frame of the edges has soap-films consisting of plane figures like the doubly covered regular tetrahedron. We give infinitely many such examples and the process

how to construct them.

Second, we discuss the Kelvin's conjecture [3,4]: The minimal surface area partition of space into cells of equal volume is a tiling by 14-faced Archimedean polyhedra (truncated octahedra) with slightly curved faces, and we give a following result related to this topic.

Theorem [2]. *The 14-faced Archimedean polyhedron has the minimal surface area among polyhedral unfoldings of the doubly covered cuboid with relation* $\sqrt{2} : \sqrt{2} : 1$ for its edge lengths.

Finally, we give conjectures on minimal surface area of polyhedral unfoldings of a 4 dimensional simplex and a 4 dimensional cube.

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